FINAL JEE–MAIN EXAMINATION – AUGUST, 2021
Held On Tuesday 31st August, 2021
TIME: 9:00 AM to 12:00 NOON

SECTION-A

1. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

(1) $* = \vee$, $\square = \vee$ (2) $* = \wedge$, $\square = \wedge$

(3) $* = \wedge$, $\square = \vee$ (4) $* = \vee$, $\square = \wedge$

Official Ans. by NTA (3)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$ is tautology

p	q	~q	p∧~q	p∨q	(p∧~q)→(p∨q)
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

2. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^{x} - 6 = 0$ is :

(1) 2 (2) 4

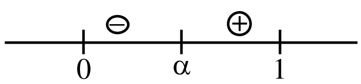
(3) 1 (4) 0

Official Ans. by NTA (3)

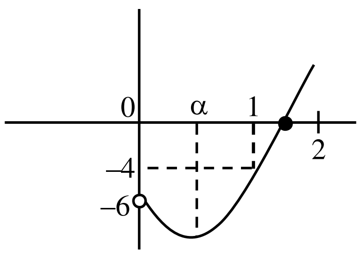
Sol. Let $e^{x} = t > 0$

$f(t) = t^4 + 2t^3 - t - 6 = 0$

$f'(t) = 4t^3 + 6t^2 - 1$



$f''(t) = 12t^2 + 12t > 0$



$f(0) = -6,\ f(1) = -4,\ f(2) = 24$

$\Rightarrow$ Number of real roots = 1

3. The sum of 10 terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots \text{ is :}$$

(1) 1 (2) $\frac{120}{121}$

(3) $\frac{99}{100}$ (4) $\frac{143}{144}$

Official Ans. by NTA (2)

Sol. $S = \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + \dots$

$= \left[\frac{1}{1^2} - \frac{1}{2^2}\right] + \left[\frac{1}{2^2} - \frac{1}{3^2}\right] + \left[\frac{1}{3^2} - \frac{1}{4^2}\right] + \dots + \left[\frac{1}{10^2} - \frac{1}{11^2}\right]$

$= 1 - \frac{1}{121}$

$= \frac{120}{121}$

4. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :

(1) –23 (2) – 15

(3) 23 (4) 15

Official Ans. by NTA (1)

Sol. Equation of plane is

$3x - 2y + 4z - 7 + \lambda(x + 5y - 2z + 9) = 0$

$(3 + \lambda)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$

passing through (1, 4, –3)

$\Rightarrow 3 + \lambda + 20\lambda - 8 - 12 + 6\lambda + 9\lambda - 7 = 0$

$\Rightarrow \lambda = \frac{2}{3}$

$\Rightarrow$ equation of plane is

$-11x - 4y - 8z + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = -23$

5. Let f be a non–negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$ and f(0) = 0, then $\lim_{x\to 0}\frac{1}{x^2}\int_0^x f(t)\,dt$:

(1) equals 0 (2) equals 1

(3) does not exist (4) equals $\frac{1}{2}$

Official Ans. by NTA (4)

Sol. $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt \quad 0 \le x \le 1$

differentiating both the sides

$\sqrt{1-(f'(x))^2} = f(x)$

$\Rightarrow 1 - (f'(x))^2 = f^2(x)$

$\frac{f'(x)}{\sqrt{1-f^2(x)}} = 1$

$\sin^{-1} f(x) = x + C$

$\because f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$

Now $\lim_{x\to 0}\frac{\int_0^x \sin t\,dt}{x^2}\left(\frac{0}{0}\right) = \frac{1}{2}$

6. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}| = |3\vec{a}+\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 4 (2) 6

(3) 5 (4) 8

Official Ans. by NTA (3)

Sol. $|3\vec{a}+\vec{b}|^2 = |2\vec{a}+3\vec{b}|^2$

$(3\vec{a}+\vec{b}).(3\vec{a}+\vec{b}) = (2\vec{a}+3\vec{b}).(2\vec{a}+3\vec{b})$

