



FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

Held On Tuesday 31st August, 2021

TIME: 3:00 PM to 06:00 PM

SECTION-A

1. If  $\alpha + \beta + \gamma = 2\pi$ , then the system of equations

$$x + (\cos \gamma)y + (\cos \beta)z = 0$$

$$(\cos \gamma)x + y + (\cos \alpha)z = 0$$

$$(\cos \beta)x + (\cos \alpha)y + z = 0$$

has :

- (1) no solution
- (2) infinitely many solution
- (3) exactly two solutions
- (4) a unique solution

Official Ans. by NTA (2)

Sol.  $\alpha + \beta + \gamma = 2\pi$

$$\begin{vmatrix} 1 & \cos \gamma & \cos \beta \\ \cos \gamma & 1 & \cos \alpha \\ \cos \beta & \cos \alpha & 1 \end{vmatrix}$$

$$\begin{aligned} &= 1 + 2\cos\alpha.\cos\beta.\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma \\ &= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha + \beta) + \cos(\alpha - \beta))\cos\gamma \\ &= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha - \beta)\cos\gamma \\ &= \sin^2\alpha - \cos^2\beta + \cos(\alpha - \beta)\cos(\alpha + \beta) \\ &= \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0 \end{aligned}$$

2. Let  $\vec{a}, \vec{b}, \vec{c}$  be three vectors mutually perpendicular to each other and have same magnitude. If a vector  $\vec{r}$  satisfies.

$$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0},$$

then  $\vec{r}$  is equal to :

- (1)  $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$
- (2)  $\frac{1}{3}(2\vec{a} + \vec{b} - \vec{c})$
- (3)  $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$
- (4)  $\frac{1}{2}(\vec{a} + \vec{b} + 2\vec{c})$

Official Ans. by NTA (3)

Sol. Suppose  $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$

$$\text{and } |\vec{a}| = |\vec{b}| = |\vec{c}| = k$$

$$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$$

$$\Rightarrow k^2(\vec{r} - \vec{b}) - k^2x\vec{a} + k^2(\vec{r} - \vec{c}) - k^2y\vec{b} + k^2(\vec{r} - \vec{a}) - k^2z\vec{c} = \vec{0}$$

$$\Rightarrow 3\vec{r} - (\vec{a} + \vec{b} + \vec{c}) - \vec{r} = \vec{0}$$

$$\Rightarrow \vec{r} = \frac{\vec{a} + \vec{b} + \vec{c}}{2}$$

3. The domain of the function

$$f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x - 1)^2}\right) + \cos^{-1}\left(\frac{x - 1}{x + 1}\right) \text{ is :}$$

- (1)  $\left[0, \frac{1}{4}\right]$
- (2)  $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$
- (3)  $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$
- (4)  $\left[0, \frac{1}{2}\right]$

Official Ans. by NTA (3)

Sol.  $f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x - 1)^2}\right) + \cos^{-1}\left(\frac{x - 1}{x + 1}\right)$

$$-1 \leq \frac{x - 1}{x + 1} \leq 1 \Rightarrow 0 \leq x < \infty \quad \dots(1)$$

$$-1 \leq \frac{3x^2 + x - 1}{(x - 1)^2} \leq 1 \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right] \cup \{0\} \quad \dots(2)$$

(1) & (2)

$$\Rightarrow \text{Domain} = \left[-\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$$

4. Let  $S = \{1, 2, 3, 4, 5, 6\}$ . Then the probability that a randomly chosen onto function  $g$  from  $S$  to  $S$  satisfies  $g(3) = 2g(1)$  is :

- (1)  $\frac{1}{10}$
- (2)  $\frac{1}{15}$
- (3)  $\frac{1}{5}$
- (4)  $\frac{1}{30}$

Official Ans. by NTA (1)



**Sol.**  $g(3) = 2g(1)$  can be defined in 3 ways  
 number of onto functions in this condition =  $3 \times 4!$   
 Total number of onto functions =  $6!$

$$\text{Required probability} = \frac{3 \times 4!}{6!} = \frac{1}{10}$$

5. Let  $f : \mathbf{N} \rightarrow \mathbf{N}$  be a function such that  
 $f(m + n) = f(m) + f(n)$  for every  $m, n \in \mathbf{N}$ . If  $f(6) = 18$ ,  
 then  $f(2) \cdot f(3)$  is equal to :

