FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021
Held On Friday 26th February, 2021
TIME: 9:00 AM to 12:00 NOON

SECTION-A

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4\vec{b}$ (3) $\vec{a}\times\vec{b}$ (4) $|\vec{a}|^4\vec{b}$

Official Ans. by NTA (4)

Sol. $\vec{a}.\vec{b}=0$

$\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\left(\vec{a}.\vec{b}\right)\vec{a}-\left(\vec{a}.\vec{a}\right)\vec{b}=-|\vec{a}|^2\vec{b}$

Now $\vec{a}\times\left(\vec{a}\times\left(-|\vec{a}|^2\vec{b}\right)\right)$

$=-|\vec{a}|^2\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)$

$=-|\vec{a}|^2\left(-|\vec{a}|^2\vec{b}\right)=|\vec{a}|^4\vec{b}$

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{14}}$

Official Ans. by NTA (1)

Sol. Let the coin be tossed n-times

$P(H) = P(T) = \frac{1}{2}$

$P(7\text{ heads}) = {}^nC_7\left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^7=\frac{{}^nC_7}{2^n}$

$P(9\text{ heads}) = {}^nC_9\left(\frac{1}{2}\right)^{n-9}\left(\frac{1}{2}\right)^9=\frac{{}^nC_9}{2^n}$

P(7 heads) = P(9 heads)

${}^nC_7 = {}^nC_9 \Rightarrow n = 16$

$P(2\text{ heads}) = {}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^2=\frac{15\times 8}{2^{16}}$

$P(2\text{ heads}) = \frac{15}{2^{13}}$

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

(1) 4 (2) 1 (3) 6 (4) 12

Official Ans. by NTA (1)

Sol. $A=\begin{pmatrix} a & b \\ b & c\end{pmatrix}$, a, b, c $\in$ I

$A^2=\begin{pmatrix} a & b \\ b & c\end{pmatrix}\begin{pmatrix} a & b \\ b & c\end{pmatrix}=\begin{pmatrix} a^2+b^2 & b(a+c) \\ b(a+c) & b^2+c^2\end{pmatrix}$

Sum of the diagonal entries of

$A^2 = a^2 + 2b^2 + c^2$

Given $a^2 + 2b^2 + c^2 = 1$, a, b, c $\in$ I

$b = 0$ & $a^2 + c^2 = 1$

Case–1 : $a = 0 \Rightarrow c = \pm 1$ (2-matrices)

Case-2 : $c = 0 \Rightarrow a = \pm 1$ (2-matrices)

Total = 4 matrices

4. In a increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to

(1) 30 (2) 26 (3) 35 (4) 32

Official Ans. by NTA (3)

Sol. a, ar, ar^2, ...

$T_2+T_6=\frac{25}{2}\Rightarrow ar\left(1+r^4\right)=\frac{25}{2}$

$a^2r^2(1+r^4)^2=\frac{625}{4}$ .... (1)

$T_3 . T_5 = 25 \Rightarrow (ar^2)(ar^4) = 25$

$a^2r^6 = 25$ .... (2)

On dividing (1) by (2)

$\frac{\left(1+r^4\right)^2}{r^4}=\frac{25}{4}$

$4r^8 - 17r^4 + 4 = 0$

$(4r^4 - 1)(r^4 - 4) = 0$

$r^4 = \frac{1}{4}, 4 \Rightarrow r^4 = 4$

(an increasing geometric series)

$a^2r^6 = 25 \Rightarrow (ar^3)^2 = 25$

$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 5(1 + 2 + 4) = 35$

5. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\le$ x, is

(1) $100(e - 1)$ (2) $100(1 - e)$

(3) $100e$ (4) $100(1 + e)$

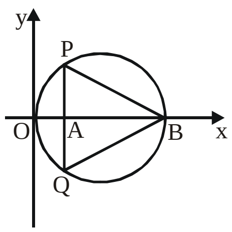
Official Ans. by NTA (1)