$9\vec{a}.\vec{a}+6\vec{a}.\vec{b}+\vec{b}.\vec{b} = 4\vec{a}.\vec{a}+12\vec{a}.\vec{b}+9.\vec{b}.\vec{b}$

$5|\vec{a}|^2 - 6\vec{a}.\vec{b} = 8|\vec{b}|^2$

$5(8)^2 - 6.8.|\vec{b}|\cos 60° = 8|\vec{b}|^2 \quad \left(\because \frac{1}{8}|\vec{a}| = 1 \Rightarrow |\vec{a}| = 8\right)$

$40 - 3|\vec{b}| = |\vec{b}|^2$

$\Rightarrow |\vec{b}|^2 + 3|\vec{b}| - 40 = 0$

$|\vec{b}| = -8, \quad |\vec{b}| = 5$

(rejected)

7. The function $f(x) = |x^2-2x-3|\cdot e^{|9x^2-12x+4|}$ is not differentiable at exactly :

(1) four points (2) three points

(3) two points (4) one point

Official Ans. by NTA (3)

Sol. $f(x) = |(x-3)(x+1)|\,.\,e^{(3x-2)^2}$

$$f(x) = \begin{cases} (x-3)(x+1).e^{(3x-2)^2} & ; \; x \in (3,\infty) \\ -(x-3)(x+1).e^{(3x-2)^2} & ; \; x \in [-1,3] \\ (x-3).(x+1).e^{(3x-2)^2} & ; \; x \in (-\infty,-1) \end{cases}$$

Clearly, non-differentiable at x = –1 & x = 3.

8. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is 3 r^2, then r^2 – d is equal to :

(1) $7 - 7\sqrt{3}$ (2) $7 + \sqrt{3}$

(3) $7 - \sqrt{3}$ (4) $7 + 3\sqrt{3}$

Official Ans. by NTA (2)

Sol. Let numbers be $\frac{a}{r}$, a, ar → G.P

$\frac{a}{r}$, 2a, ar → A.P ⇒ $4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

$r = 2 \pm \sqrt{3}$

4^{th} form of G.P = $3r^2 \Rightarrow ar^2 = 3r^2 \Rightarrow a = 3$

$r = 2 + \sqrt{3}$, a = 3, $d = 2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = (2+\sqrt{3})^2 - 3\sqrt{3}$

$= 7 + 4\sqrt{3} - 3\sqrt{3}$

$= 7 + \sqrt{3}$

9. Which of the following is **not** correct for relation R on the set of real numbers ?

(1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric.

(2) $(x, y) \in R \Leftrightarrow 0 < |x-y| \le 1$ is symmetric and transitive.

(3) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric.

(4) $(x, y) \in R \Leftrightarrow |x-y| \le 1$ is reflexive and symmetric.

Official Ans. by NTA (2)

Sol. Note that (1,2) and (2,3) satisfy $0 < |x-y| \le 1$ but (1,3) does not satisfy it so

$0 \le |x-y| \le 1$ is symmetric but not transitive

So, (2) is correct.

10. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$ (2) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$

(3) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$ (4) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

Official Ans. by NTA (3)

Sol. $\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$

$= \int \frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4}.(x-1)^2}$

put $\frac{x+2}{x-1} = t$

$= -\frac{1}{3}\int \frac{dt}{t^{5/4}}$

$= \frac{4}{3}.\frac{1}{t^{1/4}}+C$

$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$

11. If p and q are the lengths of the perpendiculars from the origin on the lines,

$x \operatorname{cosec}\alpha - y\sec\alpha = k\cot 2\alpha$ and

$x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

respectively, then k^2 is equal to :

(1) $4p^2 + q^2$ (2) $2p^2 + q^2$

(3) $p^2 + 2q^2$ (4) $p^2 + 4q^2$

Official Ans. by NTA (1)

Sol. First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$ ...(i)

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$ ...(ii)

Hence $4p^2 + q^2 = k^2$ (From (i) & (ii))

12. $\operatorname{cosec}18°$ is a root of the equation :

