FINAL JEE–MAIN EXAMINATION – APRIL, 2019
Held On Tuesday 09th APRIL, 2019
TIME: 02 : 30 PM To 05 : 30 PM

1. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to :

(1) $2\sqrt{2}+1$ (2) $\sqrt{2}-1$

(3) $\sqrt{2}+1$ (4) $2\sqrt{2}-1$

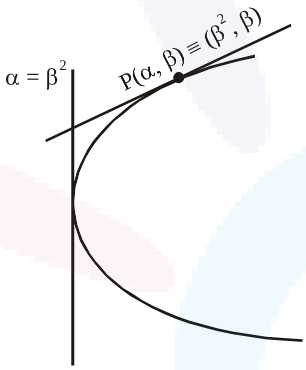
Official Ans. by NTA (3)

Sol. $T : y(\beta) = \frac{1}{2}(x+\beta^2)$

$2y\beta = x + \beta^2$

$y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$

$m = \frac{1}{2\beta}; C = \frac{\beta}{2}$



$\frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2}+\frac{1}{2}}$

$\frac{\beta^2}{4} = \frac{1}{4\beta^2}+\frac{1}{2}$

$\frac{\beta^2}{4} = \frac{1+2\beta^2}{4\beta^2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

$(\beta^2 - 1)^2 = 2$

$\beta^2 - 1 = \sqrt{2}$

$\beta^2 = \sqrt{2}+1$

2. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are addded to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :-

(1) 190 (2) 262 (3) 225 (4) 157

Official Ans. by NTA (1)

Sol. $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$n^2 + n + 198 = 2(n^2 + 4 - 4n)$

$n^2 - 9n - 190 = 0$

$n^2 - 19n + 10 - 190 = 0$

$n(n - 19) + 10(n - 19) = 0$

$n = 19$

3. If $f : R \to R$ is a differentiable function and $f(2) = 6$, then $\lim_{x\to 2} \int_6^{f(x)} \frac{2t\,dt}{(x-2)}$ is :-

(1) 0 (2) $2f'(2)$

(3) $12f'(2)$ (4) $24f'(2)$

Official Ans. by NTA (3)

Sol. $\lim_{x\to 2} \frac{\int_6^{f(x)} 2t\,dt}{x-2}$

L Hopital Rule

$\lim_{x\to 2} \frac{2f(x)f'(x)}{1} = 2f(2) = f'(2) = 12f'(2)$

4. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trival solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:-

(1) $\frac{3}{4}$ (2) -4 (3) $\frac{1}{2}$ (4) $-\frac{1}{4}$

Official Ans. by NTA (3)

Sol. $\begin{vmatrix} 2 & 3 & -1 \\ 1 & K & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$

By solving $K = \frac{9}{2}$

$2x + 3y - z = 0$ ...(1)

$x + \frac{9}{2}y - 2z = 0$ ...(2)

$2x - y + z = 0$ ...(3)

(1)-(3) $\Rightarrow 4y - 2z = 0$

$2y = z$ ...(4)

$\boxed{\frac{y}{z} = \frac{1}{2}}$ ...(5)

put z from eqn. (4) into (1)

$2x + 3y - 2y = 0$

$2x + y = 0$

$\boxed{\frac{x}{y} = -\frac{1}{2}}$ ...(6)

$\frac{(6)}{(5)} \boxed{\frac{z}{x} = -4}$

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + K = \frac{1}{2}$

5. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :-

(1) (–4, 6) (2) (6, –2)

(3) (–6, 4) (4) (4, –2)

Official Ans. by NTA (2)

Sol. Circle touches internally

$C_1(0, 0); r_1 = 2$

$C_2 : (-3, -4); r_2 = 7$

$C_1C_2 = |r_1 - r_2|$

$S_1 - S_2 = 0 \Rightarrow$ eqn. of common tangent

$6x + 8y - 20 = 0$

$3x + 4y = 10$

(6, –2) satisfy it

6. If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} term is :-

(1) –25 (2) 25

(3) –36 (4) –35

Official Ans. by NTA (1)

Sol. $a - d + a + a + d = 33 \Rightarrow a = 11$

$a(a^2 - d^2) = 1155$

$121 - d^2 = 105$

$d^2 = 16 \Rightarrow d = \pm 4$

If $d = 4$ then I^{st} term = 7

If $d = -4$ then I^{st} term = 15

$T_{11} = 7 + 40 = 47$

OR $T_{11} = 15 - 40 = -25$

7. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is :-

(1) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$

(3) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4} - \log_e 2$

Official Ans. by NTA (1)