Sol. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}dx$, period of $\{x\} = 1$

$$\sum_{n=1}^{100} \int_{0}^{1} e^{\{x\}}\,dx = \sum_{n=1}^{100} \int_{0}^{1} e^{x}dx$$

$$\sum_{n=1}^{100} (e-1) = 100(e-1)$$

6. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Official Ans. by NTA (2)

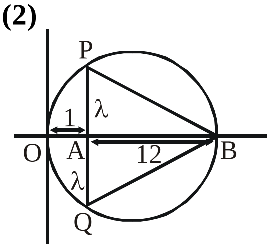
Sol. PA = AQ = λ

OA·AB

= AP . AQ

$\Rightarrow$ 1.12 = λ . λ

$\Rightarrow \lambda = 2\sqrt{3}$



Area $\Delta PQB = \frac{1}{2} \times 2\lambda \times AB$

$$\Delta = \frac{1}{2}.4\sqrt{3} \times 12$$

$$= 24\sqrt{3}$$

7. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$ is equal to

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

Official Ans. by NTA (1)

Sol. $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \ldots$

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \ldots$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \ldots + \text{up to infinite terms}$$

$$\Rightarrow S = \frac{13}{4}$$

8. The value of $\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)}\right\}$ is

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Official Ans. by NTA (1)

Sol. $L = \lim_{h\to 0} 2\left(\frac{\sqrt{3}\left(\frac{1}{2}\cosh + \frac{\sqrt{3}}{2}\sinh\right) - \left(\frac{\sqrt{3}}{2}\cosh - \frac{\sinh}{2}\right)}{\left(\sqrt{3}h\right)\left(\sqrt{3}\right)}\right)$

$$L = \lim_{h\to 0} \frac{4\sinh}{3h}$$

$$\Rightarrow L = \frac{4}{3}$$

9. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is

(1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(3) $\frac{2.10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

Official Ans. by NTA (2)

Sol. Term independent of t will be the middle term due to exect same magnitude but opposite sign powers of t in the binomial expression given

so $T_6 = {}^{10}C_5\left(tx^2 5\right)^5\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^5$

$T_6 = f(x) = {}^{10}C_5\left(x\sqrt{1-x}\right)$; for maximum

$f'(x) = 0 \Rightarrow x = \frac{2}{3}$ & $f''\left(\frac{2}{3}\right) < 0$

so $f(x)_{max.} = {}^{10}C_5\left(\frac{2}{3}\right).\frac{1}{\sqrt{3}}$

10. The rate of growth of bacteria in a culture is proportional to the number of bacteris present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4 (2) 8 (3) 2 (4) 16

Official Ans. by NTA (1)

Sol. $\frac{dB}{dt} = \lambda B \Rightarrow \int_{1000}^{1200}\frac{dB}{B} = \lambda\int_0^2 dt \Rightarrow \lambda = \frac{1}{2}\ell n\left(\frac{6}{5}\right)$

$\int_{1000}^{2000}\frac{dB}{B} = \frac{1}{2}\ell n\left(\frac{6}{5}\right)\int_0^T dt \Rightarrow T = \frac{2\ell n 2}{\ell n\left(\frac{6}{5}\right)}$

$\Rightarrow k = 2\ell n 2$

11. If (1,5,35), (7,5,5), (1,λ,7) and (2λ,1,2) are coplanar, then the sum of all possible values of λ is

(1) $\frac{39}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $-\frac{44}{5}$

Official Ans. by NTA (3)

Sol. A(1, 5, 35), B(7, 5, 5), C(1, λ, 7), D(2λ, 1, 2)

$\overline{AB} = 6\hat{i} - 30\hat{k}$, $\overline{BC} = -6\hat{i}(\lambda-5)\hat{j}+2\hat{k}$,

$\overline{CD} = (2\lambda-1)\hat{i}+(1-\lambda)\hat{j}-5\hat{k}$

Points are coplanar

$$\Rightarrow 0 = \begin{vmatrix} 6 & 0 & -30 \\ -6 & \lambda-5 & 2 \\ 2\lambda-1 & 1-\lambda & -5 \end{vmatrix}$$

$= 6(-5\lambda + 25 - 2 + 2\lambda)$

$-30(-6 + 6\lambda - (2\lambda^2 - \lambda - 10\lambda + 5))$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 11\lambda - 5 - 6 + 6\lambda)$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 17\lambda - 11)$