(1) $x^2 + 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$

(3) $x^2 - 2x + 4 = 0$ (4) $x^2 - 2x - 4 = 0$

Official Ans. by NTA (4)

Sol. $\operatorname{cosec}18° = \frac{1}{\sin 18°} = \frac{4}{\sqrt{5}-1} = \sqrt{5}+1$

Let $\operatorname{cosec}18° = x = \sqrt{5}+1$

$\Rightarrow x - 1 = \sqrt{5}$

Squaring both sides, we get

$x^2 - 2x + 1 = 5$

$\Rightarrow x^2 - 2x - 4 = 0$

13. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a = -\frac{1}{3}, b \ne \frac{7}{3}$ (2) $a \ne \frac{1}{3}, b = \frac{7}{3}$

(3) $a \ne -\frac{1}{3}, b = \frac{7}{3}$ (4) $a = \frac{1}{3}, b \ne \frac{7}{3}$

Official Ans. by NTA (4)

Sol. Here $D=\begin{vmatrix}2&1&1\\1&-1&1\\1&1&a\end{vmatrix}=2(-a-1)-1(a-1)+1+1 = 1-3a$

$D_3=\begin{vmatrix}2&1&5\\1&-1&3\\1&1&b\end{vmatrix}=2(-b-3)-1(b-3)+5(1+1) = 7-3b$

for $a=\frac{1}{3}$, $b\neq\frac{7}{3}$, system has no solutions

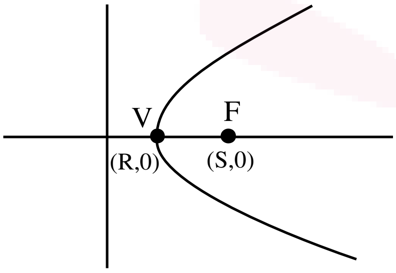
14. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x–axis at a distance R and S (>R) respectively from the origin, is :

(1) 4(S + R) (2) 2(S – R)

(3) 4(S – R) (4) 2(S + R)

Official Ans. by NTA (3)

Sol.



V $\rightarrow$ Vertex

F $\rightarrow$ focus

VF = S – R

So latus rectum = 4(S – R)

15. If the function $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , x<0\\ k & , x=0\\ \frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , x>0\end{cases}$

is continuous at x = 0, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

(1) –5 (2) 5

(3) – 4 (4) 4

Official Ans. by NTA (1)

Sol. If $f(x)$ is continuous at x = 0, RHL = LHL = $f(0)$

$\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}\cdot\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$ (Rationalisation)

$$\lim_{x\to0^+}-\frac{2\sin^2 x}{x^2}\cdot\left(\sqrt{x^2+1}+1\right)=-4$$

$$\lim_{x\to0^-} f(x)=\lim_{x\to0^-}\frac{1}{x}\ell n\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$$

$$\lim_{x\to0^-}\frac{\ell n\left(1+\frac{x}{a}\right)}{\left(\frac{x}{a}\right).a}+\frac{\ell n\left(1-\frac{x}{b}\right)}{\left(-\frac{x}{b}\right).b}$$

$$=\frac{1}{a}+\frac{1}{b}$$

So $\frac{1}{a}+\frac{1}{b}=-4=k$

$$\Rightarrow\frac{1}{a}+\frac{1}{b}+\frac{4}{k}=-4-1=-5$$

16. If $\frac{dy}{dx}=\frac{2^{x+y}-2^x}{2^y}$, y(0) = 1, then y(1) is equal to :

(1) $\log_2(2+e)$ (2) $\log_2(1+e)$

(3) $\log_2(2e)$ (4) $\log_2(1+e^2)$

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}=\frac{2^x2^y-2^x}{2^y}$

$$2^y\frac{dy}{dx}=2^x\left(2^y-1\right)$$

$$\int\frac{2^y}{2^y-1}dy=\int 2^x\,dx$$

$$\frac{\ell n\left(2^y-1\right)}{\ell n 2}=\frac{2^x}{\ell n 2}+C$$

$\Rightarrow \log_2(2^y-1)=2^x\log_2 e+C$

$\because$ y(0) = 1 $\Rightarrow 0=\log_2 e+C$

$C=-\log_2 e$

$\Rightarrow \log_2(2^y-1)=(2^x-1)\log_2 e$

put x = 1, $\log_2(2^y-1)=\log_2 e$

$2^y=e+1$

$y=\log_2(e+1)$ Ans.

17. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to :

(1) π^2 (2) $2\pi^2$

(3) $4\pi^2$ (4) 4π

Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$

$\lim\limits_{x\to 0}\dfrac{1-\cos\left(2\pi\cos^4 x\right)}{2x^4}$

$\lim\limits_{x\to 0}\dfrac{1-\cos\left(2\pi-2\pi\cos^4 x\right)}{\left[2\pi\left(1-\cos^4 x\right)\right]^2}4\pi^2.\dfrac{\sin^4 x}{2x^4}\left(1+\cos^2 x\right)^2$

$=\dfrac{1}{2}.4\pi^2.\dfrac{1}{2}(2)^2=4\pi^2$

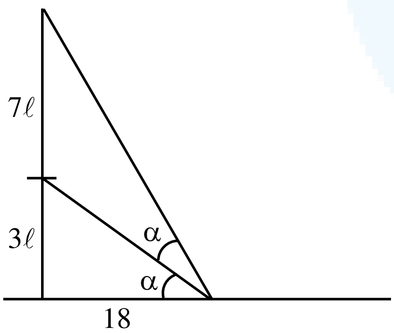
18. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

(1) $12\sqrt{15}$ (2) $12\sqrt{10}$

(3) $8\sqrt{10}$ (4) $6\sqrt{10}$

Official Ans. by NTA (2)

Sol.



Let height of pole = 10ℓ

$\tan\alpha=\dfrac{3\ell}{18}=\dfrac{\ell}{6}$

$\tan 2\alpha=\dfrac{10\ell}{18}$

$\dfrac{2\tan\alpha}{1-\tan^2\alpha}=\dfrac{10\ell}{18}$

use $\tan\alpha=\dfrac{\ell}{6}\Rightarrow \ell=\sqrt{\dfrac{72}{5}}$

height of pole = $10\ell=12\sqrt{10}$

19. If $a_r=\cos\dfrac{2r\pi}{9}+i\sin\dfrac{2r\pi}{9}$, r = 1, 2, 3,…, $i=\sqrt{-1}$,

then the determinant $\begin{vmatrix}a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}$ is equal to :

(1) $a_2a_6-a_4a_8$ (2) a_9

(3) $a_1a_9-a_3a_7$ (4) a_5

Official Ans. by NTA (3)

Sol. $a_r=e^{\frac{i2\pi r}{9}}$, r = 1, 2, 3, … a_1, a_2, a_3, … are in G.P.

$\begin{vmatrix}a_1 & a_2 & a_3\\ a_n & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}=\begin{vmatrix}a_1 & a_2^2 & a_1^3\\ a_1^4 & a_1^5 & a_1^6\\ a_1^7 & a_1^8 & a_1^9\end{vmatrix}=a_1.a_1^4.a_1^7\begin{vmatrix}1 & a_1 & a_1^2\\ 1 & a_1 & a_1^2\\ 1 & a_1 & a_1^2\end{vmatrix}=0$

Now $a_1a_9-a_3a_7=a_1^{10}-a_1^{10}=0$

20. The line 12x cosθ + 5y sinθ = 60 is tangent to which of the following curves?

(1) $x^2+y^2=169$

(2) $144x^2+25y^2=3600$

(3) $25x^2+12y^2=3600$

(4) $x^2+y^2=60$

Official Ans. by NTA (2)