- (1) 6 (2) 54  
 (3) 18 (4) 36

**Official Ans. by NTA (2)**

**Sol.**  $f(m + n) = f(m) + f(n)$

Put  $m = 1, n = 1$

$$f(2) = 2f(1)$$

Put  $m = 2, n = 1$

$$f(3) = f(2) + f(1) = 3f(1)$$

Put  $m = 3, n = 3$

$$f(6) = 2f(3) \Rightarrow f(3) = 9$$

$$\Rightarrow f(1) = 3, f(2) = 6$$

$$f(2) \cdot f(3) = 6 \times 9 = 54$$

6. The distance of the point  $(-1, 2, -2)$  from the line  
 of intersection of the planes  $2x + 3y + 2z = 0$  and  
 $x - 2y + z = 0$  is :

- (1)  $\frac{1}{\sqrt{2}}$  (2)  $\frac{5}{2}$   
 (3)  $\frac{\sqrt{42}}{2}$  (4)  $\frac{\sqrt{34}}{2}$

**Official Ans. by NTA (4)**

**Sol.**  $P_1 : 2x + 3y + 2z = 0$

$$\Rightarrow \vec{n}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$P_2 : x - 2y + z = 0$

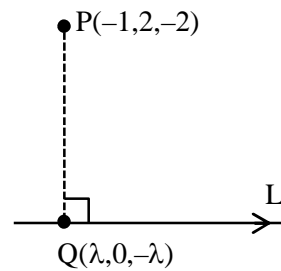
$$\Rightarrow \vec{n}_2 = \hat{i} - 2\hat{j} + \hat{k}$$

Direction vector of line L which is line of  
 intersection of  $P_1$  &  $P_2$

$$\vec{r} = \vec{n}_1 \times \vec{n}_2 = 7\hat{i} - 7\hat{k}$$

DR's of L are  $(1, 0, -1)$

$$\Rightarrow \text{Equation of L : } \frac{x}{1} = \frac{y}{0} = \frac{z}{-1} = \lambda$$



DR's of  $\overline{PQ} = (\lambda + 1, -2, 2 - \lambda)$

$$\therefore \overline{PQ} \perp \vec{r}$$

$$\Rightarrow (\lambda + 1)(1) + (-2)(0) + (2 - \lambda)(-1) = 0$$

$$\Rightarrow \lambda = \frac{1}{2} \Rightarrow Q\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$$

$$\Rightarrow PQ = \frac{\sqrt{34}}{2}$$

7. Negation of the statement  $(p \vee r) \Rightarrow (q \vee r)$  is :

- (1)  $p \wedge \sim q \wedge \sim r$  (2)  $\sim p \wedge q \wedge \sim r$   
 (3)  $\sim p \wedge q \wedge r$  (4)  $p \wedge q \wedge r$

**Official Ans. by NTA (1)**

**Sol.**  $\therefore \sim(A \Rightarrow B) = A \wedge \sim B$

$$\therefore \sim((p \vee r) \Rightarrow (q \vee r))$$

$$= (p \vee r) \wedge (\sim q \wedge \sim r)$$

$$= ((p \vee r) \wedge (\sim r)) \wedge (\sim q)$$

$$= p \wedge (\sim r) \wedge (\sim q)$$

8. If  $\alpha = \lim_{x \rightarrow \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$  and  $\beta = \lim_{x \rightarrow 0} (\cos x)^{\cot x}$  are

the roots of the equation,  $ax^2 + bx - 4 = 0$ , then the  
 ordered pair  $(a, b)$  is :

- (1)  $(1, -3)$  (2)  $(-1, 3)$   
 (3)  $(-1, -3)$  (4)  $(1, 3)$

**Official Ans. by NTA (4)**

**Sol.**  $\alpha = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ ;  $\frac{0}{0}$  form

Using L Hopital rule

$$\alpha = \lim_{x \rightarrow \frac{\pi}{4}} \frac{3 \tan^2 x \sec^2 x - \sec^2 x}{-\sin\left(x + \frac{\pi}{4}\right)}$$

$$\Rightarrow \alpha = -4$$



$$\beta = \lim_{x \rightarrow 0} (\cos x)^{\cot x} = e^{\lim_{x \rightarrow 0} \frac{(\cos x - 1)}{\tan x}}$$

$$\beta = e^{\lim_{x \rightarrow 0} \frac{-(1 - \cos x)}{x^2} \cdot \frac{x^2}{\left(\frac{\tan x}{x}\right)^x}}$$

$$\beta = e^{\lim_{x \rightarrow 0} \frac{(-1) \cdot x}{2} \cdot 1} = e^0 \Rightarrow \beta = 1$$

$$\alpha = -4 ; \beta = 1$$

If  $ax^2 + bx - 4 = 0$  are the roots then

$$16a - 4b - 4 = 0 \text{ \& } a + b - 4 = 0$$

$$\Rightarrow a = 1 \text{ \& } b = 3$$

9. The locus of mid-points of the line segments joining  $(-3, -5)$  and the points on the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \text{ is :}$$

