FINAL JEE–MAIN EXAMINATION – MARCH, 2021
Held On Wednesday 17th March, 2021
TIME: 9:00 AM to 12:00 NOON

SECTION-A

1. The inverse of $y = 5^{\log x}$ is :

(1) $x = 5^{\log y}$ (2) $x = y^{\log 5}$

(3) $x = y^{\frac{1}{\log 5}}$ (4) $x = 5^{\frac{1}{\log y}}$

Official Ans. by NTA (3)

Allen Ans. (1 or 2 or 3)

Sol. Given $y = 5^{(\log_a x)} = f(x)$

Interchanging x & y for inverse

$x = 5^{(\log_a y)} = y^{(\log_a 5)}$

option (1) or option (2)

Further, from given relation

$\log_5 y = \log_a x$

$\Rightarrow x = a^{(\log_5 y)} = y^{(\log_5 a)}$

$\Rightarrow x = y^{\left(\frac{1}{\log_a 5}\right)} = f^{-1}(y)$

option (3)

2. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}, \vec{r} \cdot \left(\hat{i} + 2\hat{j} + \hat{k}\right) = -3$, then $\vec{r} \cdot \left(2\hat{i} - 3\hat{j} + \hat{k}\right)$

is equal to :

(1) 12 (2) 8

(3) 13 (4) 10

Official Ans. by NTA (1)

Sol. $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r} \times (\vec{a} - \vec{b}) = 0$

$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b})$

$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

Also $\vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$= \vec{r}.(2\hat{i} - 3\hat{j} + \hat{k})$

$= -10 + 12 + 10 = 12$

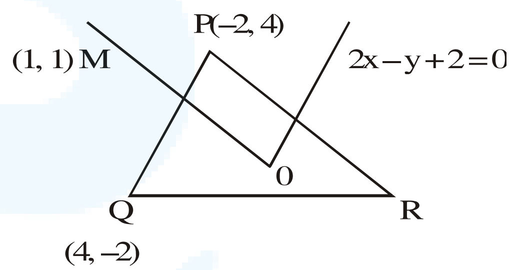
3. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively. If the equation of the perpendicular bisector of PR is 2x – y + 2 = 0, then the centre of the circumcircle of the ΔPQR is :

(1) (–1, 0) (2) (–2, –2)

(3) (0, 2) (4) (1, 4

Official Ans. by NTA (2)

Sol.



Equation of perpendicular bisector of PR is

y = x

Solving with 2x – y + 2 = 0 will give

(–2, 2)

4. The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution if k is equal to :

(1) 0 (2) 1 (3) –1 (4) –2

Official Ans. by NTA (4)

Sol. kx + y + z = 1

x + ky + z = k

x + y + zk = k^2

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K(K^2 - 1) - 1(K - 1) + 1(1 - K)$$

$= K^3 - K - K + 1 + 1 - K$

$= K^3 - 3K + 2$

$= (K - 1)^2 (K + 2)$

For $K = 1$

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for $K = -2$, at least one out of Δ_1, Δ_2, Δ_3 are not zero

Hence for no sol^n, $K = -2$

5. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + ...$ upto 100 terms, then α is :

(1) 1.01 (2) 1.00

(3) 1.02 (4) 1.03

Official Ans. by NTA (1)

Sol. $\text{Cot}^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18) + ...$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$= \tan^{-1} 201 - \tan^{-1} 1$

$$= \tan^{-1}\left(\frac{200}{202}\right)$$

$$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$$

$\alpha = 1.01$

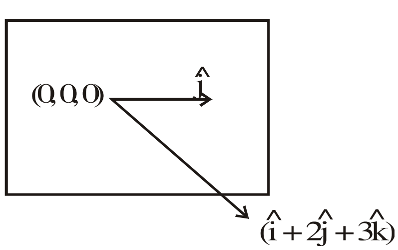
6. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is :

(1) $x + 3z = 10$ (2) $x + 3z = 0$

(3) $3x + z = 6$ (4) $3x - z = 0$

Official Ans. by NTA (4)

Sol.



$$\vec{n} = \hat{j} \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -3\hat{i} + 0\hat{j} + \hat{k}$$

So, $(-3)(x - 1) + 0(y - 2) + (1)(z - 3) = 0$

$\Rightarrow -3x + z = 0$

Option 4

Alternate :

Required plane is

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 3x - z = 0$

7. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Official Ans. by NTA (3)