Sol. $12x\cos\theta+5y\sin\theta=60$

$\dfrac{x\cos\theta}{5}+\dfrac{y\sin\theta}{12}=1$

is tangent to $\dfrac{x^2}{25}+\dfrac{y^2}{144}=1$

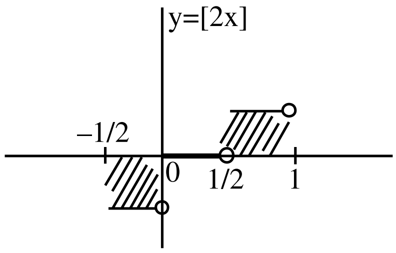
$144x^2+25y^2=3600$

SECTION-B

1. Let [t] denote the greatest integer $\le$ t. Then the value of $8\cdot\int_{-\frac{1}{2}}^{1}\left([2x]+|x|\right)dx$ is __________.

Official Ans. by NTA (5)

Sol. $I = \int_{-1/2}^{1} ([2x] + |x|)\,dx$



$= \int_{-1/2}^{1} [2x]\,dx + \int_{-1/2}^{1} |x|\,dx$

$= 0 + \int_{-1/2}^{0} (-x)\,dx + \int_{0}^{1} x\,dx$

$= \left(-\frac{x^2}{2}\right)_{-1/2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{1}$

$= \left(0 + \frac{1}{8}\right) + \frac{1}{2}$

$= \frac{5}{8}$

$8I = 5$

2. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $|z - 9\sqrt{2} - 2i|^2$ is equal to ________.

Official Ans. by NTA (98)

Sol. Let z = x + iy

$\arg\left(\frac{x-2+iy}{x+2+iy}\right) = \frac{\pi}{4}$

$\arg(x - 2 + iy) - \arg(x + 2 + iy) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

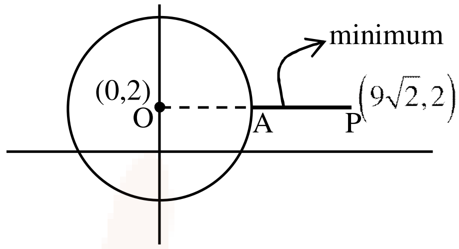
$\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \left(\frac{y}{x-2}\right)\cdot\left(\frac{y}{x+2}\right)} = \tan\frac{\pi}{4} = 1$

$\frac{xy + 2y - xy + 2y}{x^2 - 4 + y^2} = 1$

$4y = x^2 - 4 + y^2$

$x^2 + y^2 - 4y - 4 = 0$

locus is a circle with center (0, 2) & radius = $2\sqrt{2}$



min. value = $(AP)^2 = (OP - OA)^2$

$= \left(9\sqrt{2} - 2\sqrt{2}\right)^2$

$= \left(7\sqrt{2}\right)^2 = 98$

3. The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point (–1, –1, 2) is ________.

Official Ans. by NTA (61)

Sol. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$

$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 6\lambda - 1$

for point of intersection of line & plane

$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

$7\lambda = 7 \Rightarrow \lambda = 1$

point : (3, 5, 5)

$(\text{distance})^2 = (3 + 1)^2 + (5 + 1)^2 + (5 - 2)^2$

$= 16 + 36 + 9 = 61$

4. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is ________.

Official Ans. by NTA (2)

Sol. $f(x) = x^2 + ax + 1$

$f'(x) = 2x + a$

when $f(x)$ is increasing on [1, 2]

$2x + a \geq 0 \quad \forall\ x \in [1, 2]$

$a \geq -2x\ \forall\ x \in [1, 2]$

$R = -4$

when $f(x)$ is decreasing on [1, 2]

$2x + a \leq 0 \quad \forall\ x \in [1, 2]$

$a \leq -2 \quad \forall\ x \in [1, 2]$

$S = -2$

$|R - S| = |-4 + 2| = 2$

5. The mean of 10 numbers

$7 \times 8,\ 10 \times 10,\ 13 \times 12,\ 16 \times 14,\ \ldots$ is ________.