- (1)  $9x^2 + 4y^2 + 18x + 8y + 145 = 0$
- (2)  $36x^2 + 16y^2 + 90x + 56y + 145 = 0$
- (3)  $36x^2 + 16y^2 + 108x + 80y + 145 = 0$
- (4)  $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

**Official Ans. by NTA (3)**

**Sol.** General point on  $\frac{x^2}{4} + \frac{y^2}{9} = 1$  is  $A(2\cos\theta, 3\sin\theta)$

given  $B(-3, -5)$

$$\text{midpoint } C\left(\frac{2\cos\theta - 3}{2}, \frac{3\sin\theta - 5}{2}\right)$$

$$h = \frac{2\cos\theta - 3}{2}; k = \frac{3\sin\theta - 5}{2}$$

$$\Rightarrow \left(\frac{2h+3}{2}\right)^2 + \left(\frac{2k+5}{3}\right)^2 = 1$$

$$\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0$$

10. If  $\frac{dy}{dx} = \frac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y} \log_e 2}$ ,  $y(0) = 0$ , then for  $y = 1$ , the value of  $x$  lies in the interval:

- (1)  $(1, 2)$
- (2)  $\left(\frac{1}{2}, 1\right]$
- (3)  $(2, 3)$
- (4)  $\left(0, \frac{1}{2}\right]$

**Official Ans. by NTA (1)**

**Sol.**  $\frac{dy}{dx} = \frac{2^x (y + 2^y)}{2^x (1 + 2^y \ln 2)}$

$$\Rightarrow \int \frac{(1 + 2^y) \ln 2}{(y + 2^y)} dy = \int dx$$

$$\Rightarrow \ln|y + 2^y| = x + c$$

$$x = 0; y = 0 \Rightarrow c = 0$$

$$\Rightarrow x = \ln|y + 2^y|$$

$$\Rightarrow \text{at } y = 1, x = \ln 3$$

$$\therefore 3 \in (e, e^2) \Rightarrow x \in (1, 2)$$

11. An angle of intersection of the curves,  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and  $x^2 + y^2 = ab$ ,  $a > b$ , is :

(1)  $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$       (2)  $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

(3)  $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$       (4)  $\tan^{-1}(2\sqrt{ab})$

**Official Ans. by NTA (3)**

**Sol.**  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $x^2 + y^2 = ab$

$$\frac{2x_1}{a^2} + \frac{2y_1 y_1'}{b^2} = 0$$

$$\Rightarrow y_1' = \frac{-x_1}{a^2} \cdot \frac{b^2}{y_1} \quad \dots(1)$$

$$\therefore 2x_1 + 2y_1 y_1' = 0$$

$$\Rightarrow y_2' = \frac{-x_1}{y_1} \quad \dots(2)$$

Here  $(x_1, y_1)$  is point of intersection of both curves

$$\therefore x_1^2 = \frac{a^2 b}{a+b}, y_1^2 = \frac{ab^2}{a+b}$$

$$\therefore \tan \theta = \left| \frac{y_1' - y_2'}{1 + y_1' y_2'} \right| = \left| \frac{\frac{-x_1 b^2}{a^2 y_1} + \frac{x_1}{y_1}}{1 + \frac{x_1^2 b^2}{a^2 y_1^2}} \right|$$



$$\tan \theta = \left| \frac{-b^2 x_1 y_1 + a^2 x_1 y_1}{a^2 y_1^2 + b^2 x_1^2} \right|$$

$$\tan \theta = \left| \frac{a - b}{\sqrt{ab}} \right|$$

12. If  $y \frac{dy}{dx} = x \left[ \frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)} \right]$ ,  $x > 0$ ,  $\phi > 0$ , and  $y(1) = -1$ ,

then  $\phi\left(\frac{y^2}{4}\right)$  is equal to :

- (1)  $4\phi(2)$                       (2)  $4\phi(1)$   
 (3)  $2\phi(1)$                       (4)  $\phi(1)$

