

**FINAL JEE–MAIN EXAMINATION – JANUARY, 2019**

**Held On Thursday 10th JANUARY, 2019**

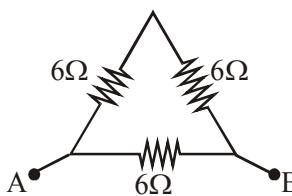
**TIME: 09 : 30 AM To 12 : 30 PM**

1. A uniform metallic wire has a resistance of  $18\Omega$  and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is :

(1)  $8\Omega$     (2)  $12\Omega$     (3)  $4\Omega$     (4)  $2\Omega$

**Ans. (3)**

**Sol.**



$R_{eq}$  between any two vertex will be

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} \Rightarrow R_{eq} = 4\Omega$$

2. A satellite is moving with a constant speed  $v$  in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is :

(1)  $\frac{3}{2}mv^2$     (2)  $mv^2$

(3)  $2mv^2$     (4)  $\frac{1}{2}mv^2$

**Ans. (2)**

**Sol.** At height  $r$  from center of earth. orbital velocity

$$= \sqrt{\frac{GM}{r}}$$

∴ By energy conservation

$$KE \text{ of } 'm' + \left( -\frac{GMm}{r} \right) = 0 + 0$$

(At infinity, PE = KE = 0)

$$\Rightarrow KE \text{ of } 'm' = \frac{GMm}{r} = \left( \sqrt{\frac{GM}{r}} \right)^2 m = mv^2$$

3. A solid metal cube of edge length 2 cm is moving in a positive y direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is :

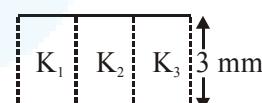
(1) 6 mV    (2) 1 mV    (3) 12 mV    (4) 2 mV

**Ans. (3)**

**Sol.** Potential difference between two faces perpendicular to x-axis will be

$$\ell \cdot (\vec{V} \times \vec{B}) = 12 \text{ mV}$$

4. A parallel plate capacitor is of area  $6 \text{ cm}^2$  and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants  $K_1 = 10$ ,  $K_2 = 12$  and  $K_3 = 14$ . The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be :



$$(1) 12 \quad (2) 4 \quad (3) 36 \quad (4) 14$$

**Ans. (1)**

**Sol.** Let dielectric constant of material used be  $K$ .

$$\therefore \frac{10 \epsilon_0 A / 3}{d} + \frac{12 \epsilon_0 A / 3}{d} + \frac{14 \epsilon_0 A / 3}{d} = \frac{K \epsilon_0 A}{d}$$

$$\Rightarrow K = 12$$

5. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is :

(1) 63 mA    (2) 0.4 mA  
(3) 100 mA    (4) 20 mA

**Ans. (4)**

**Sol.**  $P = i^2 R$ .

∴ for  $i_{max}$ ,  $R$  must be minimum  
from color coding  $R = 50 \times 10^2 \Omega$   
 $\therefore i_{max} = 20 \text{ mA}$



10. If the magnetic field of a plane electromagnetic wave is given by (The speed of light =  $3 \times 10^8$  m/s)

$$B = 100 \times 10^{-6} \sin \left[ 2\pi \times 2 \times 10^{15} \left( t - \frac{x}{c} \right) \right] \text{ then the}$$

maximum electric field associated with it is :

(1)  $4 \times 10^4$  N/C      (2)  $4.5 \times 10^4$  N/C  
 (3)  $6 \times 10^4$  N/C      (4)  $3 \times 10^4$  N/C

**Ans. (4)**

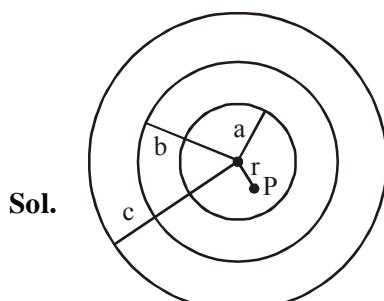
$$\begin{aligned} \text{Sol. } E_0 &= B_0 \times C \\ &= 100 \times 10^{-6} \times 3 \times 10^8 \\ &= 3 \times 10^4 \text{ N/C} \end{aligned}$$