$= 6(-3\lambda + 23 + 10\lambda^2 - 85\lambda + 55)$

$= 6(10\lambda^2 - 88\lambda + 78) = 12(5\lambda^2 - 44\lambda + 39)$

$\Rightarrow 0 = 12(5\lambda^2 - 44\lambda + 39)$

$\lambda_1 + \lambda_2 = \frac{44}{5}$

12. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}; 0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1 - y^2$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

Official Ans. by NTA (3)

Sol. $\frac{\sin^{-1}x}{r} = a$, $\frac{\cos^{-1}x}{r} = b$, $\frac{\tan^{-1}y}{r} = c$

So, $a + b = \frac{\pi}{2r}$

$$\cos\left(\frac{\pi c}{a+b}\right)=\cos\left(\frac{\pi \tan^{-1} y}{\frac{\pi}{2r} r}\right)$$

$= \cos(2\tan^{-1}y)$, let $\tan^{-1}y = \theta$

$= \cos(2\theta)$

$$=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-y^2}{1+y^2}$$

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 42 (2) 82 (3) 77 (4) 35

Official Ans. by NTA (3)

Sol. **(I)** First possiblity is 1, 1, 1, 1, 1, 2, 3

required number = $\frac{7!}{5!}$ = 7 × 6 = 42

(II) Second possiblity is 1, 1, 1, 1, 2, 2, 2

required number = $\frac{7!}{4!\ 3!}=\frac{7\times6\times5}{6}=35$

Total = 42 + 35 = 77

14. Let f be any function defined on R and let it satisfy the condition :

$|f(x) - f(y)| \le |(x - y)^2|, \forall (x,y) \in R$

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0, \forall x \in R$

(3) $f(x) = 0, \forall x \in R$

(4) $f(x) > 0, \forall x \in R$

Official Ans. by NTA (4)

Sol. $\left|\frac{f(x)-f(y)}{(x-y)}\right| \le |(x-y)|$

$x - y = h$ let $\Rightarrow x = y + h$

$$\lim_{x\to0}\left|\frac{f(y+h)-f(y)}{h}\right|\le 0$$

$\Rightarrow |f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = k$ (constant)

and $f(0) = 1$ given

So, $f(y) = 1 \Rightarrow f(x) = 1$

15. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point

(1) (2,2) (2) (0,0)

(3) (2,9) (4) $\left(3,\frac{21}{2}\right)$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Since, slope is maximum so,

$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$x = 2, 3$

at $x = 2$

$\frac{d^3y}{dx^3}=12x-30$

at $x = 2$, $\frac{d^3y}{dx^3}<0$

So, maxima

$y = \frac{1}{2} \times 16 - 5 \times 8 + 18 \times 4 - 19 \times 2$

$= 8 - 40 + 72 - 38 = 80 - 78 = 2$

point (2, 2)

16. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

(1) Right angled triangle

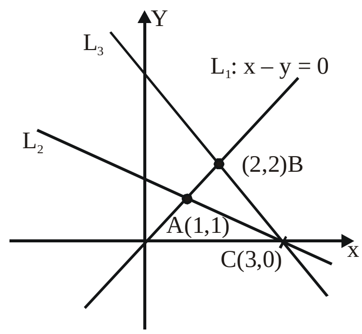
(2) Equilateral triangle

(3) Isosceles triangle

(4) None of the above

Official Ans. by NTA (3)

Sol.



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : x + y = 6$

on solving L_1 and L_2 :

y = L and x = 1

L_1 and L_3 :

x = 2

y = 2

L_2 and L_3 :

x + y = 3

2x + y = 6

x = 3

y = 0

$$AC = \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{4+1} = \sqrt{5}$$

$$AB = \sqrt{1+1} = \sqrt{2}$$

so its an isosceles triangle

17. Consider the three planes

$P_1 : 3x + 15y + 21z = 9,$

$P_2 : x - 3y - z = 5,$ and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true ?

