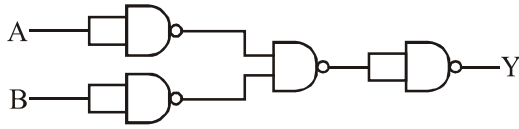




FINAL JEE–MAIN EXAMINATION – MARCH, 2021
Held On Tuesday 16th March, 2021
TIME: 3:00 PM to 06:00 PM

SECTION-A

1. The following logic gate is equivalent to :



- (1) NOR Gate (2) OR Gate
 (3) AND Gate (4) NAND Gate

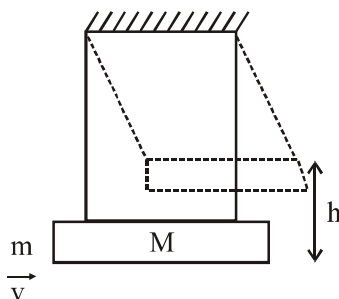
Official Ans. by NTA (1)

Sol. Truth table for the given logic gate :

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

The truth table is similar to that of a NOR gate.

2. A large block of wood of mass $M = 5.99 \text{ kg}$ is hanging from two long massless cords. A bullet of mass $m = 10\text{g}$ is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h = 9.8 \text{ cm}$ before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is : (Take $g = 9.8 \text{ ms}^{-2}$)



- (1) 841.4 m/s (2) 811.4 m/s
 (3) 831.4 m/s (4) 821.4 m/s

Official Ans. by NTA (3)

Sol. From energy conservation,

[after bullet gets embedded till the system comes momentarily at rest]

$$(M + m)g h = \frac{1}{2}(M + m)v_1^2$$

[v_1 is velocity after collision]

$$\therefore v_1 = \sqrt{2gh}$$

Applying momentum conservation, (just before and just after collision)

$$mv = (M + m)v_1$$

$$v = \left(\frac{M+m}{m}\right)v_1 = \frac{6}{10 \times 10^{-3}} \times \sqrt{2 \times 9.8 \times 9.8 \times 10^{-2}}$$

$$\approx 831.55 \text{ m/s}$$

3. A charge Q is moving \vec{dl} distance in the magnetic field \vec{B} . Find the value of work done by \vec{B} .

- (1) 1 (2) Infinite
 (3) Zero (4) -1

Official Ans. by NTA (3)

Sol. Since force on a point charge by magnetic field is always perpendicular to \vec{v} [$\vec{F} = q\vec{V} \times \vec{B}$]

\therefore Work by magnetic force on the point charge is zero.

4. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min ? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s , respectively.

(Density of water : 10^3 kg/m^3)

- (1) Unsteady to steady flow
 (2) Remains steady flow
 (3) Remains turbulent flow
 (4) Steady flow to unsteady flow

Official Ans. by NTA (4)



Sol. The nature of flow is determined by Reynolds Number.

$$R_e = \frac{\rho v D}{\eta}$$

$$\left[\begin{array}{l} \rho \rightarrow \text{density of fluid} \quad ; \quad \eta \rightarrow \text{coefficient of} \\ v \rightarrow \text{velocity of flow} \quad \quad \quad \text{viscosity} \\ D \rightarrow \text{Diameter of pipe} \end{array} \right]$$

From NCERT

If $R_e < 1000 \rightarrow$ flow is steady

$1000 < R_e < 2000 \rightarrow$ flow becomes unsteady

$R_e > 2000 \rightarrow$ flow is turbulent

$$R_{e\text{initial}} = 10^3 \times \frac{0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$$= 382.16$$

$$R_{e\text{final}} = 10^3 \times \frac{0.48 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$$= 1019.09$$

5. A mosquito is moving with a velocity $\vec{v} = 0.5t^2 \hat{i} + 3t \hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s ?