Sol. $I = \int_0^1 x\tan\left(\frac{1}{1+x^2(x^2-1)}\right)dx$

$I = \int_0^1 x\left(\tan^{-1}x^2 - \tan^{-1}(x^2-1)\right)dx$

$x^2 = t \Rightarrow 2xdx = dt$

$I = \frac{1}{2}\int_0^1\left(\tan^{-1}t - \tan^{-1}(t-1)\right)dx$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt - \frac{1}{2}\int_0^1 \tan^{-1}(t-1)\,dt$

$= \frac{1}{2}\int_0^1 \tan^{-1}t\,dt - \frac{1}{2}\int_0^1 \tan^{-1}dt = \int_0^1 \tan^{-1}dt$

$\tan^{-1}t = \theta \Rightarrow t = \tan\theta$

$dt = \sec^2\theta d\theta$

$\int_0^{\pi/4} \theta\cdot\sec^2\theta\,d\theta$

$I = (\theta.\tan\theta)\Big|_0^{\pi/4} - \int_0^{\pi/4}\tan\theta\,d\theta$

$= \left(\frac{\pi}{4} - 0\right) - \ln(\sec\theta)\Big|_0^{\pi/4}$

$= \frac{\pi}{4} - \left(\ell n\sqrt{2} - 0\right)$

$= \frac{\pi}{4} - \frac{1}{2}\ell n 2$

8. The value of sin 10º sin30º sin50º sin70º is :-

(1) $\frac{1}{36}$ (2) $\frac{1}{32}$

(3) $\frac{1}{18}$ (4) $\frac{1}{16}$

Official Ans. by NTA (4)

Sol. (sin 10º sin 30º sin 70º) sin 30º

$$\frac{1}{4}(\sin 30^\circ)^2 = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$$

9. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then:-

(1) 5Im(ω) < 1 (2) 4Im(ω) > 5

(3) 5Re(ω) > 1 (4) 5Re(ω) > 4

Official Ans. by NTA (3)

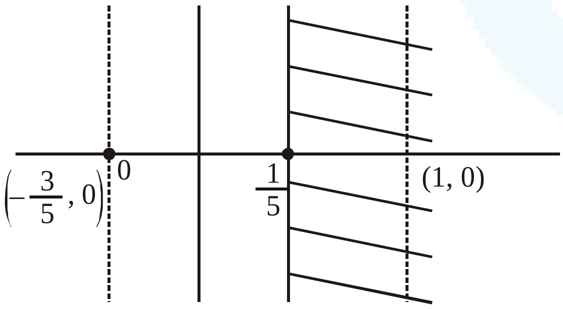
Sol. $|z| < 1$

$5\omega(1 - z) = 5 + 3z$

$5\omega - 5\omega z = 5 + 3z$

$$z = \frac{5\omega - 5}{3+5\omega}$$

$$|z| = 5\left|\frac{\omega-1}{3+5\omega}\right| < 1$$



$5|\omega - 1| < |3 + 5\omega|$

$$5|\omega-1| < 5\left|\omega+\frac{3}{5}\right|$$

$$|\omega-1| < 5\left|\omega-\left(-\frac{3}{5}\right)\right|$$

10. If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :-

(1) 964 (2) 625

(3) 227 (4) 232

Official Ans. by NTA (4)

Sol. $$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$$

$$\frac{r}{n-r+1} = \frac{2}{15}$$

$15r = 2n - 2r + 2$

$$\boxed{17r = 2n+2}$$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$$

$$\frac{r+1}{n-r} = \frac{3}{14}$$

$14r + 14 = 3n - 3r$

$$\begin{array}{l} 3n - 17r = 14 \\ 2n - 17r = -2 \\ \hline n = 16 \end{array}$$

$17r = 34$, $r = 2$

${}^{16}C_1$, ${}^{16}C_2$, ${}^{16}C_3$

$$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$$

$$\frac{680+16}{3} = \frac{696}{3} = 232$$

11. If $\cos x \dfrac{dy}{dx} - y\sin x = 6x, (0 < x < \dfrac{\pi}{2})$ and $y\left(\dfrac{\pi}{3}\right) = 0$, then $y\left(\dfrac{\pi}{6}\right)$ is equal to :-