**Official Ans. by NTA (2)**

**Sol.** Let,  $y = tx$

$$\frac{dy}{dx} = t + x \frac{dt}{dx}$$

$$\therefore tx \left( t + x \frac{dt}{dx} \right) = x \left( t^2 + \frac{\phi(t^2)}{\phi'(t^2)} \right)$$

$$t^2 + xt \frac{dt}{dx} = t^2 + \frac{\phi(t^2)}{\phi'(t^2)}$$

$$\int \frac{t\phi'(t^2)}{\phi(t^2)} dt = \int \frac{dx}{x}$$

Let  $\phi(t^2) = p$

$$\therefore \phi'(t^2) 2t dt = dp$$

$$\Rightarrow \int \frac{dy}{2p} = \int \frac{dx}{x}$$

$$\frac{1}{2} \ln \phi(t^2) = \ln x + \ln c$$

$$\phi(t^2) = x^2 k$$

$$\phi\left(\frac{y^2}{x^2}\right) = kx^2, \phi(1) = k$$

$$\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$$

13. The sum of the roots of the equation  $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$ , is :

- (1)  $\log_2 14$                       (2)  $\log_2 11$   
 (3)  $\log_2 12$                       (4)  $\log_2 13$

**Official Ans. by NTA (2)**

**Sol.**  $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$   
 $\log_2(2^{x+1}) - \log_2(3 + 2^x)^2 + \log_2(10 - 2^{-x}) = 0$

$$\log_2 \left( \frac{2^{x+1} \cdot (10 - 2^{-x})}{(3 + 2^x)^2} \right) = 0$$

$$\frac{2(10 \cdot 2^x - 1)}{(3 + 2^x)^2} = 1$$

$$\Rightarrow 20 \cdot 2^x - 2 = 9 + 2^{2x} + 6 \cdot 2^x$$

$$\therefore (2^x)^2 - 14(2^x) + 11 = 0$$

Roots are  $2^{x_1}$  &  $2^{x_2}$

$$\therefore 2^{x_1} \cdot 2^{x_2} = 11$$

$$x_1 + x_2 = \log_2(11)$$

14. If  $z$  is a complex number such that  $\frac{z-i}{z-1}$  is purely imaginary, then the minimum value of  $|z - (3 + 3i)|$  is :

- (1)  $2\sqrt{2} - 1$                       (2)  $3\sqrt{2}$   
 (3)  $6\sqrt{2}$                           (4)  $2\sqrt{2}$

**Official Ans. by NTA (4)**

**Sol.**  $\frac{z-i}{z-1}$  is purely Imaginary number

Let  $z = x + iy$

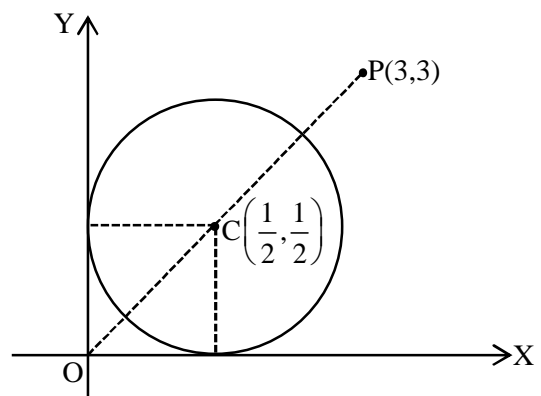
$$\therefore \frac{x + i(y-1)}{(x-1) + i(y)} \times \frac{(x-1) - iy}{(x-1) - iy}$$

$$\Rightarrow \frac{x(x-1) + y(y-1) + i(-y-x+1)}{(x-1)^2 + y^2} \text{ is purely}$$

Imaginary number

$$\Rightarrow x(x-1) + y(y-1) = 0$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$$





$$\begin{aligned} \therefore |z - (3 + 3i)|_{\min} &= |PC| - \frac{1}{\sqrt{2}} \\ &= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2} \end{aligned}$$

15. Let  $a_1, a_2, a_3, \dots$  be an A.P. If  $\frac{a_1 + a_2 + \dots + a_{10}}{a_1 + a_2 + \dots + a_p} = \frac{100}{p^2}$ ,  $p \neq 10$ , then  $\frac{a_{11}}{a_{10}}$  is equal

to :

- (1)  $\frac{19}{21}$                                       (2)  $\frac{100}{121}$   
 (3)  $\frac{21}{19}$                                       (4)  $\frac{121}{100}$