∴ correct answer is  $3 \times 10^4$  N/C

11. A charge Q is distributed over three concentric spherical shells of radii a, b, c ( $a < b < c$ ) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where  $r < a$ , would be :

(1)  $\frac{Q}{4\pi\epsilon_0(a+b+c)}$   
 (2)  $\frac{Q(a+b+c)}{4\pi\epsilon_0(a^2+b^2+c^2)}$   
 (3)  $\frac{Q}{12\pi\epsilon_0} \frac{ab+bc+ca}{abc}$   
 (4)  $\frac{Q}{4\pi\epsilon_0} \frac{(a^2+b^2+c^2)}{(a^3+b^3+c^3)}$

**Ans. (2)**



$$\text{Potential at point P, } V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$$

$$\therefore Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$$

{since  $\sigma_a = \sigma_b = \sigma_c$ }

$$\therefore Q_a = \left[ \frac{a^2}{a^2 + b^2 + c^2} \right] Q$$

$$Q_b = \left[ \frac{b^2}{a^2 + b^2 + c^2} \right] Q$$

$$Q_c = \left[ \frac{c^2}{a^2 + b^2 + c^2} \right] Q$$

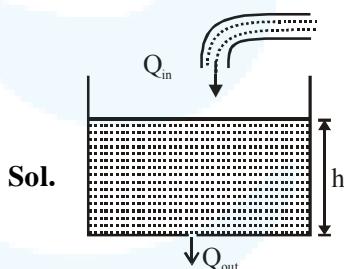
$$V = \frac{Q}{4\pi\epsilon_0} \left[ \frac{(a+b+c)}{a^2 + b^2 + c^2} \right]$$

∴ correct answer is (2)

12. Water flows into a large tank with flat bottom at the rate of  $10^{-4}$  m<sup>3</sup>s<sup>-1</sup>. Water is also leaking out of a hole of area 1 cm<sup>2</sup> at its bottom. If the height of the water in the tank remains steady, then this height is:

(1) 4 cm      (2) 2.9 cm      (3) 1.7 cm      (4) 5.1 cm

**Ans. (4)**



Since height of water column is constant therefore, water inflow rate ( $Q_{in}$ )  
 = water outflow rate

$$Q_{in} = 10^{-4} \text{ m}^3\text{s}^{-1}$$

$$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$$

$$10^{-4} = 10^{-4} \sqrt{20 \times h}$$

$$h = \frac{1}{20} \text{ m}$$

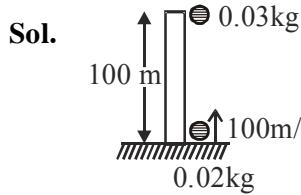
$$h = 5 \text{ cm}$$

∴ correct answer is (4)

13. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity  $100 \text{ ms}^{-1}$ , from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : ( $g = 10 \text{ ms}^{-2}$ )

(1) 30 m (2) 10 m (3) 40 m (4) 20 m

**Ans. (3)**



Time taken for the particles to collide,

$$t = \frac{d}{V_{\text{rel}}} = \frac{100}{100} = 1 \text{ sec}$$

Speed of wood just before collision =  $gt = 10 \text{ m/s}$   
& speed of bullet just before collision  $v-gt = 100 - 10 = 90 \text{ m/s}$