(1) $-\dfrac{\pi^2}{4\sqrt{3}}$ (2) $-\dfrac{\pi^2}{2}$

(3) $-\dfrac{\pi^2}{2\sqrt{3}}$ (4) $\dfrac{\pi^2}{2\sqrt{3}}$

Official Ans. by NTA (3)

Sol. $\dfrac{dy}{dx} - y\tan x = 6x\sec x$

$y\left(\dfrac{\pi}{3}\right) = 0 ; y\left(\dfrac{\pi}{6}\right) = ?$

$e^{\int pdx} = e^{-\int \tan x dx} = e^{\ell n \cos x} = \cos x$

$y \cdot \cos x = \int 6x \sec x \cos x \, dx$

$y.\cos x = \dfrac{6x^2}{2} + C$

$y = 3x^2 \sec x + C \sec x$

$0 = 3\cdot\dfrac{\pi^2}{9}\cdot(2) + C(2)$

$2C = \dfrac{-2\pi^2}{3} \Rightarrow \boxed{C = -\dfrac{\pi^2}{3}}$

$y(\pi/6) = 3\cdot\dfrac{\pi^2}{36}\cdot\left(\dfrac{2}{\sqrt{3}}\right) + \left(\dfrac{2}{\sqrt{3}}\right)\cdot\left(-\dfrac{\pi^2}{3}\right)$

$\Rightarrow y = -\dfrac{\pi^2}{2\sqrt{3}}$

12. If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1 (a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is :-

(1) $\dfrac{2}{5}$ (2) $\dfrac{2}{\sqrt{5}}$ (3) $\dfrac{\sqrt{2}}{5}$ (4) $\sqrt{\dfrac{2}{5}}$

Official Ans. by NTA (4)

Sol. $\left(\dfrac{-1}{a-1}\right)\left(\dfrac{-2}{a^2}\right) = -1$

$2 = -(a^2)(a - 1)$

$a^3 - a^2 + 2 = 0$

$(a + 1)(a^2 - 2a + 2) = 0$

$\therefore a = -1$

$\left.\begin{array}{l} L_1 : x - 2y + 1 = 0 \\ L_2 : 2x + y - 1 = 0 \end{array}\right\}$

$0(0, 0) \quad P\left(\dfrac{1}{5}, \dfrac{3}{5}\right)$

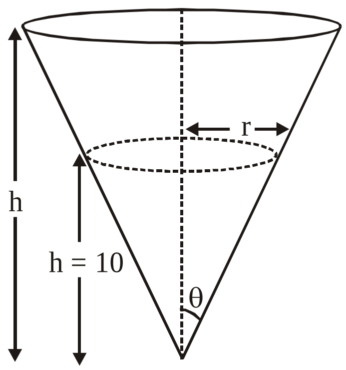
$OP = \sqrt{\dfrac{1}{25} + \dfrac{9}{25}} = \sqrt{\dfrac{10}{25}} = \sqrt{\dfrac{2}{5}}$

13. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\dfrac{1}{2}\right)$. Water is poured into it at a constant rage of 5 cubic meter per minute. The the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10m; is :-

(1) $2/\pi$ (2) $1/5\pi$ (3) $1/10\pi$ (4) $1/15\pi$

Official Ans. by NTA (2)

Sol. $\tan\theta = \dfrac{1}{2} = \dfrac{r}{h}$



$r = \dfrac{h}{2}$

$V = \dfrac{1}{3}\pi r^2 h$

$V = \dfrac{1}{3}\pi.\dfrac{h^3}{4}$

$\dfrac{dV}{dt} = \dfrac{\pi}{12}(3h)^2\left(\dfrac{dh}{dt}\right)$

$5 = \dfrac{\pi}{4}.(100)\dfrac{dh}{dt} \Rightarrow \dfrac{dh}{dt} = \dfrac{1}{5\pi}$

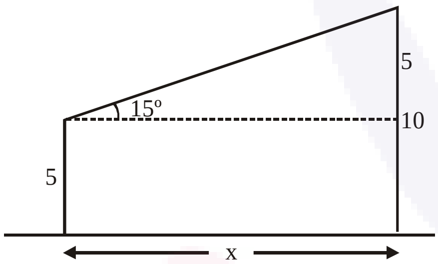
14. Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15° with ground. Then the distance (in m) between the poles, is :-

(1) $\frac{5}{2}(2+\sqrt{3})$ (2) $5(\sqrt{3}+1)$

(3) $5(2+\sqrt{3})$ (4) $10(\sqrt{3}-1)$

Official Ans. by NTA (3)