**Official Ans. by NTA (3)**

**Sol.** 
$$\frac{\frac{10}{2}(2a_1 + 9d)}{\frac{p}{2}(2a_1 + (p-1)d)} = \frac{100}{p^2}$$

$$(2a_1 + 9d)p = 10(2a_1 + (p-1)d)$$

$$9dp = 20a_1 - 2pa_1 + 10d(p-1)$$

$$9p = (20 - 2p) \frac{a_1}{d} + 10(p-1)$$

$$\frac{a_1}{d} = \frac{(10-p)}{2(10-p)} = \frac{1}{2}$$

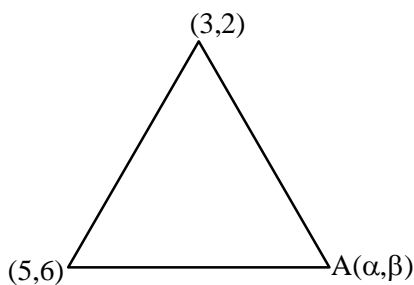
$$\therefore \frac{a_{11}}{a_{10}} = \frac{a_1 + 10d}{a_1 + 9d} = \frac{\frac{1}{2} + 10}{\frac{1}{2} + 9} = \frac{21}{19}$$

16. Let A be the set of all points  $(\alpha, \beta)$  such that the area of triangle formed by the points  $(5, 6)$ ,  $(3, 2)$  and  $(\alpha, \beta)$  is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

- (1)  $\frac{4}{\sqrt{5}}$                                       (2)  $\frac{16}{\sqrt{5}}$   
 (3)  $\frac{8}{\sqrt{5}}$                                       (4)  $\frac{12}{\sqrt{5}}$

**Official Ans. by NTA (3)**

**Sol.**



$$\begin{vmatrix} 5 & 6 & 1 \\ 3 & 2 & 1 \\ \alpha & \beta & 1 \end{vmatrix} = 12$$

$$4\alpha - 2\beta = \pm 24 + 8$$

$$\Rightarrow 4\alpha - 2\beta = +24 + 8 \Rightarrow 2\alpha - \beta = 16$$

$$2x - y - 16 = 0 \quad \dots(1)$$

$$\Rightarrow 4\alpha - 2\beta = -24 + 8 \Rightarrow 2\alpha - \beta = -8$$

$$2x - y + 8 = 0 \quad \dots(2)$$

perpendicular distance of (1) from  $(0, 0)$

$$\left| \frac{0-0-16}{\sqrt{5}} \right| = \frac{16}{\sqrt{5}}$$

perpendicular distance of (2) from  $(0, 0)$  is

$$\left| \frac{0-0+8}{\sqrt{5}} \right| = \frac{8}{\sqrt{5}}$$

17. The number of solutions of the equation

$$32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \leq x \leq \frac{\pi}{4} \text{ is :}$$

- (1) 3    (2) 1  
 (3) 0    (4) 2

**Official Ans. by NTA (2)**

**Sol.**  $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$$

$$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$$

In interval  $\left[0, \frac{\pi}{4}\right]$  only one solution

18. Let  $f$  be any continuous function on  $[0, 2]$  and twice differentiable on  $(0, 2)$ . If  $f(0) = 0$ ,  $f(1) = 1$  and  $f(2) = 2$ , then

- (1)  $f''(x) = 0$  for all  $x \in (0, 2)$   
 (2)  $f''(x) = 0$  for some  $x \in (0, 2)$   
 (3)  $f'(x) = 0$  for some  $x \in [0, 2]$   
 (4)  $f''(x) > 0$  for all  $x \in (0, 2)$

**Official Ans. by NTA (2)**

**Sol.**  $f(0) = 0$   $f(1) = 1$  and  $f(2) = 2$

Let  $h(x) = f(x) - x$  has three roots



By Rolle's theorem  $h'(x) = f'(x) - 1$  has at least two roots

$h''(x) = f''(x) = 0$  has at least one roots

19. If  $[x]$  is the greatest integer  $\leq x$ , then

$\pi^2 \int_0^2 \left( \sin \frac{\pi x}{2} \right) (x - [x])^{[x]} dx$  is equal to :

- (1)  $2(\pi - 1)$                       (2)  $4(\pi - 1)$   
 (3)  $4(\pi + 1)$                       (4)  $2(\pi + 1)$