Sol. $A^2 = \sin^2\alpha\ I$

So, $\left|A^2 - \frac{I}{2}\right| = \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{2}}$

8. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $\sim q \Rightarrow p$

(3) $p \Rightarrow \sim q$ (4) $p \Rightarrow q$

Official Ans. by NTA (1)

Sol. $\because p \to q \equiv \sim p \vee q$

So, $* \equiv \vee$

Thus, $p * (\sim q) \equiv p \vee (\sim q)$

$\equiv q \to p$

9. Two dices are rolled. If both dices have six faces numbered 1,2,3,5,7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

(1) $\frac{4}{9}$ (2) $\frac{17}{36}$ (3) $\frac{5}{12}$ (4) $\frac{1}{2}$

Official Ans. by NTA (2)

Sol. n(E) = 5 + 4 + 4 + 3 + 1 = 17

So, $P(E) = \frac{17}{36}$

10. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where $x \in N$ is equal to :

(1) 2

(2) 4

(3) 3

(4) 1

Official Ans. by NTA (1)

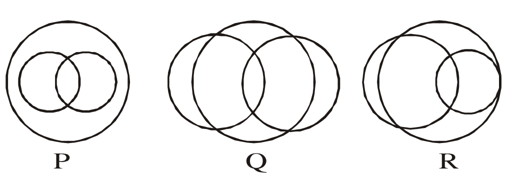
Sol. $^7C_3 x^4 \; x^{(3\log_2^x)} = 4480$

$\Rightarrow \; x^{(4+3\log_2^x)} = 2^7$

$\Rightarrow \; (4+3t)t = 7; t = \log_2^x$

$\Rightarrow \; t = 1, \frac{-7}{3} \Rightarrow x = 2$

11. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?



(1) P and Q

(2) P and R

(3) None of these

(4) Q and R

Official Ans. by NTA (3)

Sol. $A \cap B \cap C$ is visible in all three venn diagram

Hence, Option (3)

12. The sum of possible values of x for $\tan^{-1}(x + 1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ is :

(1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$

(3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

Official Ans. by NTA (1)

Sol. $\tan^{-1}(x + 1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\frac{8}{31}$

Taking tangent both sides :-

$\frac{(x+1)+(x-1)}{1-(x^2-1)} = \frac{8}{31}$

$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$

$\Rightarrow \; 4x^2 + 31x - 8 = 0$

$\Rightarrow \; x = -8, \frac{1}{4}$

But, if $x = \frac{1}{4}$

$\tan^{-1}(x+1) \in \left(0, \frac{\pi}{2}\right)$

& $\cot^{-1}\left(\frac{1}{x-1}\right) \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow$ LHS $> \frac{\pi}{2}$ & RHS $< \frac{\pi}{2}$

(Not possible)

Hence, x = – 8

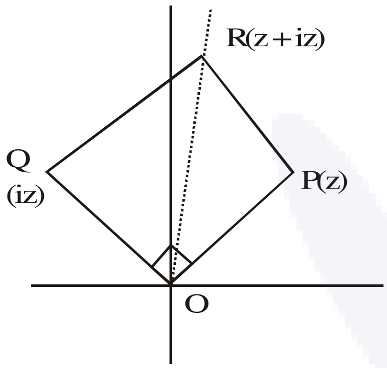
13. The area of the triangle with vertices A(z), B(iz) and C (z + iz) is :

(1) 1 (2) $\frac{1}{2}|z|^2$

(3) $\frac{1}{2}$ (4) $\frac{1}{2}|z+iz|^2$

Official Ans. by NTA (2)

Sol.



$A=\frac{1}{2}|z|\,|iz|$

$=\frac{|z|^2}{2}$

14. The line 2x – y + 1 = 0 is a tangent to the circle at the point (2, 5) and the centre of the circle lies on x – 2y = 4. Then, the radius of the circle is:

(1) $3\sqrt{5}$

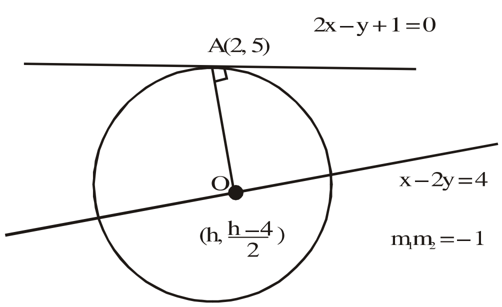
(2) $5\sqrt{3}$

(3) $5\sqrt{4}$

(4) $4\sqrt{5}$

Official Ans. by NTA (1)