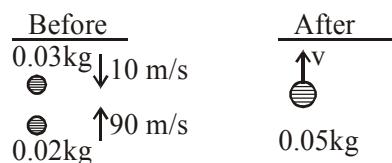
Now, conservation of linear momentum just before and after the collision -

$$-(0.02)(1v) + (0.02)(9v) = (0.05)v$$

$$\Rightarrow 150 = 5v$$

$$\Rightarrow v = 30 \text{ m/s}$$

$$\text{Max. height reached by body } h = \frac{v^2}{2g}$$



$$h = \frac{30 \times 30}{2 \times 10} = 45 \text{ m}$$

$\therefore$  Height above tower = 40 m

14. The density of a material in SI units is  $128 \text{ kg m}^{-3}$ . In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is :

(1) 410 (2) 640 (3) 16 (4) 40

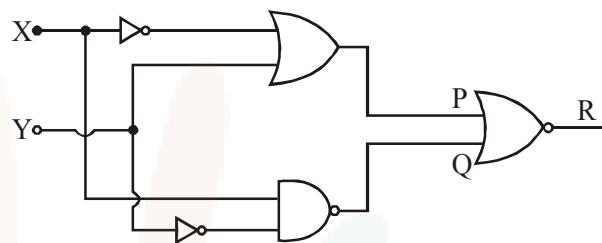
**Ans. (4)**

**Sol.** 
$$\frac{128 \text{ kg}}{\text{m}^3} = \frac{125(50 \text{ g})(20)}{(25 \text{ cm})^3 (4)^3}$$

$$= \frac{128}{64} (20) \text{ units}$$

$$= 40 \text{ units}$$

15. To get output '1' at R, for the given logic gate circuit the input values must be :



(1)  $X = 0, Y = 1$  (2)  $X = 1, Y = 1$   
(3)  $X = 0, Y = 0$  (4)  $X = 1, Y = 0$

**Ans. (4)**

**Sol.**  $p = \bar{x} + y$

$$Q = \bar{y} \cdot x = y + \bar{x}$$

$$O/P = \bar{P} + Q$$

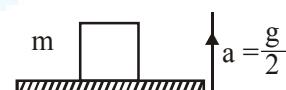
To make O/P

$P + Q$  must be 'O'

SO,  $y = 0$

$$x = 1$$

16. A block of mass m is kept on a platform which starts from rest with constant acceleration  $g/2$  upward, as shown in fig. Work done by normal reaction on block in time t is :



(1) 0 (2)  $\frac{3mg^2t^2}{8}$

(3)  $-\frac{mg^2t^2}{8}$  (4)  $\frac{mg^2t^2}{8}$

**Ans. (2)**

**Sol.**  $N - mg = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

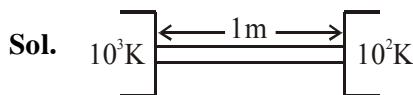
Now, work done  $W = \bar{N} \bar{S} = \left( \frac{3mg}{2} \right) \left( \frac{1}{2} gt^2 \right)$

$$\Rightarrow W = \frac{3mg^2t^2}{4}$$

17. A heat source at  $T = 10^3$  K is connected to another heat reservoir at  $T = 10^2$  K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is  $0.1 \text{ WK}^{-1} \text{ m}^{-1}$ , the energy flux through it in the steady state is :

(1)  $90 \text{ Wm}^{-2}$       (2)  $200 \text{ Wm}^{-2}$   
(3)  $65 \text{ Wm}^{-2}$       (4)  $120 \text{ Wm}^{-2}$

**Ans. (1)**



$$\left( \frac{dQ}{dt} \right) = \frac{kA\Delta T}{\ell}$$

$$\Rightarrow \frac{1}{A} \left( \frac{dQ}{dt} \right) = \frac{(0.1)(900)}{1} = 90 \text{ W/m}^2$$

18. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS(Line of Sight) mode ? (Given : radius of earth =  $6.4 \times 10^6$  m).

