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#### FINAL JEE–MAIN EXAMINATION – JANUARY, 2019 Held On Wednesday 09th JANUARY, 2019 TIME: 9:30 AM To 12:30 PM

1. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to :

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(1) $1.0 \times 10^{-5} \text{ T}$	(2) $1.5 \times 10^{-5} \text{ T}$
(3) $1.0 \times 10^{-7} \text{ T}$	(4) $1.0 \times 10^{-7} \text{ T}$
Ans. (1)	



 $\vec{B} = \frac{\mu_0 i}{4\pi} \theta \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] \hat{k}$   $r_1 = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$   $r_2 = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$   $\theta = \frac{\pi}{4}, \text{ i} = 10 \text{ A}$   $\Rightarrow \vec{B} = \frac{4\pi \times 10^{-7}}{16} \times 10 \left[ \frac{1}{3 \times 10^{-2}} - \frac{1}{5 \times 10^{-2}} \right] \hat{k}$   $\Rightarrow \left| \vec{B} \right| = \frac{\pi}{3} \times 10^{-5} \text{ T}$   $\approx 1 \times 10^{-5} \text{ T}$ 

**2.** A gas can be taken from A to B via two different processes ACB and ADB.



When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is : (1) 80 J (2) 20 J (3) 100 J (4) 40 J Ans. (4)

Sol.  $P \qquad C \qquad B \qquad A \qquad D \qquad V \qquad \Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB} \qquad \Rightarrow 60 \ J = 30 \ J + \Delta U_{ACB} \qquad \Rightarrow \Delta U_{ACB} = 30 \ J \qquad \Rightarrow \Delta U_{ACB} = \Delta U_{ACB} = 30 \ J \qquad \Rightarrow \Delta U_{ACB} = \Delta U_{ACB} = 30 \ J \qquad \Rightarrow \Delta U_{ACB} = \Delta U_{ACB} + \Delta W_{ADB} \qquad = 10 \ J + 30 \ J = 40 \ J$ 

- 3. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive xdirection. At a particular point in space and time,  $\vec{E} = 6.3\hat{j}V/m$ . The corresponding magnetic field  $\vec{B}$ , at that point will be:
  - (1)  $18.9 \times 10^{-8} \text{kT}$  (2)  $6.3 \times 10^{-8} \text{kT}$

(3) 
$$2.1 \times 10^{-8} \text{kT}$$
 (4)  $18.9 \times 10^{8} \text{kT}$ 

Ans. (3)

Sol. 
$$|\mathbf{B}| = \frac{|\mathbf{E}|}{C} = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8} \text{ T}$$
  
and  $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{C}}$   
 $\hat{\mathbf{j}} \times \hat{\mathbf{B}} = \hat{\mathbf{i}}$   
 $\hat{\mathbf{B}} = \hat{\mathbf{k}}$   
 $\vec{\mathbf{B}} = |\mathbf{B}| \hat{\mathbf{B}} = 2.1 \times 10^8 \hat{\mathbf{k}} \text{ T}$ 

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- Two coherent sources produce waves of 4. different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:
  - (1) 4 : 1(2) 25 : 9(3) 16:9(4) 5 : 3
- Ans. (2)

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 $\frac{I_{max}}{I_{min}} = 16$ Sol.

 $\Rightarrow \frac{A_{max}}{A_{min}} = 4$ 

$$\Rightarrow \frac{\mathbf{A}_1 + \mathbf{A}_2}{\mathbf{A}_1 - \mathbf{A}_2} = \frac{4}{1}$$

Using componendo & dividendo.

$$\frac{A_1}{A_2} = \frac{5}{3} \Longrightarrow \frac{I_1}{I_2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

5. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is  $\theta$ , then :



(1) 
$$\tan \theta = \frac{2}{\sqrt{3}}$$
  
(2)  $\tan \theta = \frac{1}{3}$   
(3)  $\tan \theta = \frac{1}{2}$ 

(4)  $\tan \theta = \frac{1}{2\sqrt{3}}$ 

Sol. mg Μ mg

> Let mass of one rod is m. Balancing torque about hinge point.  $mg (C_1P) = mg (C_2N)$

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2} \operatorname{mgL} \sin \theta = \frac{\operatorname{mgL}}{2} \cos \theta$$
$$\Rightarrow \tan \theta = \frac{1}{2}$$

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A mixture of 2 moles of helium gas (atomic mass 6. = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio

of their rms speeds 
$$\left[\frac{V_{rms}(helium)}{V_{rms}((argon)}\right]$$
, is close to

Ans. (4)