Sol.



$$\left(\frac{h-\frac{(h-4)}{2}}{2-h}\right)(2)=-1$$

h = 8

center (8, 2)

$$\text{Radius}=\sqrt{(8-2)^2+(2-5)^2}=3\sqrt{5}\ )$$

15. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

(1) 5 (2) 2 (3) 4 (4) 6

Official Ans. by NTA (3)

Sol. Total matches between boys of both team

$= {}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both

team $= {}^nC_1\ {}^6C_1 = 6n$

Now, 28 + 6n = 52

$\Rightarrow$ n = 4

16. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\ldots\infty}}}}$ is :

(1) $2+\frac{2}{5}\sqrt{30}$ (2) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(3) $4+\frac{4}{\sqrt{5}}\sqrt{30}$ (4) $5+\frac{2}{5}\sqrt{30}$

Official Ans. by NTA (1)

Sol. $y=4+\cfrac{1}{\left(5+\cfrac{1}{y}\right)}$

$$y-4=\frac{y}{(5y+1)}$$

$5y^2 - 20y - 4 = 0$

$$y=\frac{20+\sqrt{480}}{10}$$

$$y=\frac{20-\sqrt{480}}{10}\rightarrow \text{rejected}$$

$$y=2+\sqrt{\frac{480}{100}}$$

Correct with Option (A)

17. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles.

(2) Both circles' centres lie inside region of one another.

(3) Both circles pass through the centre of each other.

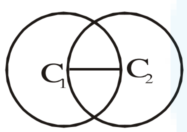
(4) Circles have two intersection points.

Official Ans. by NTA (2)

Sol. $r_1 = 3,\ c_1\ (5,\ 5)$

$r_2 = 3,\ c_2\ (8,\ 5)$

$C_1C_2 = 3,\ r_1 = 3,\ r_2 = 3$



18. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in R$ such that

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^{\alpha}x}{\cos^{\alpha}x+\sin^{\alpha}x}dx$$

(1) $g(\alpha)$ is a strictly increasing function

(2) $g(\alpha)$ has an inflection point at $\alpha = -\frac{1}{2}$

(3) $g(\alpha)$ is a strictly decreasing function

(4) $g(\alpha)$ is an even function

Official Ans. by NTA (4)

Allen Answer (1 or 2 or 3/Bonus)

Sol. $g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3}\frac{\sin^{\alpha}x}{(\sin^{\alpha}x+\cos^{\alpha}x)}$ .....(i)

$g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3}\frac{\cos^{\alpha}x}{(\sin^{\alpha}x+\cos^{\alpha}x)}$ .....(ii)

(1) + (2)

$2g\ (\alpha) = \frac{\pi}{6}$

$g(\alpha)=\frac{\pi}{12}$

Constant and even function

Due to typing mistake it must be bonus.

19. Which of the following is true for y(x) that satisfies the differential equation

$\frac{dy}{dx}=xy-1+x-y\ ;\ y(0)=0:$

(1) $y(1)=e^{-\frac{1}{2}}-1$

(2) $y(1)=e^{\frac{1}{2}}-e^{-\frac{1}{2}}$

(3) $y(1) = 1$

(4) $y(1)=e^{\frac{1}{2}}-1$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}=(1+y)(x-1)$

$\frac{dy}{(y+1)}=(x-1)dx$

Integrate $\ln(y + 1) = \frac{x^2}{2}-x+c$

$(0,0) \Rightarrow c = 0 \Rightarrow\ y=e^{\left(\frac{x^2}{2}-x\right)}-1$

20. The value of

$$\lim_{x\to 0^+}\frac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3},$$ where

[x] denotes the greatest integer $\le x$ is :

(1) π

(2) 0

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{2}$

Official Ans. by NTA (4)

Sol. $\lim_{x\to 0^+}\frac{\cos^{-1}x}{(1-x^2)}\times\frac{\sin^{-1}x}{x}=\frac{\pi}{2}$