(1) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

(2) $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis

(3) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(4) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

Official Ans. by NTA (2)

Official Ans. by ALLEN (Bonus)

Sol. Given :

$$\vec{v} = 0.5t^2 \hat{i} + 3t \hat{j} + 9\hat{k}$$

$$\vec{v}_{\text{at } t=2} = 2\hat{i} + 6\hat{j} + 9\hat{k}$$

\therefore Angle made by direction of motion of mosquito will be,

$$\cos^{-1} \frac{2}{11} \text{ (from x-axis)} = \tan^{-1} \frac{\sqrt{117}}{2}$$

$$\cos^{-1} \frac{6}{11} \text{ (from y-axis)} = \tan^{-1} \frac{\sqrt{85}}{6}$$

$$\cos^{-1} \frac{9}{11} \text{ (from z-axis)} = \tan^{-1} \frac{\sqrt{40}}{9}$$

None of the option is matching.

Hence this question should be bonus.

6. Find out the surface charge density at the intersection of point $x = 3$ m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.

(1) 0.424 nC m⁻² (2) 47.88 C/m

(3) 0.07 nC m⁻² (4) 4.0 nC m⁻²

Official Ans. by NTA (1)

Sol. $\frac{2K\lambda}{r} = \frac{\sigma}{\epsilon_0}$ ($x = 3$ m)

$$\sigma = 0.424 \times 10^{-9} \frac{C}{m^2}$$

7. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths ? ($m_p = 1.00727$ u, $m_e = 0.00055$ u)

(1) 1860 : 1 (2) (1860)² : 1

(3) 41.4 : 1 (4) 43 : 1

Official Ans. by NTA (4)

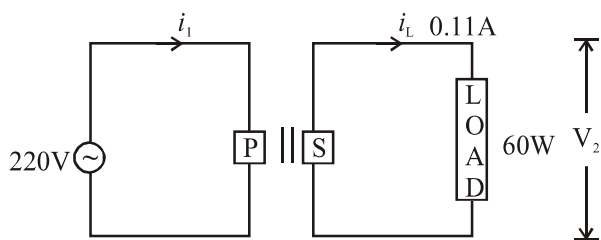
Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1831.4} = 42.79$$



8. For the given circuit, comment on the type of transformer used :



- (1) Auxilliary transformer
- (2) Auto transformer
- (3) Step-up transformer
- (4) Step down transformer

Official Ans. by NTA (3)

Sol. $V_S = \frac{P}{i} = \frac{60}{0.11} = 545.45$
 $V_P = 220$
 $V_S > V_P$
 \Rightarrow Step up transformer

9. The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol⁻¹ is, ($N_A = 6 \times 10^{23}/mol$)

- (1) 240 Ci
- (2) 357 Ci
- (3) 535 Ci
- (4) 252 Ci

Official Ans. by NTA (2)

Sol. $A = \lambda N$
 $N = nN_A$ $\left(t_{1/2} = \frac{\ln 2}{\lambda} \right)$
 $N = \left(\frac{1.5 \times 10^{-3}}{198} \right) N_A$
 $A = \left(\frac{\ln 2}{t_{1/2}} \right) N$
 1 Curie = 3.7×10^{10} Bq
 $A = 365$ Bq

10. Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k = 1.38 \times 10^{-23}$ JK⁻¹)

- (1) 58 nm
- (2) 32 nm
- (3) 86 nm
- (4) 102 nm

Official Ans. by NTA (4)

Sol. $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$
 $\lambda = 102$ nm

11. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index ? (Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively)

- (1) 1
- (2) Infinite
- (3) $\frac{R_1 R_2}{R_1 - R_2}$
- (4) Zero

Official Ans. by NTA (2)

Sol. $\frac{1}{F} = \left[\frac{\mu_L}{\mu_S} - 1 \right] \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$

If $\mu_L = \mu_S \Rightarrow \frac{1}{F} = 0 \Rightarrow F = \infty$

12. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment ?

