FINAL JEE–MAIN EXAMINATION – JULY, 2021

Held On Thursday 22nd July, 2021

TIME: 3:00 PM to 06:00 PM

SECTION-A

1. Let L be the line of intersection of planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})=2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point (1,2,0), then the value of $35(\alpha+\beta+\gamma)$ is equal to :

(1) 101 (2) 119 (3) 143 (4) 134

Official Ans. by NTA (2)

2. Let S_n denote the sum of first n-terms of an arithmetic progression. If S_{10} = 530, S_5 = 140, then $S_{20} - S_6$ is equal to :

(1) 1862 (2) 1842 (3) 1852 (4) 1872

Official Ans. by NTA (1)

3. Let $f: \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases} -\frac{4}{3}x^3+2x^2+3x & , \quad x>0 \\ 3xe^x & , \quad x \le 0 \end{cases}.$$ Then f is increasing function in the interval

(1) $\left(-\frac{1}{2},2\right)$ (2) (0,2)

(3) $\left(-1,\frac{3}{2}\right)$ (4) (–3, –1)

Official Ans. by NTA (3)

4. Let y = y(x) be the solution of the differential equation $\text{cosec}^2 x\,dy + 2dx = (1 + y\cos 2x)\,\text{cosec}^2 x\,dx$, with $y\left(\frac{\pi}{4}\right)=0$. Then, the value of $(y(0) + 1)^2$ is equal to :

(1) $e^{1/2}$ (2) $e^{-1/2}$ (3) e^{-1} (4) e

Official Ans. by NTA (3)

5. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 × 2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :

(1) $\frac{45}{162}$ (2) $\frac{23}{81}$ (3) $\frac{22}{81}$ (4) $\frac{43}{162}$

Official Ans. by NTA (4)

6. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$, and $|\vec{a}|=\sqrt{10}$. Then a possible value of $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{b}\ \vec{d}]+[\vec{a}\ \vec{c}\ \vec{d}]$ is equal to :

(1) – 42 (2) – 40 (3) – 29 (4) – 38

Official Ans. by NTA (1)

7. If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx = \frac{\alpha\pi^3}{1+4\pi^2}, \alpha \in \mathbf{R}$ where [x] is the greatest integer less than or equal to x, then the value of α is :

(1) $200(1-e^{-1})$ (2) $100(1-e)$

(3) $50(e-1)$ (4) $150(e^{-1}-1)$

Official Ans. by NTA (1)

8. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following is **not** true ?

(1) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))=\vec{0}$

(2) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2

(3) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=8$

(4) $|3\vec{a}+\vec{b}-2\vec{c}|^2=51$

Official Ans. by NTA (4)

9. The values of λ and μ such that the system of equations x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + λz = μ has no solution, are :

(1) $\lambda = 3, \mu = 5$ (2) $\lambda = 3, \mu \ne 10$

(3) $\lambda \ne 2, \mu = 10$ (4) $\lambda = 2, \mu \ne 10$

Official Ans. by NTA (4)

10. If the shortest distance between the straight lines 3(x – 1) = 6(y – 2) = 2(z – 1) and 4(x – 2) = 2(y – λ) = (z – 3), $\lambda \in \mathbf{R}$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

(1) 3 (2) 2 (3) 5 (4) –1

Official Ans. by NTA (1)

11. Which of the following Boolean expressions is **not** a tautology ?

(1) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(2) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

(3) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

(4) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Official Ans. by NTA (4)

12. Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to :

(1) 2 (2) 1 (3) 3 (4) 9

Official Ans. by NTA (3)

13. Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in \mathbf{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

(1) $\left[0, \frac{1}{e}\right)$ (2) $[\log_e 2, \log_e 3)$

(3) $[1, e)$ (4) $[0, \log_e 2)$

Official Ans. by NTA (4)

14. Let the circle $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

(1) $\frac{25}{9} < C < \frac{13}{3}$ (2) $100 < C < 165$

(3) $81 < C < 156$ (4) $100 < C < 156$

Official Ans. by NTA (4)

15. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to

(1) 1 (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) 2

Official Ans. by NTA (2)

16. The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to

(1) 11 (2) 7 (3) 5 (4) 9

Official Ans. by NTA (3)

17. If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to :

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{1}{2}$ (4) 1

Official Ans. by NTA (1)

18. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , x \neq 0 \\ \alpha & , x = 0 \end{cases}.$$

If f is continuous at $x = 0$, then α is equal to :

(1) 1 (2) 3 (3) 0 (4) 2

Official Ans. by NTA (1)

19. Let a line $L : 2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to :

(1) 12 (2) –12 (3) 24 (4) –24

Official Ans. by NTA (4)

20. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is :

(1) $\frac{-1+\sqrt{5}}{2}$ (2) $\frac{-1+\sqrt{8}}{2}$

(3) $\frac{-1+\sqrt{3}}{2}$ (4) $\frac{-1+\sqrt{6}}{2}$

Official Ans. by NTA (1)

SECTION-B

1. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

Official Ans. by NTA (720)

2. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _______.

Official Ans. by NTA (96)

3. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3 × 3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is _______.

Official Ans. by NTA (3125)

4. Consider the following frequency distribution :

Class :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to _______.

Official Ans. by NTA (4)

5. The sum of all the elements in the set {n ∈ {1, 2, ….., 100} | H.C.F. of n and 2040 is 1} is equal to _______.

Official Ans. by NTA (1251)

6. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is _______.

Official Ans. by NTA (2)

7. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a function defined as

$$f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$$

Let $g : \mathbf{R} \rightarrow \mathbf{R}$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in **R** where g is not continuous and not differentiable, respectively, then n + m is equal to _______.

Official Ans. by NTA (4)

8. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _______.

Official Ans. by NTA (8)

9. Let y = y(x) be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right) dx = (x + 2)\, dy$, y(1) = 1. If the domain of y = y(x) is an open interval (α, β), then |α + β| is equal to _______.

Official Ans. by NTA (4)

10. The number of elements in the set $\{n \in \{1, 2, 3, \ldots, 100\} \mid (11)^n > (10)^n + (9)^n\}$ is _______.

Official Ans. by NTA (96)