(1) 80 km      (2) 48 km  
(3) 40 km      (4) 65 km

**Ans. (4)**

**Sol.** Maximum distance upto which signal can be broadcasted is

$$d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

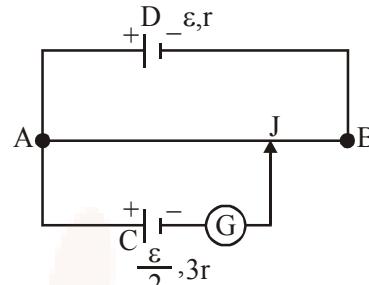
where  $h_T$  and  $h_R$  are heights of transmitter tower and height of receiver respectively.

Putting all values -

$$d_{\max} = \sqrt{2 \times 6.4 \times 10^6} \left[ \sqrt{104} + \sqrt{40} \right]$$

on solving,  $d_{\max} = 65 \text{ km}$

19. A potentiometer wire AB having length L and resistance  $12r$  is joined to a cell D of emf  $\varepsilon$  and internal resistance  $r$ . A cell C having emf  $\frac{\varepsilon}{2}$  and internal resistance  $3r$  is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is :



(1)  $\frac{5}{12}L$       (2)  $\frac{11}{24}L$       (3)  $\frac{11}{12}L$       (4)  $\frac{13}{24}L$

**Ans. (4)**

$$i = \frac{\varepsilon}{13r}$$

$$i \left( \frac{x}{L} 12r \right) = \frac{\varepsilon}{2}$$

$$\frac{\varepsilon}{13r} \left[ \frac{x}{L} \cdot 12r \right] = \frac{\varepsilon}{2} \Rightarrow x = \frac{13L}{24}$$

20. An insulating thin rod of length  $\ell$  has a linear charge density  $p(x) = \rho_0 \frac{x}{\ell}$  on it. The rod is rotated about an axis passing through the origin ( $x = 0$ ) and perpendicular to the rod. If the rod makes  $n$  rotations per second, then the time averaged magnetic moment of the rod is :

(1)  $\frac{\pi}{4} n \rho \ell^3$       (2)  $n \rho \ell^3$       (3)  $\pi n \rho \ell^3$       (4)  $\frac{\pi}{3} n \rho \ell^3$

**Ans. (1)**

**Sol.**  $\because M = NIA$

$$dq = \lambda dx \text{ & } A = \pi x^2$$

$$\int dm = \int (x) \frac{\rho_0 x}{\ell} dx \cdot \pi x^2$$

$$M = \frac{n \rho_0 \pi}{\ell} \int_0^{\ell} x^3 dx = \frac{n \rho_0 \pi}{\ell} \cdot \left[ \frac{L^4}{4} \right]$$

$$M = \frac{n \rho_0 \pi \ell^3}{4} \text{ or } \frac{\pi}{4} n \rho \ell^3$$



26. A plano convex lens of refractive index  $\mu_1$  and focal length  $f_1$  is kept in contact with another plano concave lens of refractive index  $\mu_2$  and focal length  $f_2$ . If the radius of curvature of their spherical faces is  $R$  each and  $f_1 = 2f_2$ , then  $\mu_1$  and  $\mu_2$  are related as :

- (1)  $\mu_1 + \mu_2 = 3$
- (2)  $2\mu_1 - \mu_2 = 1$
- (3)  $2\mu_2 - \mu_1 = 1$
- (4)  $3\mu_2 - 2\mu_1 = 1$

**Ans. (2)**

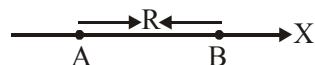
$$\text{Sol. } \frac{1}{2f_2} = \frac{1}{f_1} = (\mu_1 - 1) \left( \frac{1}{\infty} - \frac{1}{-R} \right)$$

$$\frac{1}{f_2} = (\mu_2 - 1) \left( \frac{1}{-R} - \frac{1}{\infty} \right)$$

$$\frac{(\mu_1 - 1)}{R} = \frac{(\mu_2 - 1)}{2R}$$

$$2\mu_2 - \mu_1 = 1$$

27. Two electric dipoles, A, B with respective dipole moments  $\vec{d}_A = -4qa\hat{i}$  and  $\vec{d}_B = -2qa\hat{i}$  placed on the x-axis with a separation  $R$ , as shown in the figure