Sol. $\tan 15^\circ = \frac{5}{x}$



$2-\sqrt{13} = \frac{5}{x}$

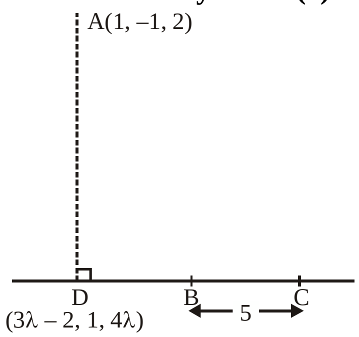
$x = 5(2+\sqrt{3})$

15. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is :-

(1) $2\sqrt{34}$ (2) $\sqrt{34}$ (3) 6 (4) $5\sqrt{17}$

Official Ans. by NTA (2)

Sol.



$\overrightarrow{AD}\cdot(3\hat{i}+4\hat{k}) = 0$

$3(3\lambda - 3) + 0 + 4(4\lambda - 2) = 0$

$(9\lambda - 9) + (16\lambda - 8) = 0$

$25\lambda = 17 \Rightarrow \lambda = \frac{17}{25}$

$\therefore\ \overrightarrow{AD} = \left(\frac{51}{25}-3\right)\hat{i} + 2\hat{j} + \left(\frac{68}{25}-2\right)\hat{k}$

$= \frac{24}{25}\hat{i} + 2\hat{j} + \frac{18}{25}\hat{k}$

$|\overrightarrow{AD}| = \sqrt{\frac{576}{625}+4+\frac{324}{625}}$

$= \sqrt{\frac{900}{625}+4} = \sqrt{\frac{3400}{625}}$

$= \sqrt{34}\cdot\frac{10}{25} = \frac{2}{5}\sqrt{34}$

Area of $\Delta = \frac{1}{2}\times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$

16. The total number of matrices

$A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}, (x, y \in R, x \neq y)$ for which

$A^TA = 3I_3$ is :-

(1) 6 (2) 2 (3) 3 (4) 4

Official Ans. by NTA (4)

Sol. $A^TA = 3I_3$

$$\begin{pmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$8x^2 = 3$

$6y^2 = 3$

$x^2 = 3/8$

$y^2 = 1/2$

$x = \pm\sqrt{\frac{3}{8}}; y = \pm\sqrt{\frac{1}{2}}$

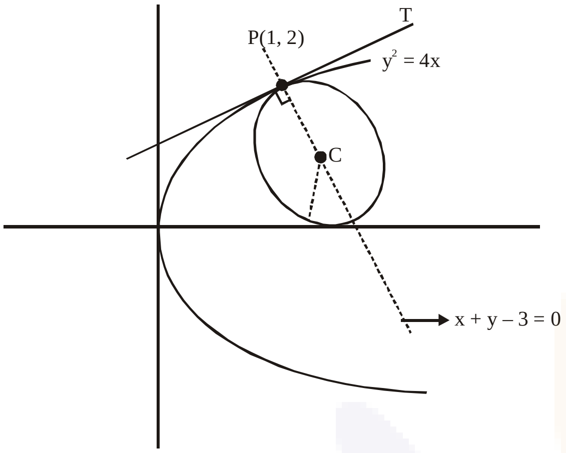
17. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :-

(1) $4\pi(2-\sqrt{2})$ (2) $8\pi(3-2\sqrt{2})$

(3) $4\pi(3+\sqrt{2})$ (4) $8\pi(2-\sqrt{2})$

Official Ans. by NTA (2)

Sol.



Equation of circle is

$(x-1)^2 + (y-2)^2 + \lambda(x-y+1) = 0$

$\Rightarrow x^2 + y^2 + x(\lambda-2) + y(-4-\lambda) + (5+\lambda) = 0$

As cirlce touches x axis then $g^2 - c = 0$

$$\frac{(\lambda-2)^2}{4} = (5+\lambda)$$

$\lambda^2 + 4 - 4\lambda = 20 + 4\lambda$

$\lambda^2 - 8\lambda - 16 = 0$

$$\lambda = \frac{8 \pm \sqrt{128}}{2}$$

$$\lambda = 4 \pm 4\sqrt{2}$$

$$\text{Radius} = \left|\frac{(-4-\lambda)}{2}\right|$$

Put λ and get least radius.

