FINAL JEE–MAIN EXAMINATION – JANUARY, 2019

Held On Friday 11th JANUARY, 2019

TIME: 02 : 30 PM To 05 : 30 PM

1. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is :-

(1) 2.3×10^{-2} (2) 3.3×10^{-2}

(3) 3.3×10^{-4} (4) 4.3×10^{-2}

Ans. **(3)**

Sol. $\chi = \frac{I}{H}$

$$I = \frac{\text{Magnetic moment}}{\text{Volume}}$$

$$I = \frac{20 \times 10^{-6}}{10^{-6}} = 20 \text{ N/m}^2$$

$$\chi = \frac{20}{60 \times 10^{+3}} = \frac{1}{3} \times 10^{-3}$$

$= 0.33 \times 10^{-3} = 3.3 \times 10^{-4}$

2. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is :-

(1) $2\sqrt{\frac{p}{k}}$ (2) $\sqrt{\frac{2p}{k}}$ (3) $\sqrt{\frac{2k}{p}}$ (4) $2\sqrt{\frac{k}{p}}$

Ans. **(1)**

Sol. $\frac{dp}{dt} = F = kt$

$$\int_P^{3P} dP = \int_O^T kt\, dt$$

$$2p = \frac{KT^2}{2}$$

$$T = 2\sqrt{\frac{P}{K}}$$

3. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu F$. Which of the combinations, shown in figures below, will achieve the desired value ?

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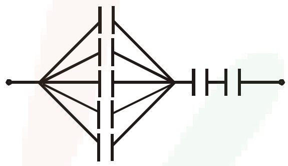

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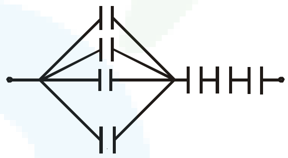

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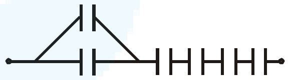

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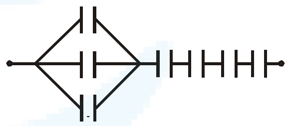


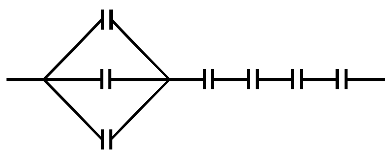
Ans. **(4)**

Sol. $C_{eq} = \frac{6}{13}\mu F$

Therefore three capacitors most be in parallel to get 6 in

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F$$



4. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole ?

(1) -9×10^{-20} J

(2) -7×10^{-27} J

(3) -10×10^{-29} J

(4) -20×10^{-18} J

Ans. (2)

Sol. $U = -\vec{P}.\vec{E}$

$= -PE \cos\theta$

$= -(10^{-29})(10^{3}) \cos 45°$

$= -0.707 \times 10^{-26}$ J

$= -7 \times 10^{-27}$ J.

5. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :-

(1) 10^{-3} rad/s

(2) 10^{-1} rad/s

(3) 1 rad/s

(4) 10^{-5} rad/s

Ans. (1)

Sol. Angular frequency of pendulum

$$\omega = \sqrt{\frac{g_{eff}}{\ell}}$$

$$\therefore \quad \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$$

$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$$

[ω_s = angular frequency of support]

$$\Delta\omega = \frac{1}{2} \times \frac{2A\omega_s^2}{100} \times 100$$

$\Delta\omega = 10^{-3}$ rad/sec.

6. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is :-

(1) 270°C (2) 230°C

(3) 250°C (4) 200°C

Ans. (2)

Sol. $\Delta\ell_1 = \Delta\ell_2$

$$\ell\alpha_1\Delta T_1 = \ell\alpha_2\Delta T_2$$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta T_1}{\Delta T_2}$$

$$\frac{4}{3} = \frac{T-30}{180-30}$$

$$\boxed{T = 230°C}$$

7. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is :-

(1) 09 (2) 10

(3) 04 (4) 05

Ans. (4)