- (1) 0.14%
- (2) 0.9%
- (3) 9%
- (4) 1.4%

Official Ans. by NTA (4)

Sol. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{Al} = \frac{mg \cdot L}{\pi R^2 \cdot \ell}$

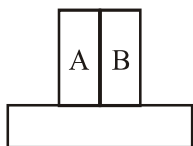
$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2 \cdot \frac{\Delta R}{R} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta Y}{Y} \times 100 = 100 \left[\frac{1}{1000} + \frac{1}{1000} + 2 \left(\frac{0.001}{0.2} \right) + \frac{0.001}{0.5} \right]$$

$$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$$



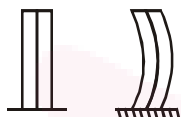
13. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold bath, it will :



- (1) Bend towards the right
- (2) Not bend but shrink
- (3) Neither bend nor shrink
- (4) Bend towards the left

Official Ans. by NTA (4)

- Sol.** $\alpha_A > \alpha_B$
Length of both strips will decrease
 $\Delta L_A > \Delta L_B$



14. A resistor develops 500 J of thermal energy in 20s when a current of 1.5 A is passed through it. If the current is increased from 1.5 A to 3A, what will be the energy developed in 20 s.

- (1) 1500 J
- (2) 1000 J
- (3) 500 J
- (4) 2000 J

Official Ans. by NTA (4)

- Sol.** $500 = (1.5)^2 \times R \times 20$
 $E = (3)^2 \times R \times 20$
 $E = 2000 \text{ J}$

15. **Statement I :** A cyclist is moving on an unbanked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve ($g = 9.8 \text{ m/s}^2$)

Statement II : If the road is banked at an angle of 45° , cyclist can cross the curve of 2m radius with the speed of 18.5 kmh^{-1} without slipping.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Statement I is incorrect and statement II is correct
- (2) Statement I is correct and statement II is incorrect
- (3) Both statement I and statement II are false
- (4) Both statement I and statement II are true

Official Ans. by NTA (4)

Sol. Statement I :

$$v_{\max} = \sqrt{\mu Rg} = \sqrt{(0.2) \times 2 \times 9.8}$$

$$v_{\max} = 1.97 \text{ m/s}$$

$$7 \text{ km/h} = 1.944 \text{ m/s}$$

Speed is lower than v_{\max} , hence it can take safe turn.

Statement II

$$v_{\max} = \sqrt{Rg \left[\frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right]}$$

$$= \sqrt{2 \times 9.8 \left[\frac{1 + 0.2}{1 - 0.2} \right]} = 5.42 \text{ m/s}$$

$$18.5 \text{ km/h} = 5.14 \text{ m/s}$$

Speed is lower than v_{\max} , hence it can take safe turn.

16. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight ? (Assume radius of earth is 6400 km)

- (1) 19.77 m
- (2) 39.55 m
- (3) 79.1 m
- (4) 158.2 m

Official Ans. by NTA (2)

Sol. $D = 2\sqrt{2Rh}$

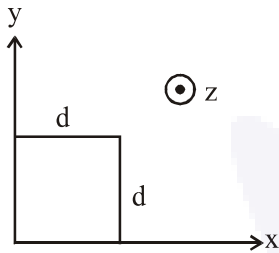
$$h = \frac{D^2}{8R} = \frac{45^2}{8 \times 6400} \text{ km} \cong 39.55 \text{ m}$$



17. The magnetic field in a region is given by

$$\vec{B} = B_0 \left(\frac{x}{a} \right) \hat{k}$$

A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v} = v_0 \hat{i}$. The emf induced in the loop is :



- (1) $\frac{B_0 v_0^2 d}{2a}$ (2) $\frac{B_0 v_0 d}{2a}$
 (3) $\frac{B_0 v_0 d^2}{a}$ (4) $\frac{B_0 v_0 d^2}{2a}$

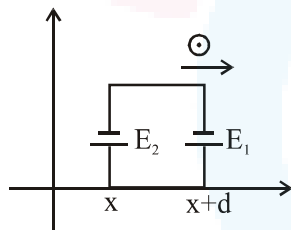
Official Ans. by NTA (3)

Sol. $E_1 = \frac{B_0(x+d)}{a} v_0 d$

$E_2 = \frac{B_0(x)}{a} v_0 d$

$E_{net} = E_1 - E_2$

$E_{net} = \frac{B_0 v_0 d^2}{a}$



18. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value ? ($\ln 2 = 0.693$)

- (1) 34.65 s (2) 17.32 s
 (3) 0.034 s (4) 15.01 s

Official Ans. by NTA (1)

Sol. $A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$

$\frac{A_0}{2} = A_0 e^{-\frac{bt}{2m}}$

$\frac{bt}{2m} = \ln 2$

$t = \frac{2m}{b} \ln 2 = \frac{2 \times 500 \times 0.693}{20}$

$t = 34.65$ second.

19. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.

- (1) 60 minutes (2) 20 minutes
 (3) 40 minutes (4) 13 minutes

Official Ans. by NTA (2)

Sol. $N_1 = N_0 e^{-\lambda t_1}$

$\frac{N_1}{N_0} = e^{-\lambda t_1}$

$0.67 = e^{-\lambda t_1}$

$\ln(0.67) = -\lambda t_1$

$N_2 = N_0 e^{-\lambda t_2}$

$\frac{N_2}{N_0} = e^{-\lambda t_2}$

$0.33 = e^{-\lambda t_2}$

$\ln(0.33) = -\lambda t_2$

$\ln(0.67) - \ln(0.33) = \lambda t_1 - \lambda t_2$

$\lambda(t_1 - t_2) = \ln\left(\frac{0.67}{0.33}\right)$

$\lambda(t_1 - t_2) \cong \ln 2$

$t_1 - t_2 \cong \frac{\ln 2}{\lambda} = t_{1/2}$

Half life = $t_{1/2} = 20$ minutes.

20. Red light differs from blue light as they have :

- (1) Different frequencies and different wavelengths
 (2) Different frequencies and same wavelengths
 (3) Same frequencies and same wavelengths
 (4) Same frequencies and different wavelengths

Official Ans. by NTA (1)

Sol. Red light and blue light have different wavelength and different frequency.

SECTION-B

1. The energy dissipated by a resistor is 10 mJ in 1s when an electric current of 2 mA flows through it. The resistance is _____ Ω . (Round off to the Nearest Integer)

Official Ans. by NTA (2500)

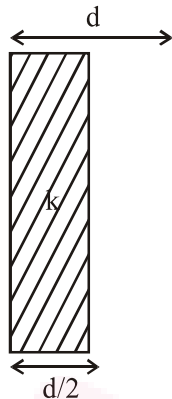
Sol. Ans. (2500)

$Q = i^2 RT$

$R = \frac{Q}{i^2 t} = \frac{10 \times 10^{-3}}{4 \times 10^{-6} \times 1} = 2500 \Omega$



2. In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1 m . If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 m^2 (see fig.) the capacitance of the set-up will be _____ ϵ_0 .
 (Dielectric constant of the material = 3.2)
 (Round off to the Nearest Integer)



Official Ans. by NTA (3)

Sol. Ans. (3)

$$C = \frac{\epsilon_0 A}{\frac{d}{2K} + \frac{d}{2}} = \frac{2\epsilon_0 A}{\frac{d}{K} + d}$$

$$= \frac{2 \times 2\epsilon_0}{\frac{1}{3.2} + 1} = \frac{4 \times 3.2}{4.2} \epsilon_0$$

$$= 3.04 \epsilon_0$$

3. A force $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point of $x = 2$ plane and x -axis. The magnitude of torque of this force about a point $(2, 3, 4)$ is _____. (Round off to the Nearest Integer)

Official Ans. by NTA (20)

Sol. Ans. (20)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{r} = (2\hat{i}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = -3\hat{j} - 4\hat{k}$$

$$\& \vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(-12 + 12) - \hat{j}(0 + 16) + \hat{k}(0 + 12)$$

$$= -16\hat{j} + 12\hat{k}$$

$$\therefore |\vec{\tau}| = \sqrt{16^2 + 12^2} = 20$$

4. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x \text{ GM}^2}{5 \text{ R}}$ where x is ____ (Round off to the Nearest Integer)
 (M is the mass of earth, R is the radius of earth, G is the gravitational constant)

Official Ans. by NTA (3)

Sol. Ans. (3)

$$\text{Energy given} = U_f - U_i$$

$$= 0 - \left(-\frac{3 \text{ GM}^2}{5 \text{ R}} \right)$$

$$= \frac{3 \text{ GM}^2}{5 \text{ R}}$$

$$x = 3$$

5. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass are 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be _____ $^\circ$ (in degree)
 (Round off to the Nearest Integer)

Official Ans. by NTA (12)