18. If the function $f(x) = \begin{cases} a|\pi - x| + 1, x \le 5 \\ b|x-\pi| + 3, x > 5 \end{cases}$ is continuous at x = 5, then the value of a – b is :-

(1) $\frac{2}{5-\pi}$ (2) $\frac{2}{\pi-5}$

(3) $\frac{2}{\pi+5}$ (4) $\frac{-2}{\pi+5}$

Official Ans. by NTA (1)

Sol.

$$f(x) \begin{cases} a|\pi - x| + 1; x \ge 5 \\ b|\pi - x| + 3; x > 5 \end{cases}$$

$a|\pi - 5| + 1 = b|5 - \pi| + 3$

$a(5 - \pi) + 1 = b(5 - \pi) + 3$

$(a - b)(5 - \pi) = 2$

$$a - b = \frac{2}{5-\pi}$$

19. If $f(x) = [x] - \left[\frac{x}{4}\right], x \in R$, where [x] denotes the greatest integer function, then :

(1) Both $\lim\limits_{x\to 4-} f(x)$ and $\lim\limits_{x\to 4+} f(x)$ exist but are not equal

(2) $\lim\limits_{x\to 4-} f(x)$ exists but $\lim\limits_{x\to 4+} f(x)$ does not exist

(3) $\lim\limits_{x\to 4+} f(x)$ exists but $\lim\limits_{x\to 4-} f(x)$ does not exist

(4) f is continuous at x = 4

Official Ans. by NTA (4)

Sol. $f(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x\to 4+} f(x) = \lim_{x\to 4+}\left(\left([x] - \left[\frac{x}{4}\right]\right)\right) = 4 - 1 = 3$$

$$\lim_{x\to 4-} f(x) = \lim_{x\to 4-}\left([x] - \frac{x}{4}\right) = 3 - 0 = 3$$

f(x) = 3

$\therefore$ continuous at x = 4

20. If $\int e^{\sec x}(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)dx = e^{\sec x} f(x) + C$, then a possible choice of f(x) is :-

(1) $\sec x - \tan x - \frac{1}{2}$ (2) $x \sec x + \tan x + \frac{1}{2}$

(3) $\sec x + x \tan x - \frac{1}{2}$ (4) $\sec x + \tan x + \frac{1}{2}$

Official Ans. by NTA (4)

Sol. $\int e^{\sec x}(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)dx$

$= e^{\sec x} f(x) + C$

Diff. both sides w.r.t. 'x'

$e^{\sec x}(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x))$

$= e^{\sec x} \cdot \sec x \tan x f(x) + e^{\sec x} f'(x)$

$f'(x) = \sec^2 x + \tan x \sec x$

$\Rightarrow f(x) = \tan x + \sec x + c$

21. If m is chosen in the quadratic equation $(m^2 + 1) x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :-

(1) $8\sqrt{3}$ (2) $4\sqrt{3}$

(3) $10\sqrt{5}$ (4) $8\sqrt{5}$

Official Ans. by NTA (4)

Sol. $SOR = \frac{3}{m^2+1} \Rightarrow (S.O.R)_{max} = 3$

when m = 0

$x^2 - 3x + 1 = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha + \beta = 3$

$\alpha\beta = 1$

$|\alpha^3 - \beta^2| = ||\alpha - \beta|(\alpha^2 + \beta^2 + \alpha\beta)|$

$= \left|\sqrt{(\alpha-\beta)^2 - \alpha\beta} \quad ((\alpha+\beta)^2 - \alpha\beta)\right|$

$= \left|\sqrt{9-4}\,(9-1)\right|$

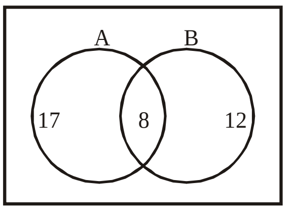
$= \sqrt{5} \times 8$

22. Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :-

(1) 12.8 (2) 13.5 (3) 13.9 (4) 13

Official Ans. by NTA (3)

Sol. Let population = 100



n(A) = 25

n(B) = 20

$n(A \cap B) = 8$

$n(A \cap \bar{B}) = 17$

$n(\bar{A} \cap B) = 12$

$\frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$

5.1 + 4.8 + 4 = 13.9

23. Let P be the plane, which contains the line of intersection of the planes, x + y + z – 6 = 0 and 2x + 3y + z + 5 = 0 and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :-

(1) $63\sqrt{5}$ (2) $205\sqrt{5}$

(3) $17/\sqrt{5}$ (4) $11/\sqrt{5}$

Official Ans. by NTA (4)