**Sol.** 
$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$$

7. When the switch S, in the circuit shown, is closed, then the value of current *i* will be :

$$\begin{array}{c} 20V i_1 \\ A \\ 2\Omega \\ \downarrow i \\ 4\Omega \\ \downarrow i \\ 4\Omega \\ B \\ \downarrow i \\ 4\Omega \\ B \\ \downarrow 2\Omega \\ \downarrow s \\ I \\ \overline{z} \\ V=0 \end{array}$$
(1) 3 A (2) 5 A (3) 4 A (4) 2 A

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- Ans. (2) Sol.  $20 \lor i_1$ A  $20 \lor i_2$   $10 \lor 40$   $20 \lor i_2$   $10 \lor 40$   $20 \lor i_2$   $10 \lor 40$   $20 \lor 40$   $10 \lor 40$   $10 \lor 6$   $20 \lor 12$   $10 \lor 6$   $10 \lor 12$   $10 \lor 12$  $10 \lor$
- **8.** A resistance is shown in the figure. Its value and tolerance are given respectively by:

(1) 27 KΩ, 20%	(2) 270 KΩ, 5%
(3) 270 KΩ, 10%	(4) 27 KΩ, 10%

Ans. (4)

Sol. Color code :

Red violet orange silver  $R = 27 \times 10^3 \Omega \pm 10\%$   $= 27 K\Omega \pm 10\%$ 

**9.** A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turas, and carrying a current of 5.2 A. The coercivity of the bar magnet is :

(1) 1200 A/m	(2) 2600 A/m
(3) 520 A/jm	(4) 285 A/m

Ans. (2)

Sol. Coercivity =  $H = \frac{B}{\mu_0}$ = ni =  $\frac{N}{\ell}i$  =  $\frac{100}{0.2} \times 5.2$ 

$$= 2600 \text{ A/m}$$

10. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/ °C. It is observed that an external compressive force F, is applied on

each of its ends, prevents any change in the length of the rod, when its temperature rises by  $\Delta T$  K. Young's modulus, Y, for this metal is :

(1) 
$$\frac{F}{2A\alpha\Delta T}$$
 (2)  $\frac{F}{A\alpha(\Delta T - 273)}$ 

(4)  $\frac{2F}{A\alpha\Lambda T}$ 

$$(3) \frac{F}{A\alpha\Delta T}$$

**Ans.** (3)

Sol. Young's modulus 
$$y = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{F/A}{(\Delta \ell / \ell)}$$
$$= \frac{F}{A(\alpha \Delta T)}$$

11. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is :





Ans. (3)

Sol. Maximum speed is at mean position (equilibrium). F = kx

$$x = \frac{F}{k}$$

$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v_{max} = \frac{F}{\sqrt{mk}}$$

12. Three charges +Q, q, +Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at x = 0, Ls zero, then value of q is :

(1) + Q/2 $(2) -Q/2 \quad (3) -Q/4$ (4) + Q/4Ans. (3)

Sol. 
$$F_{a} \xrightarrow{F_{b}} F_{b} \xrightarrow{F_{b}} \frac{1}{Q} \frac$$

$$\Rightarrow q = -\frac{Q}{4}$$

13. A conducting circular loop made of a thill wire, has area  $3.5 \times 10^{-3} \text{ m}^2$  and resistance  $10\Omega$  . It is placed perpendicular to a time dependent magnetic field  $B(t) = (0.4T)\sin(50\pi t)$ . The field is uniform in

space. Then the net charge flowing through the loop during t = 0 s and t = 10 ms is close to: (1) 14mC (2) 21 mC (3) 6 mC (4) 7 mC

Ans. (1)

Sol. 
$$Q = \frac{\Delta \phi}{R} = \frac{1}{10} A (B_r - B_i) = \frac{1}{10} \times 3.5 \times 10^{-3} \left( 0.4 \sin \frac{\pi}{2} - 0 \right)$$
  
 $= \frac{1}{10} \left( 3.5 \times 10^{-3} \right) \left( 0.4 - 0 \right)$   
 $= 1.4 \times 10^{-4} = 0.14 \text{ mC}$ 

Two masses m and  $\frac{m}{2}$  are connected at the two 14.

ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is  $\tau = k\theta$  for angular displacement 0. If the rod is rota ted by  $\theta_0$  and released, the tension in it when it passes through its mean position will be:



Ans. (4)

**Sol.** 
$$\omega = \sqrt{\frac{k}{I}}$$

$$\omega = \sqrt{\frac{3k}{m\ell^2}}$$



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 $\Omega = \omega \theta_0 = \text{ average velocity}$   $T = m\Omega^2 r_1$   $T = m\Omega^2 \frac{\ell}{3}$   $= m\omega^2 \theta_0^2 \frac{\ell}{3}$   $= m \frac{3k}{m\ell^2} \theta_0^2 \frac{\ell}{3}$   $= \frac{k\theta_0^2}{\ell}$   $I = \mu\ell^2 = \frac{\frac{m^2}{2}}{\frac{3m}{2}}\ell^2$   $= \frac{m\ell^2}{3}$   $m/2 \underbrace{\ell}_{r_2} \underbrace{r_1}_{r_1} \underbrace{r_1}_{r_2} = \frac{1}{2} \Rightarrow r_1 = \frac{\ell}{3}$ 

15. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is: (1) 2.5% (2) 0.5% (3) 1.0% (4) 2.0%
Ans. (3)

Sol.  $R = \frac{\rho \ell}{A}$  and volume (V) =  $A\ell$ .  $R = \frac{\rho \ell^2}{V}$  $\Rightarrow \frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell} = 1\%$ 

16. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure.</p>

Capacitance of this capacitor is :

(1) 
$$\frac{1}{2} \frac{k \in_0 a^2}{d}$$
 (2)  $\frac{k \in_0 a^2}{d} \ln K$   
(3)  $\frac{k \in_0 a^2}{d(K-1)} \ln K$  (4)  $\frac{k \in_0 a^2}{2d(K+1)}$ 



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17. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is  $10^{19}$ m<sup>-3</sup> and their mobility is 1.6 m<sup>2</sup>/(V.s) then the resistivity of the semiconductor (since it is an n-type semiconductor

contribution of holes is ignored) is close to:

(1)  $2\Omega m$  (2)  $0.4\Omega m$ (3)  $4\Omega m$  (4)  $0.2\Omega m$ 

(3)  $4\Omega m$ 

Ans. (2)

**Sol.**  $j = \sigma E = nev_d$ 

$$\sigma = ne \frac{v_d}{E}$$

$$\frac{1}{\sigma} = \rho = \frac{1}{n_e e \mu_e}$$

$$=\frac{1}{10^{19}\times1.6\times10^{-19}\times1.6}$$

=  $0.4 \Omega m$ 

18. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is :

(1)  $\frac{4L}{m}$  (2)  $\frac{L}{m}$  (3)  $\frac{L}{2m}$  (4)  $\frac{2L}{m}$ 

Ans. (3)

**Sol.** 
$$\frac{dA}{dt} = \frac{L}{2m}$$

**19.** A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward ?







**20.** Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-

section as AB and length  $\frac{3L}{2}$ , is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to :



(1) 60°C (2) 75°C (3) 35°C (4) 45°C Ans. (4)



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21. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is 60 ms<sup>-1</sup>. When the car has acceleration a, the wave-speed increases to 60.5 ms<sup>-1</sup>. The value of a, in terms of gravitational acceleration g, is closest to :

(1) 
$$\frac{g}{5}$$
 (2)  $\frac{g}{20}$   
(3)  $\frac{g}{10}$  (4)  $\frac{g}{30}$ 

Ans. (1)

**Sol.**  $60 = \sqrt{\frac{Mg}{\mu}}$ 

$$60.5 = \sqrt{\frac{M(g^2 + a^2)^{1/2}}{\mu}} \Rightarrow \frac{60.5}{60} = \sqrt{\sqrt{\frac{g^2 + a^2}{g^2}}}$$
$$\left(1 + \frac{0.5}{60}\right)^4 = \frac{g^2 + a^2}{g^2} = 1 + \frac{2}{60}$$
$$\Rightarrow g^2 + a^2 = g^2 + g^2 \times \frac{2}{60}$$
$$a = g\sqrt{\frac{2}{60}} = \frac{g}{\sqrt{30}} = \frac{g}{5.47}$$
$$\approx \frac{g}{5}$$

22. A sample of radioactive material A, that has an activity of 10 mCi(1 Ci =  $3.7 \times 10^{10}$  decays/s), has twice the number of nuclei as another sample of a different radioactive maternal B which has an activity of 20 mCi. The correct choices for hall-lives of A and B would then be respectively :

(1) 20 days and 5 days (2) 20 days and 10 days (3) 5 days and 10 days (4) 10 days and 40 days

Ans. (1)

Sol. Activity 
$$A = \lambda N$$
  
For  $A$  10 =  $(2N_0)\lambda_A$   
For  $B$  20 =  $N_0\lambda_B$   
 $\therefore \lambda_B = 4\lambda_A \Longrightarrow (T_{1/2})_A = 4(T_{1/2})_B$ 