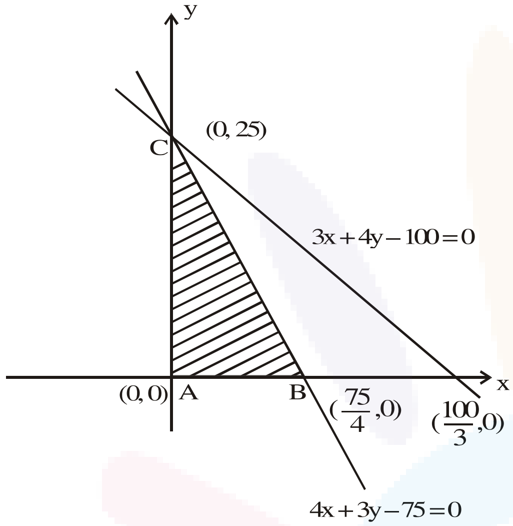
SECTION-B

1. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \le 100$ and $4x + 3y \le 75$ for $x \ge 0$ and $y \ge 0$ is _______ .

Official Ans. by NTA (904)

Allen Answer (904 or 904.01 or 904.02)

Sol.



$z = 6xy + y^2 = y(6x + y)$

$3x + 4y \le 100$ .....(i)

$4x + 3y \le 75$ ......(ii)

$x \ge 0$

$y \ge 0$

$x \le \frac{75-3y}{4}$

$Z = y(6x + y)$

$Z \le y\left(6.\left(\frac{75-3y}{4}\right)+y\right)$

$Z \le \frac{1}{2}(225y-7y^2) \le \frac{(225)^2}{2\times4\times7}$

$= \frac{50625}{56}$

≈ 904.0178

≈ 904.02

It will be attained at $y = \frac{225}{14}$

2. If the function $f(x) = \frac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is _________ .

Official Ans. by NTA (6)

Sol. $\lim_{x\to0}\frac{\cos(\sin x)-\cos x}{x^4} = f(0)$

$\Rightarrow \lim_{x\to0}\frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4} = \frac{1}{K}$

$\Rightarrow \lim_{x\to0} 2\left(\frac{\sin x + x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right) = \frac{1}{K}$

$\Rightarrow 2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6} = \frac{1}{K}$

$\Rightarrow K = 6$

3. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is _______ .

Official Ans. by NTA (481)

Sol. $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ at x= 1 ; $2^{2x} = 4$

for $\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$;

Let $\tan^{-1} x = \theta$; $\theta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore \sin\left(\cos^{-1}\cos 2\theta\right) = \sin 2\theta$

$$\left\{\begin{array}{l}\text{If} \quad x>1 \Rightarrow \frac{\pi}{2}>\theta>\frac{\pi}{4} \\ \therefore \quad \pi>2\theta>\frac{\pi}{2}\end{array}\right\}$$

$$=2\sin\theta\cos\theta=\frac{2\tan\theta}{1+\tan^2\theta}$$

$$=\frac{2x}{1+x^2}$$

Hence, $f(x) = \frac{2\cdot 2^x}{1+2^{2x}}$

$$\therefore \; f'(x)=\frac{(1+2^{2x})(2.2^x\ln 2)-2^{2x}\cdot 2\cdot\ln 2\cdot 2\cdot 2^x}{(1+2^{2x})}$$

$$\therefore \; f^1(1)=\frac{20\ln 2-32\ln 2}{25}=-\frac{12}{25}\ln 2$$

So, $a = 25$, $b = 12 \Rightarrow |a^2 - b^2| = 25^2 - 12^2$

$= 625 - 144$

$= 481$

4. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to _____.

Official Ans. by NTA (6)

Sol. Let $P(E_1) = P_1$; $P(E_2) = P_2$; $P(E_3) = P_3$

$$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3)=\alpha=P_1(1-P_2)(1-P_3)......(1)$$

$$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3)=\beta=(1-P_1)P_2(1-P_3)......(2)$$

$$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3)=\gamma=(1-P_1)(1-P_2)P_3......(3)$$

$$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3)=P=(1-P_1)(1-P_2)(1-P_3)......(4)$$

Given that, $(\alpha - 2\beta)\, P = \alpha\beta$

$\Rightarrow (P_1(1-P_2)(1-P_3)-2(1-P_1)P_2(1-P_3))P=P_1P_2(1-P_1)(1-P_2)(1-P_3)^2$

$\Rightarrow (P_1(1-P_2) - 2(1-P_1)P_2) = P_1P_2$

$\Rightarrow (P_1 - P_1P_2 - 2P_2 + 2P_1P_2) = P_1P_2$

$\Rightarrow P_1 = 2P_2$ ......(1)

and similarly, $(\beta - 3\gamma)P = 2B\gamma$

$P_2 = 3P_3$ ......(2)

So, $P_1 = 6P_3 \Rightarrow \boxed{\frac{P_1}{P_3}=6}$

5. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

$\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

$\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

such that $\vec{a}\cdot\vec{b}=1$ and $\vec{b}\cdot\vec{c}=-3$, then

$\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to _______.