Official Ans. by NTA (398)

Sol. $7 \times 8,\ 10 \times 10,\ 13 \times 12,\ 16 \times 14 \ldots\ldots$

$T_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3)$

$= 2(3n^2 + 13n + 12) = 6n^2 + 26n + 24$

$$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$$

$$= \frac{6(10\times11\times21)}{6} + 26\times\frac{10\times11}{2} + 24\times10$$

$= 10 \times 11\ (21 + 13) + 240$

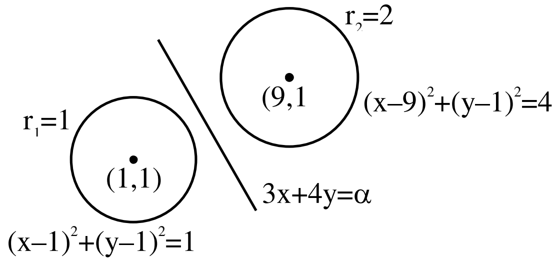
$= 3980$

$$\text{Mean} = \frac{S_{10}}{10} = \frac{3980}{10} = 398$$

6. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x - 1)^2 + (y - 1)^2 = 1$ and $(x - 9)^2 + (y - 1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is ________.

Official Ans. by NTA (165)

Sol.



Both centres should lie on either side of the line as well as line can be tangent to circle.

$(3 + 4 - \alpha) . (27 + 4 - \alpha) < 0$

$(7 - \alpha) . (31 - \alpha) < 0 \Rightarrow \alpha \in (7, 31) \quad \ldots(1)$

d_1 = distance of (1, 1) from line

d_2 = distance of (9, 1) from line

$$d_1 \geq r_1 \Rightarrow \frac{|7-\alpha|}{5}\ 1 \Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty) \quad \ldots(2)$$

$$d_2 \geq r_2 \Rightarrow \frac{|31-\alpha|}{5} \geq 2 \Rightarrow \alpha \in (-\infty, 21] \cup [41, \infty)$$

$\ldots(3)$

$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12, 21]$

Sum of integers = 165

7. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is ________.

Official Ans. by NTA (576)

Sol. VOWELS → 2 Vowels, 4 Consonants

All Consonants should not be together

= Total – All consonants together,

$= 6! - 3!\ 4! = 576$

8. If $x\,\phi(x) = \int_5^x \left(3t^2 - 2\phi'(t)\right) dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is ________.

Official Ans. by NTA (4)

Sol. $x\phi(x) = \int_5^x 3t^2 - 2\phi'(t)\,dt$

$x\phi(x) = x^3 - 125 - 2[\phi(x) - \phi(5)]$

$x\phi(x) = x^3 - 125 - 2\phi(x) - 2\phi(5)$

$\phi(0) = 4 \Rightarrow \phi(5) = -\dfrac{133}{2}$

$\phi(x) = \dfrac{x^3 + 8}{x + 2}$

$\phi(2) = 4$

9. If $\left(\dfrac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\dfrac{x}{4} - \dfrac{12}{x^2}\right)^{12}$, then k is equal to ________.

Official Ans. by NTA (55)

Sol. $\left(\dfrac{x}{4} - \dfrac{12}{x^2}\right)^{12}$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\dfrac{x}{4}\right)^{12-r} \left(\dfrac{12}{x^2}\right)^r$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\dfrac{1}{4}\right)^{12-r} (12)^r.(x)^{12-3r}$

Term independent of x $\Rightarrow 12 - 3r = 0 \Rightarrow r = 4$

$T_5 = (-1)^4 \cdot {}^{12}C_4 \left(\dfrac{1}{4}\right)^8 (12)^4 = \dfrac{3^6}{4^4}.k$

$\Rightarrow k = 55$

10. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to ________.

Official Ans. by NTA (28)

Sol. I_1 = first unit is functioning

I_2 = second unit is functioning

$P(I_1) = 0.9$, $P(I_2) = 0.8$

$P(\bar{I}_1) = 0.1$, $P(\bar{I}_2) = 0.2$

$P = \dfrac{0.8 \times 0.1}{0.1 \times 0.2 + 0.9 \times 0.2 + 0.1 \times 0.8} = \dfrac{8}{28}$

$98P = \dfrac{8}{28} \times 98 = 28$