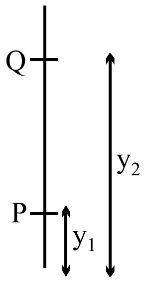
Sol. For diffraction

location of 1st minime

$$y_1 = \frac{D\lambda}{a} = 0.2469 D\lambda$$

location of 2nd minima

$$y_2 = \frac{2D\lambda}{a} = 0.4938 D\lambda$$



Now for interference

Path difference at P.

$$\frac{dy}{D} = 4.8\lambda$$

path difference at Q

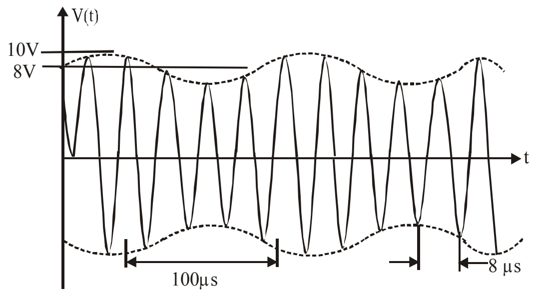
$$\frac{dy}{D} = 9.6\lambda$$

So orders of maxima in between P & Q is

5, 6, 7, 8, 9

So 5 bright fringes all present between P & Q.

8. An amplitude modulated signal is plotted below :-



Which one of the following best describes the above signal ?

(1) $(9 + \sin(2.5\pi \times 10^5\ t))\sin(2\pi \times 10^4 t)$V

(2) $(9 + \sin(4\pi \times 10^4\ t))\sin(5\pi \times 10^5 t)$V

(3) $(1 + 9\sin(2\pi \times 10^4\ t))\sin(2.5\pi \times 10^5 t)$V

(4) $(9 + \sin(2\pi \times 10^4\ t))\sin(2.5\pi \times 10^5 t)$V

Ans. **(4)**

Sol. Analysis of graph says

(1) Amplitude varies as 8 – 10 V or 9 ± 1

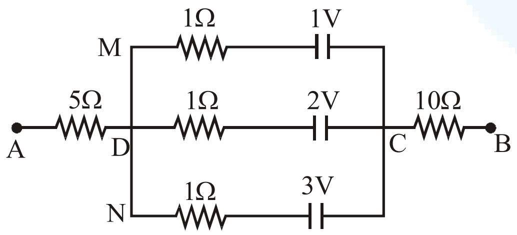
(2) Two time period as

100 μs (signal wave) & 8 μs (carrier wave)

Hence signal is $\left[9 \pm 1\sin\left(\frac{2\pi t}{T_1}\right)\right]\sin\left(\frac{2\pi t}{T_2}\right)$

$= 9 \pm 1\sin(2\pi \times 10^4 t)\sin 2.5\pi \times 10^5\ t$

9. In the circuit , the potential difference between A and B is :-



(1) 6 V (2) 1 V (3) 3 V (4) 2 V

Ans. **(4)**

Sol. Potential difference across AB will be equal to battery equivalent across CD

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} = \frac{\frac{1}{1} + \frac{2}{1} + \frac{3}{1}}{\frac{1}{1} + \frac{1}{1} + \frac{1}{1}}$$

$$= \frac{6}{3} = 2V$$

10. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by [Given permittivity of space $\epsilon_0 = 9 \times 10^{-12}$ SI units, Speed of light $c = 3 \times 10^8$ m/s]:-

(1) 1 kV/m (2) 2 kV/m

(3) 1.4 kV/m (4) 0.7 kV/m

Ans. **(3)**

Sol. Intensity of EM wave is given by

$$I = \frac{\text{Power}}{\text{Area}} = \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$= \frac{27 \times 10^{-3}}{10 \times 10^{-6}} = \frac{1}{2} \times 9 \times 10^{-12} \times E^2 \times 3 \times 10^8$$

$$E = \sqrt{2} \times 10^3 \text{ kv/m}$$

$= 1.4$ kv/m

11. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :-

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$

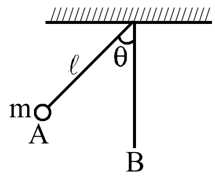
(3) $K_2 = 2K_1$ (4) $K_2 = K_1$

Ans. **(3)**

Sol. Maximum kinetic energy at lowest point B is given by

$K = mgl(1 - \cos\theta)$

where θ = angular amp.