Sol. Ans. (12)

$$\omega_1 = 0.02 ; \mu_1 = 1.5 ; \omega_2 = 0.03 ; \mu_2 = 1.6$$

Achromatic combination

$$\therefore \theta_{\text{net}} = 0$$

$$\theta_1 - \theta_2 = 0$$

$$\theta_1 = \theta_2$$

$$\omega_1 \delta_1 = \omega_2 \delta_2$$

$$\& \delta_{\text{net}} = \delta_1 - \delta_2 = 2^\circ$$

$$\delta_1 - \frac{\omega_1 \delta_1}{\omega_2} = 2^\circ$$

$$\delta_1 \left(1 - \frac{\omega_1}{\omega_2} \right) = 2^\circ$$

$$\delta_1 \left(1 - \frac{2}{3} \right) = 2^\circ$$

$$\delta_1 = 6^\circ$$

$$\delta_1 = (\mu_1 - 1) A_1$$

$$6^\circ = (1.5 - 1) A_1$$

$$A_1 = 12^\circ$$

6. A body of mass 2kg moves under a force of $(2\hat{i} + 3\hat{j} + 5\hat{k})\text{N}$. It starts from rest and was at the origin initially. After 4s, its new coordinates are (8, b, 20). The value of b is _____.
(Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol. Ans. (12)

$$\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$$

$$= \hat{i} + 1.5\hat{j} + 2.5\hat{k}$$

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$= 0 + \frac{1}{2}(\hat{i} + 1.5\hat{j} + 2.5\hat{k})(16)$$

$$= 8\hat{i} + 12\hat{j} + 20\hat{k}$$

$$b = 12$$

7. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _____°. (Round off to the Nearest Integer) (find the angle in degree)

Official Ans. by NTA (120)

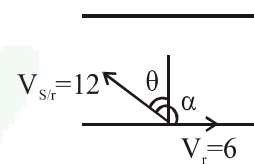
Sol. Ans. (12)

$$12\sin\theta = v_r$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\therefore \alpha = 120^\circ$$



8. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is

$$\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}} \text{ where } x \text{ is } \underline{\hspace{2cm}}. \text{ (Round off to the Nearest Integer)}$$

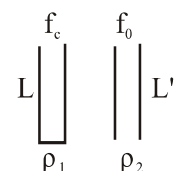
Official Ans. by NTA (4)

Sol. Ans. (4)

$$f_c = f_o$$

$$\frac{3V_c}{4L} = \frac{2V_o}{2L'}$$

$$\frac{3V_c}{4L} = \frac{V_o}{L'}$$



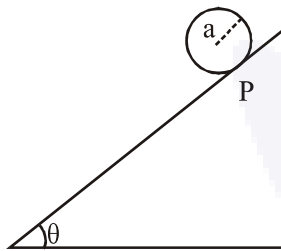
$$L' = \frac{4L}{3} \frac{V_o}{V_c} = \frac{4L}{3} \sqrt{\frac{B \cdot \rho_1}{\rho_2 \cdot B}} \text{ (B is bulk modulus)}$$

$$= \frac{4L}{3} \sqrt{\frac{\rho_1}{\rho_2}}$$

$$x = 4$$



9. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$ where b is _____. (Round off to the Nearest Integer) (g = acceleration due to gravity) (θ = angle as shown in figure)



Official Ans. by NTA (3)

Sol. Ans. (3)

$$a = \frac{g \sin \theta}{1 + \frac{I}{mR^2}} = \frac{g \sin \theta}{1 + \frac{1}{2}} = \frac{2}{3}g \sin \theta$$

b = 3

10. For an ideal heat engine, the temperature of the source is 127°C . In order to have 60% efficiency the temperature of the sink should be _____ $^\circ\text{C}$. (Round off to the Nearest Integer)

Official Ans. by NTA (113)

Official Ans. by ALLEN (-113)

Sol. Ans. (-113)

$$\eta = 0.60 = 1 - \frac{T_L}{T_H}$$

$$\frac{T_L}{T_H} = 0.4 \Rightarrow T_L = 0.4 \times 400$$

$$= 160 \text{ K}$$

$$= -113^\circ\text{C}$$