(1) P_1 and P_2 are parallel

(2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel

(4) P_1,P_2 and P_3 all are parallel

Official Ans. by NTA (2)

Sol. $P_1 : x + 5y + 7z = 3,$

$P_2 : x - 3y - z = 5$

$$P_3 : x + 5y + 7z = \frac{5}{2}$$

so P_1 and P_3 are parallel.

18. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) (a + 2) (a + 3) (a + 4)

(2) –2

(3) (a + 1) (a + 2) (a + 3)

(4) 0

Official Ans. by NTA (2)

Sol. $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ a^2+7a+12-a^2-3a-2 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

= 4(a + 2) – 4a – 10

= 4a + 8 – 4a – 10 = –2

19. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is

(1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

Official Ans. by NTA (1)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ (using king)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+3^x)\cos^2 x}{1+3^x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \cos^2 x dx = 2\int_0^{\pi/2} \cos^2 x\, dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos^2 x\, dx = \frac{\pi}{4}$$

20. Let R = {(P,Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set :

(1) S = {(x,y) | $x^2 + y^2 = 4$}

(2) S = {(x,y) | $x^2 + y^2 = 1$}

(3) S = {(x,y) | $x^2 + y^2 = \sqrt{2}$ }

(4) S = {(x,y) | $x^2 + y^2 = 2$}

Official Ans. by NTA (4)

Sol. Equivalence class of (1, –1) is a circle with centre at (0,0) and radius = $\sqrt{2}$

$\Rightarrow x^2 + y^2 = 2$

$S = \{(x,y)| \ x^2 + y^2 = 2\}$

SECTION-B

1. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is

Official Ans. by NTA (2)

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) = ax + \frac{a^{3/2}}{2}$ ...(1)

$\Rightarrow 2yy' = a$

put in equation (1)

$$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$$

$$(y^2 - 2xyy') = \frac{(2yy')^{3/2}}{2}$$

squaring

$$(y^2 - 2xyy')^2 = \frac{y^3(y')^3}{2}$$

$\therefore$ order = 1

degree = 3

Degree – order = 3 – 1 = 2

2. The number of integral values of 'k' for which the equation 3sinx + 4 cosx = k + 1 has a solution, k $\in$ R is

Official Ans. by NTA (11)

Sol. 3 sinx + 4 cosx = k + 1

$$\Rightarrow k + 1 \in \left[-\sqrt{3^2 + 4^2}, \sqrt{3^2 + 4^2}\right]$$

$$\Rightarrow k + 1 \in [-5, 5]$$

$$\Rightarrow k \in [-6, 4]$$

No. of integral values of k = 11

3. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is

Official Ans. by NTA (1)

Sol. $\log_4(x - 1) = \log_2(x - 3)$

$$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$$

$$\Rightarrow \log_2(x-1)^{1/2} = \log_2(x-3)$$

$$\Rightarrow (x-1)^{1/2} = x - 3$$

$\Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x - 2)(x - 5) = 0$

$\Rightarrow x = 2,5$

But $x \neq 2$ because it is not satisfying the domain of given equation i.e $\log_2(x - 3) \rightarrow$ its domain $x > 3$

finally x is 5

$\therefore$ No. of solutions = 1.

4. The sum of 162[th] power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

Official Ans. by NTA (3)

Sol. $x^3 - 2x^2 + 2x - 1 = 0$

x = 1 satisfying the equation

$\therefore$ x– 1 is factor of

$x^3 - 2x^2 + 2x - 1$

$= (x - 1)(x^2 - x + 1) = 0$

$$x = 1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$$

$x = 1, -\omega^2, -\omega$

sum of 162[th] power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + (\omega)^{324} + (\omega)^{162}$

$= 1 + 1 + 1 = 3$

5. Let m,n $\in$ N and gcd (2,n) = 1. If

$$30\binom{30}{0}+29\binom{30}{1}+\ldots+2\binom{30}{28}+1\binom{30}{29}=n.2^m,$$

then n + m is equal to

(Here $\binom{n}{k}={}^nC_k$)

Official Ans. by NTA (45)

Sol. $30({}^{30}C_0) + 29({}^{30}C_1) + \ldots + 2({}^{30}C_{28}) + 1({}^{30}C_{29})$