The distance from A at which both of them produce the same potential is :

$$(1) \frac{\sqrt{2}R}{\sqrt{2}+1}$$

$$(2) \frac{R}{\sqrt{2}+1}$$

$$(3) \frac{\sqrt{2} R}{\sqrt{2}-1}$$

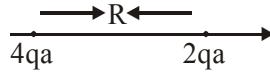
$$(4) \frac{R}{\sqrt{2}-1}$$

**Ans. (3)**

$$\text{Sol. } V = \frac{4qa}{(R+x)} = \frac{2qa}{(x^2)}$$

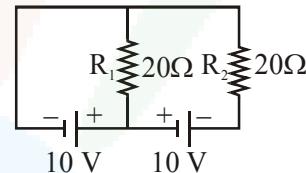
$$\sqrt{2}x = R + x$$

$$x = \frac{R}{\sqrt{2}-1}$$



$$\text{dist} = \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}$$

28. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance  $R_1$ , and  $R_2$  respectively, are:



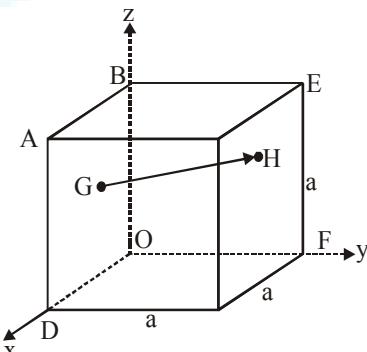
- (1) 2, 2
- (2) 0, 1
- (3) 1,2
- (4) 0.5,0

**Ans. (4)**

$$\text{Sol. } i_1 = \frac{10}{20} = 0.5A$$

$$i_2 = 0$$

29. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be:



$$(1) \frac{1}{2}a(\hat{i} - \hat{k}) \quad (2) \frac{1}{2}a(\hat{j} - \hat{i})$$

$$(3) \frac{1}{2}a(\hat{k} - \hat{i}) \quad (4) \frac{1}{2}a(\hat{j} - \hat{k})$$

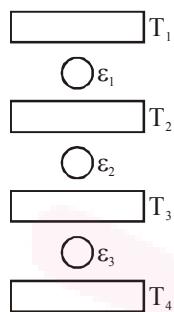
**Ans. (2)**

**Sol.**  $\vec{r}_g = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}$

$$\vec{r}_H = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$$

$$\vec{r}_H - \vec{r}_g = \frac{a}{2}(\hat{j} - \hat{i})$$

**30.** Three Carnot engines operate in series between a heat source at a temperature  $T_1$  and a heat sink at temperature  $T_4$  (see figure). There are two other reservoirs at temperature  $T_2$ , and  $T_3$ , as shown, with  $T_2 > T_2 > T_3 > T_4$ . The three engines are equally efficient if:



(1)  $T_2 = (T_1 T_4)^{1/3}; T_3 = (T_1 T_4)^{1/3}$

(2)  $T_2 = (T_1 T_4)^{1/3}; T_3 = (T_1 T_4)^{1/3}$

(3)  $T_2 = (T_1 T_4)^{1/4}; T_3 = (T_1 T_4)^{1/4}$

(4)  $T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1 T_4)^{1/3}$

**Ans. (1)**

**Sol.**  $t_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{T_2}{T_2} = 1 - \frac{T_4}{T_3}$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_4}{T_3}$$

$$\Rightarrow T_2 = \sqrt{T_1 T_3} = \sqrt{T_1 \sqrt{T_2 T_4}}$$

$$T_3 = \sqrt{T_2 T_4}$$

$$T_2^{3/4} = \sqrt{T_1^{1/2} T_4^{1/4}}$$

$$T_2 = T_1^{2/3} T_4^{1/3}$$