$K_1 = mg\ell(1 - \cos\theta)$

$K_2 = mg(2\ell)(1 - \cos\theta)$

$K_2 = 2K_1$.

12. In a hydrogen like atom, when an electron jumps from the M - shell to the L - shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be :-

(1) $\frac{27}{20}\lambda$ (2) $\frac{16}{25}\lambda$ (3) $\frac{20}{27}\lambda$ (4) $\frac{25}{16}\lambda$

Ans. (3)

Sol. For M $\rightarrow$ L steel

$$\frac{1}{\lambda}=K\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{K\times 5}{36}$$

for N $\rightarrow$ L

$$\frac{1}{\lambda'}=K\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\frac{K\times 3}{16}$$

$$\lambda'=\frac{20}{27}\lambda$$

13. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be :-

(1) $V^{-2}A^2F^2$ (2) $V^{-4}A^2F$

(3) $V^{-4}A^{-2}F$ (4) $V^{-2}A^2F^{-2}$

Ans. (2)

Sol. $\frac{F}{A}=y.\frac{\Delta\ell}{\ell}$

$$[Y]=\frac{F}{A}$$

Now from dimension

$$F=\frac{ML}{T^2}$$

$$L=\frac{F}{M}.T^2$$

$$L^2=\frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \quad \because T=\frac{V}{A}$$

$$L^2=\frac{F^2}{M^2A^2}\frac{v^4}{A^2} \qquad F=MA$$

$$L^2=\frac{V^4}{A^2}$$

$$[Y]=\frac{[F]}{[A]}=F^1V^{-4}A^2$$

14. A particle moves from the point $(2.0\hat{i}+4.0\hat{j})$ m, at t = 0, with an initial velocity $(5.0\hat{i}+4.0\hat{j})$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})$ ms^{-2}. What is the distance of the particle from the origin at time 2 s ?

(1) $20\sqrt{2}$ m (2) $10\sqrt{2}$ m

(3) 5 m (4) 15 m

Ans. (1)

Sol. $\vec{S}=(5\hat{i}+4\hat{j})2+\frac{1}{2}(4\hat{i}+4\hat{j})4$

$$=10\hat{i}+8\hat{j}+8\hat{i}+8\hat{j}$$

$$\vec{r}_f-\vec{r}_i=18\hat{i}+16\hat{j}$$

$$\vec{r}_f=20\hat{i}+20\hat{j}$$

$$|\vec{r}_f|=20\sqrt{2}$$

15. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is :-

(1) 30° (2) 45°

(3) 90° (4) 60°

Ans. (4)

Sol. i = e

$$r_1=r_2=\frac{A}{2}=30^\circ$$

by Snell's law

$$1\times\sin i=\sqrt{3}\times\frac{1}{2}=\frac{\sqrt{3}}{2}$$

i = 60

16. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is :-

(1) 80 Ω (2) 120 Ω

(3) 125 Ω (4) 100 Ω

Ans. **(1)**

Sol. $R_g = 20\Omega$

$N_L = N_R = N = 30$

$$FOM = \frac{I}{\phi} = 0.005 \text{ A/Div.}$$

$$\text{Current sentivity} = CS = \left(\frac{1}{0.005}\right) = \frac{\phi}{I}$$

$Ig_{max} = 0.005 \times 30$

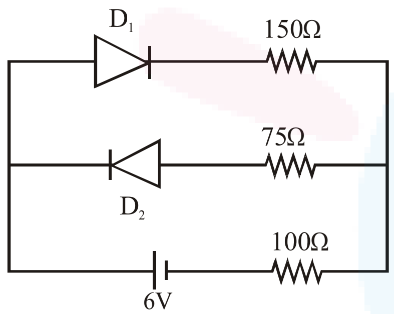
$= 15 \times 10^{-2} = 0.15$

$15 = 0.15\ [20 + R]$

$100 = 20 + R$

$R = 80$

17. The circuit shown below contains two ideal diodes, each with a forward resistance of 50Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in Amperes) is :-



(1) 0.027 (2) 0.020

(3) 0.030 (4) 0.036

Ans. **(2)**

Sol. $I = \frac{6}{300} = 0.002$ (D_2 is in reverse bias)

18. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be :-

(1) 80°C (2) 60°C

(3) 70°C (4) 85°C

Ans. **(1)**

Sol. $100 \times S_A \times [100 - 90] = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5\ S_B$

$$S_A = \frac{3}{4} S_B$$

Now, $100 \times S_A \times [100 - T] = 50 \times S_B\ (T - 50)$

$$2 \times \left(\frac{3}{4}\right)(100 - T) = (T - 50)$$

$300 - 3T = 2T - 100$

$400 = 5T$

$T = 80$

19. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be :-

(1) $\frac{2}{\sqrt{3}}$s (2) $2\sqrt{3}$ s

(3) $\frac{\sqrt{3}}{2}$s (4) $\frac{3}{2}$s

Ans. **(2)**

Sol. $\because g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_e}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also $T \propto \frac{1}{\sqrt{g}}$

$$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$$\Rightarrow T_p = 2\sqrt{3}\text{s}$$

20. The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is :-

(1) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(2) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{\sqrt{2}}\hat{j}\right)$

(3) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$

(4) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$

Ans. (BONUS)

21. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice.

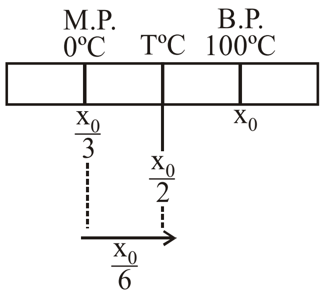
What is the temperature of an object in 0 °C, if this thermometer in the contact with the object reads $x_0/2$?

(1) 35 (2) 25

(3) 60 (4) 40

Ans. (2)

Sol.

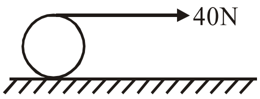


$$\Rightarrow T^\circ C = \frac{x_0}{6} \ \& \ \left(x_0 - \frac{x_0}{3}\right) = (100 - 0^\circ C)$$

$$x_0 = \frac{300}{2}$$

$$\Rightarrow T^\circ C = \frac{150}{6} = 25^\circ C$$

22. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) :-

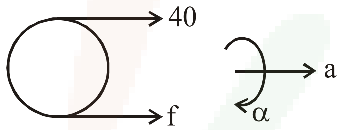


(1) 12 rad/s^2 (2) 16 rad/s^2

(3) 10 rad/s^2 (4) 20 rad/s^2

Ans. (2)

Sol.



$40 + f = m(R\alpha)$ .....(i)

$40 \times R - f \times R = mR^2\alpha$

$40 - f = mR\alpha$ ...... (ii)

From (i) and (ii)

$$\alpha = \frac{40}{mR} = 16$$

23. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where K is a constant. In this process the temperature of the gas is incresed by ΔT. The amount of heat absorbed by gas is (R is gas constant) :

(1) $\frac{1}{2}R\Delta T$ (2) $\frac{3}{2}R\Delta T$

(3) $\frac{1}{2}KR\Delta T$ (4) $\frac{2K}{3}\Delta T$

Ans. (1)

Sol. VT = K

$$\Rightarrow V\left(\frac{PV}{nR}\right) = k \Rightarrow PV^2 = K$$

$$\because \ C = \frac{R}{1-x} + C_V \ \text{(For polytropic process)}$$

$$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$$

$\therefore \ \Delta Q = nC\,\Delta T$

$$= \frac{R}{2} \times \Delta T$$

24. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to : $\left(\frac{hc}{e} = 1240\,\text{nm} - \text{V}\right)$

(1) 0.5 V (2) 1.0 V
(3) 2.0 V (4) 1.5 V

Ans. **(2)**

Sol. $\frac{hc}{\lambda_1} = \phi + eV_1$ .... (i)

$\frac{hc}{\lambda_2} = \phi + eV_2$ .... (ii)

(i) – (ii)

$$hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e(V_1 - V_2)$$

$$\Rightarrow V_1 - V_2 = \frac{hc}{e}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}\right)$$

$$= (1240\text{nm} - \text{V})\frac{100\text{nm}}{300\text{nm} \times 400\text{nm}}$$