Official Ans. by NTA (2)

Sol. $\vec{a}.\vec{b}=1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow -2\alpha\beta = 4 \Rightarrow \boxed{\alpha\beta=-2}$ ....(1)

$\vec{b}.\vec{c}=-3 \Rightarrow -\beta+2\alpha+1=-3$

$\boxed{\beta-2\alpha=4}$ ....(2)

Solving (1) & (2), $(\alpha,\beta) = (-1, 2)$

$$\frac{1}{3}[\vec{a}\;\vec{b}\;\vec{c}]=\frac{1}{3}\begin{vmatrix}\alpha & \beta & 3\\ -\beta & -\alpha & -1\\ 1 & -2 & -1\end{vmatrix}$$

$$=\frac{1}{3}\begin{vmatrix}-1 & 2 & 3\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}$$

$$=\frac{1}{3}\begin{vmatrix}0 & 0 & 2\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}=\frac{1}{3}[2(4-1)]=2$$

6. If $A=\begin{bmatrix}2 & 3\\0 & -1\end{bmatrix}$, then the value of $\det(A^4)+\det\left(A^{10}-(\text{Adj}(2A))^{10}\right)$ is equal to ________ .

Official Ans. by NTA (16)

Sol. $2A\ \text{adj}\ (2A) = |2A|I$

$\Rightarrow A\ \text{adj}\ (2A) = -4I$ ....(i)

Now, $E = |A^4| + |A^{10} - (\text{adj}(2A))^{10}|$

$= (-2)^4 + \frac{|A^{20}-A^{10}(\text{adj } 2A)^{10}|}{|A|^{10}}$

$= 16 + \frac{|A^{20}-(A\,\text{adj}(2A))^{10}|}{|A|^{10}}$

$= 16 + \frac{|A^{20}-2^{10}I|}{2^{10}}$ (from (1))

Now, characteristic roots of A are 2 and –1.

So, characteristic roots of A^{20} are 2^{10} and 1.

Hence, $(A^{20} - 2^{10} I)(A^{20} - I) = 0$

$\Rightarrow |A^{20} - 2^{10}I| = 0$ (as $A^{20} \neq I$)

$\Rightarrow E = 16$ Ans.

7. If $[\cdot]$ represents the greatest integer function, then the value of

$$\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[\left[x^2\right]-\cos x\right]dx\right|$$ is ________ .

Official Ans. by NTA (1)

Sol. $I = \int_0^{\sqrt{\pi/2}}\left([x^2]+[-\cos x]\right)dx$

$= \int_0^1 0\,dx + \int_1^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}}(-1)\,dx$

$= \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} = -1$

$\Rightarrow |I| = 1$

8. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

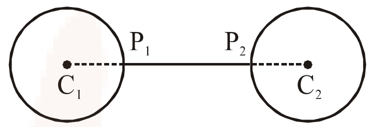
$x^2 + y^2 - 24x - 10y + 160 = 0$ is ______ .

Official Ans. by NTA (1)

Sol. Given $C_1(5, 5)$, $r_1 = 3$ and $C_2(12, 5)$, $r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{min} = 7 - 6 = 1$



9. If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point (–2, 1, 3) is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is _____ .

Official Ans. by NTA (4)

Sol. Required plane is

$p_1 + \lambda p_2 = (2 + 3\lambda)x - (7 + 5\lambda)y + (4 + 4\lambda)z - 3 + 11\lambda = 0$;

which is satisfied by (–2, 1, 3).

Hence, $\lambda = \frac{1}{6}$

Thus, plane is $15x - 47y + 28z - 7 = 0$

So, $2a + b + c - 7 = 4$

10. If $(2021)^{3762}$ is divided by 17, then the remainder is _______ .

Official Ans. by NTA (4)

Sol. $(2023 - 2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$ (as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17 - 1)^{940}$

$= 17k_2 + 4(17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder $= 4$