$= 30({}^{30}C_{30}) + 29({}^{30}C_{29}) + \ldots\ldots + 2({}^{30}C_2) + 1({}^{30}C_1)$

$$=\sum_{r=1}^{30} r\left({}^{30}C_r\right)$$

$$=\sum_{r=1}^{30} r\left(\frac{30}{r}\right)\left({}^{29}C_{r-1}\right)$$

$$=30\sum_{r=1}^{30} {}^{29}C_{r-1}$$

$$=30\left({}^{29}C_0+{}^{29}C_1+{}^{29}C_2+\ldots+{}^{29}C_{29}\right)$$

$$=30\left(2^{29}\right)=15\left(2\right)^{30}=n\left(2\right)^m$$

$\therefore$ n = 15, m = 30

n + m = 45

6. If y = y(x) is the solution of the equaiton

$$e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x,\ y(0)=0;$$

then $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to

Official Ans. by NTA (1)

Sol. Put $e^{\sin y} = t$

$$\Rightarrow e^{\sin y}\cos y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow \text{D.E is } \frac{dt}{dx}+t\cos x=\cos x$$

$$\text{I.F.} = e^{\int \cos x\,dx}=e^{\sin x}$$

$$\Rightarrow \text{solution is } t.e^{\sin x}=\int \cos x e^{\sin x}$$

$$\Rightarrow e^{\sin y}e^{\sin x}=e^{\sin x}+c$$

$\because$ x = 0, y = 0 $\Rightarrow$ c = 0

$$\Rightarrow e^{\sin y}=1$$

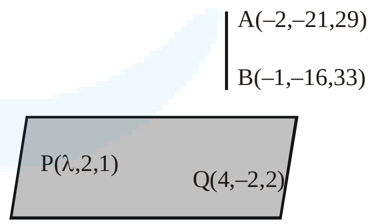
$\Rightarrow$ y = 0

$$\Rightarrow 1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)=1$$

7. Let $(\lambda,2,1)$ be a point on the plane which passes through the ponit (4,–2,2). If the plane is perpendicular to the line joining the points (–2,–21,29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4$ is equal to

Official Ans. by NTA (8)

Sol.



$$\overline{AB}.\,\overline{PQ}=0$$

$$\Rightarrow\left(\hat{i}+5\hat{j}-6\hat{k}\right).\left((4-\lambda)\hat{i}-4\hat{j}+\hat{k}\right)=0$$

$$\Rightarrow 4-\lambda-20-6=0$$

$$\Rightarrow \lambda=-22$$

$$\Rightarrow\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4=4+8-4=8$$

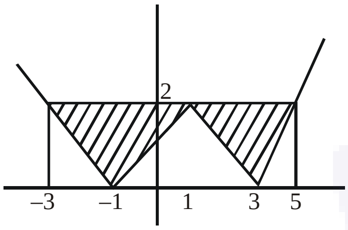
8. The area bounded by the lines y = ||x – 1| –2| is

Official Ans. by NTA (8)

Ans. By ALLEN (BONUS)

Sol. **Remark :**

Question is incomplete it should be area bounded by y = |x – 1| – 2| and y = 2



Area $= 2\left(\frac{1}{2}.4.2\right)$

9. The value of the integral $\int_0^{\pi}|\sin 2x|\,dx$ is

Official Ans. by NTA (2)

Sol. Put 2x = t ⇒ 2dx = dt

$\Rightarrow\ I = \frac{1}{2}\int_0^{2\pi}|\sin t|\,dt$

$= \int_0^{\pi}|\sin t|\,dt$

= 2

10. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

Official Ans. by NTA (1)

Sol. $\sqrt{3}\left(\cos x\right)^2 - \sqrt{3}\cos x + \cos x - 1 = 0$

$\Rightarrow \left(\sqrt{3}\cos x + 1\right)\left(\cos x - 1\right) = 0$

$\Rightarrow \cos x = 1$ or $\cos x = -\frac{1}{\sqrt{3}}$ (reject)

$\Rightarrow x = 0$ only