= 1V

25. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water ? [Specific Heat Capacities of water and metal are, respectively, 4200 $Jkg^{-1}K^{-1}$ and 400 $JKg^{-1}K^{-1}$]

(1) 30% (2) 20%
(3) 25% (4) 15%

Ans. **(2)**

Sol. 0.1 × 400 × (500 – T) = 0.5 × 4200 × (T – 30) + 800 (T – 30)

⇒ 40(500 – T) = (T – 30) (2100 + 800)

⇒ 20000 – 40T = 2900 T – 30 × 2900

⇒ 20000 + 30 × 2900 = T(2940)

T = 30.4°C

$$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100$$

$\simeq 20\%$

26. The magnitude of torque on a particle of mass 1kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians) :-

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

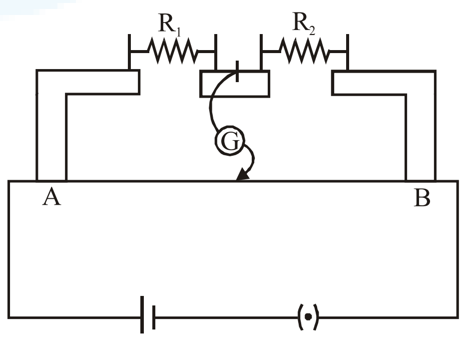
Ans. **(2)**

Sol. 2.5 = 1 × 5 sin θ

sin θ = 0.5 = $\frac{1}{2}$

θ = $\frac{\pi}{6}$

27. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is



(1) 40 Ω (2) 60 Ω (3) 20 Ω (4) 30Ω

Ans. (2)

Sol. $\frac{R_1}{R_2} = \frac{2}{3}$ ……(i)

$\frac{R_1 + 10}{R_2} = 1 \Rightarrow R_1 + 10 = R_2$ ……(ii)

$\frac{2R_2}{3} + 10 = R_2$

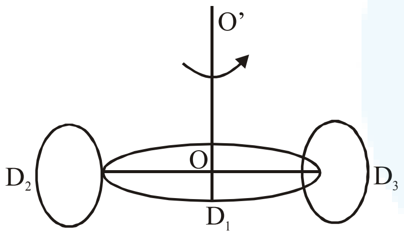
$10 = \frac{R_2}{3} \Rightarrow R_2 = 30\Omega$

& $R_1 = 20\Omega$

$$\frac{\frac{30 \times R}{30 + R}}{30} = \frac{2}{3}$$

$R = 60\ \Omega$

28. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be:-



(1) $3MR^2$ (2) $\frac{2}{3}MR^2$

(3) MR^2 (4) $\frac{4}{5}MR^2$

Ans. (1)

Sol. $I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$

$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2$

$= 3\ MR^2$

29. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil :

(1) Decreases by a factor of $9\sqrt{3}$

(2) Increases by a factor of 3

(3) Decreases by a factor of 9

(4) Increases by a factor of 27

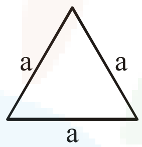
Ans. (2)

Sol. Total length L will remain constant

$L = (3a)\ N$ (N = total turns)

and length of winding = (d) N

(d = diameter of wire)



self inductance $= \mu_0 n^2 A\ell$

$= \mu_0 n^2 \left(\frac{\sqrt{3}a^2}{4}\right) dN$

$\propto a^2 N \propto a$

So self inductance will become 3 times

30. A particle of mass m and charge q is in an electric and magnetic field given by

$\vec{E} = 2\hat{i} + 3\hat{j}$; $\vec{B} = 4\hat{j} + 6\hat{k}$.

The charged particle is shifted from the origin to the point P(x = 1 ; y = 1) along a straight path. The magnitude of the total work done is :-

(1) (0.35)q (2) (0.15)q

(3) (2.5)q (4) 5q

Ans. (4)

Sol. $\vec{F}_{net} = q\vec{E} + q(\vec{v} \times \vec{B})$

$= (2q\hat{i} + 3q\hat{j}) + q(\vec{v} \times \vec{B})$

$W = \vec{F}_{net}.\vec{S}$

$= 2q + 3q$

$= 5q$