Sol. $\lambda(x + y + z - 6) + 2x + 3y + z + 5 = 0$

$(\lambda + 2)x + (\lambda + 3)y + (\lambda + 1)z + 5 - 6\lambda = 0$

$\lambda + 1 = 0 \Rightarrow \lambda = -1$

$P : x + 2y + 11 = 0$

perpendicular distance $= \frac{11}{\sqrt{5}}$

24. If $P \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively :-

(1) F, T, T (2) T, F, F

(3) T, T, F (4) F, F, F

Official Ans. by NTA (2)

Sol. $P \Rightarrow (q \vee r)$: F

P : T $q \vee r$: F

P : T : q : F : r : F

25. The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :-

(1) $(1, 2) \cup (2, \infty)$

(2) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

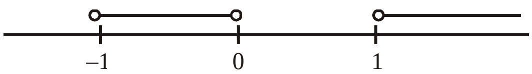
(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

Official Ans. by NTA (3)

Sol. $4 - x^2 \neq 0$; $x^3 - x > 0$

$x = \pm 2$ $\quad x(x - 1)(x + 1) > 0$



$\therefore D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

26. The sum of the series 1 + 2 × 3 + 3 × 5+4 × 7 +.... upto 11th term is :-

(1) 915 (2) 946 (3) 945 (4) 916

Official Ans. by NTA (2)

Sol. $T_r = r(2r - 1)$

$S = \Sigma 2r^2 - \Sigma r$

$S = \frac{2.n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$

$S_{11} = \frac{2}{6}.(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946$

27. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :-

(1) 7/3 (2) 9/4 (3) 7/2 (4) 8/3

Official Ans. by NTA (1)

Sol. $\frac{34+x}{2}=35$

$x = 36$

$$42=\frac{10+22+26+29+34+36+42+67+70+y}{10}$$

$420 - 336 = y \Rightarrow y = 84$

$\frac{y}{x}=\frac{84}{36}=\frac{7}{3}$

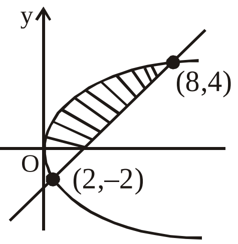
28. The area (in sq. units) of the region

$A=\{(x,y):\frac{y^2}{2}\le x\le y+4\}$ is :-

(1) $\frac{53}{3}$ (2) 18 (3) 30 (4) 16

Official Ans. by NTA (2)

Sol.



$y^2 = 2x$

$x - y - 4 = 0$

$(x - 4)^2 = 2x$

$x^2 + 16 - 8x - 2x = 0$

$x^2 - 10x + 16 = 0$

$x = 8, 2$

$y = 4, -2$

$$A=\int_{-2}^{4}\left(y+4-\frac{y^2}{2}\right)dy$$

$$=\left.\frac{y^2}{2}\right|_{-2}^{4}+\left.4y\right|_{-2}^{4}-\left.\frac{y^3}{6}\right|_{-2}^{4}$$

$$=(8-2)+4(6)-\frac{1}{6}(64+8)$$

$= 6 + 24 - 12 = 18$

29. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and $\theta\in(0,\pi)$ with $\hat{k}$, then a value of θ is :-

(1) $\frac{5\pi}{12}$ (2) $\frac{5\pi}{6}$ (3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{4}$

Official Ans. by NTA (3)

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{1}{4}+\frac{1}{2}+\cos^2\gamma=1$

$\cos^2\gamma=1-\frac{3}{4}=\frac{1}{4}$

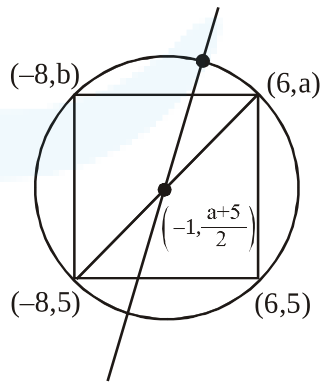
$\cos^2\gamma=\pm\frac{1}{2}\Rightarrow\gamma=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

30. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :-

(1) 72 (2) 84 (3) 98 (4) 56

Official Ans. by NTA (2)

Sol.



$\frac{3(a+5)}{2}=-1+7$

$a+5=\frac{2(6)}{3}$

$a = -1$

sides = 6 and 14

$\Rightarrow$ A = 84