23. Consider a tank made of glass(reiractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index  $\mu$ . A student finds that, irrespective of what the incident angle *i* (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of  $\mu$  is :



slab  $\mu = 1.5$ 

24. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d»a). If the loop applies a force F on the wire then :





Ans. (3)

Sol.   
solution long wire  
Eqvilent dipole of given loop  

$$F = m \cdot \frac{dB}{dr}$$
  
Now  $\frac{dB}{dx} = \frac{d}{dx} \left( \frac{\mu_0 I}{2\pi x} \right)$   
 $\propto \frac{1}{x^2}$   
 $\Rightarrow \text{ So } F \propto \frac{M}{x^2} [\because M = \text{NIA}]$   
 $\therefore F \propto \frac{a^2}{d^2}$ 

25. Surface of certain metal is first illuminated with light of wavelength  $\lambda_1 = 350$  nm and then, by light of wavelength  $\lambda_2 = 54D$  nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to :

(Energir of photon =  $\frac{1240}{\lambda(\text{in nm})}$  eV ) (1) 1.8 (2) 1.4 (3) 2.5 (4) 5.6

Ans. (1)

Sol. 
$$\frac{hc}{\lambda_{1}} = \phi + \frac{1}{2}m(2v)^{2}$$
$$\frac{hc}{\lambda_{2}} = \phi + \frac{1}{2}mv^{2}$$
$$\Rightarrow \frac{hc}{\lambda_{1}} - \phi}{\frac{hc}{\lambda_{2}} - \phi} = 4 \Rightarrow \frac{hc}{\lambda_{1}} - \phi = \frac{4hc}{\lambda_{2}} - 4\phi$$
$$\Rightarrow \frac{4hc}{\lambda_{2}} - \frac{hc}{\lambda_{1}} = 3\phi$$
$$\Rightarrow \phi = \frac{1}{3}hc\left(\frac{4}{\lambda_{2}} - \frac{1}{\lambda_{1}}\right)$$
$$= \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right)$$
$$= 1.8 \text{ eV}$$

- 26. A particle is moving with a velocity  $\overline{v} = K(y\hat{i} + x\hat{j})$ , where K is a constant. The general equation for its path is:
  - (1) xy = constant (2)  $y^2 = x^2 + constant$ (3)  $y = x^2 + constant$  (4)  $y^2 = x + constant$

Sol. 
$$\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$$
  
Now,  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$   
 $\Rightarrow ydy = xdx$   
Integrating both side  
 $y^2 = x^2 + c$ 

27. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is :

(1) 0.55 cm away from the lens

(2) 1.1 cm away from the lens

(3) 0.55 cm towards the lens

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**28.** For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is :

(1) 
$$\frac{R}{\sqrt{5}}$$
 (2) R (3)  $\frac{R}{\sqrt{2}}$  (4)  $R\sqrt{2}$ 

Ans. (3)

Sol. Electric field on axis of ring

 $E = \frac{kQh}{\left(h^2 + R^2\right)^{3/2}}$ 

for maximum electric field

$$\frac{dE}{dh} = 0$$

$$\Rightarrow$$
 h =  $\frac{R}{\sqrt{2}}$ 

29. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an brutal speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with

C, also perfectly inelastically  $\frac{5}{6}$  th of the initial kinetic energy is lost in whole process. What is value of M/m ?

$$(1) 4 (2) 5 (3) 3 (4) 2$$

Ans. (1)

**Sol.** 
$$k_i = \frac{1}{2}mv_0^2$$

From linear momentum conservation  $mv_0 = (2m + M) v_f$ 

$$\Rightarrow v_{f} = \frac{mv_{0}}{2m + M}$$

$$\frac{k_{i}}{k_{f}} = 6$$

$$\Rightarrow \frac{\frac{1}{2}mv_{0}^{2}}{\frac{1}{2}(2m + M)\left(\frac{mv_{0}}{2m + M}\right)^{2}} = 6$$

$$\Rightarrow \frac{2m + M}{m} = 6$$

$$\Rightarrow \frac{M}{m} = 4$$

**30.** Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm2, is v. If the electron density in copper is  $9 \times 10^{28}$  /m<sup>3</sup> the value of v in mm/s is close to (Take charge of electron to be = $1.6 \times 10^{-19}$ C)

Ans. (4)

**Sol.** 
$$I = neAv_d$$

$$\Rightarrow v_{d} = \frac{I}{neA} = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$$
  
= 0.02 m/s