**Official Ans. by NTA (2)**

**Sol.**  $\pi^2 \left[ \int_0^1 \sin \frac{\pi x}{2} dx + \int_1^2 \sin \frac{\pi x}{2} (x-1) dx \right]$   
 $= \pi^2 \left[ -\frac{2}{\pi} \left( \cos \frac{\pi x}{2} \right) + \left( (x-1) \left( -\frac{2}{\pi} \cos \frac{\pi x}{2} \right) \right)_1^2 - \int_1^2 -\frac{2}{\pi} \cos \frac{\pi x}{2} dx \right]$   
 $= \pi^2 \left[ 0 + \frac{2}{\pi} + \frac{2}{\pi} + \frac{2}{\pi} \cdot \frac{2}{\pi} \left( \sin \frac{\pi x}{2} \right)_1^2 \right]$   
 $= 4\pi - 4 = 4(\pi - 1)$

20. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :

- (1)  $\frac{92}{5}$                                       (2)  $\frac{134}{5}$   
 (3)  $\frac{536}{25}$                                       (4)  $\frac{112}{5}$

**Official Ans. by NTA (3)**

**Sol.** Let 8, 16,  $x_1, x_2, x_3, x_4, x_5$  be the observations.

Now  $\frac{x_1 + x_2 + \dots + x_5 + 14}{7} = 8$

$\Rightarrow \sum_{i=1}^5 x_i = 42$                       ... (1)

Also  $\frac{x_1^2 + x_2^2 + \dots + x_5^2 + 8^2 + 6^2}{7} - 64 = 16$

$\Rightarrow \sum_{i=1}^5 x_i^2 = 560 - 100 = 460$                       ... (2)

So variance of  $x_1, x_2, \dots, x_5$

$= \frac{460}{5} - \left( \frac{42}{5} \right)^2 = \frac{2300 - 1764}{25} = \frac{536}{25}$

**SECTION-B**

1. If the coefficient of  $a^7 b^8$  in the expansion of  $(a + 2b + 4ab)^{10}$  is  $K \cdot 2^{16}$ , then K is equal to \_\_\_\_\_.

**Official Ans. by NTA (315)**

**Sol.**  $\frac{10!}{\alpha! \beta! \gamma!} a^\alpha (2b)^\beta \cdot (4ab)^\gamma$

$\frac{10!}{\alpha! \beta! \gamma!} a^{\alpha+\gamma} \cdot b^{\beta+\gamma} \cdot 2^\beta \cdot 4^\gamma$

$\alpha + \beta + \gamma = 10$                       ... (1)

$\alpha + \gamma = 7$                                       ... (2)

$\beta + \gamma = 8$                                       ... (3)

$(2) + (3) - (1) \Rightarrow \gamma = 5$

$\alpha = 2$

$\beta = 3$

so coefficients =  $\frac{10!}{2!3!5!} 2^3 \cdot 2^{10}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{2 \times 3 \times 2 \times 5!} \times 2^{13}$

$= 315 \times 2^{16} \Rightarrow k = 315$

2. Suppose the line  $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$  lies on the plane  $x + 3y - 2z + \beta = 0$ . Then  $(\alpha + \beta)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (7)**

**Sol.** Point (2, 2, -2) also lies on given plane

So  $2 + 3 \times 2 - 2(-2) + \beta = 0$

$\Rightarrow 2 + 6 + 4 + \beta = 0 \Rightarrow \beta = -12$

Also  $\alpha \times 1 - 5 \times 3 + 2 \times -2 = 0$

$\Rightarrow \alpha - 15 - 4 = 0 \Rightarrow \alpha = 19$

$\therefore \alpha + \beta = 19 - 12 = 7$

3. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is \_\_\_\_\_.

**Official Ans. by NTA (5143)**

**Sol.** A = 4 - digit numbers divisible by 3

$A = 1002, 1005, \dots, 9999.$

$9999 = 1002 + (n - 1)3$



$$\Rightarrow (n - 1)3 = 8997 \Rightarrow n = 3000$$

B = 4 - digit numbers divisible by 7

$$B = 1001, 1008, \dots, 9996$$

$$\Rightarrow 9996 = 1001 + (n - 1)7$$

$$\Rightarrow n = 1286$$

$$A \cap B = 1008, 1029, \dots, 9996$$

$$9996 = 1008 + (n - 1)21$$

$$\Rightarrow n = 429$$

So, no divisible by either 3 or 7

$$= 3000 + 1286 - 429 = 3857$$

$$\text{total 4-digits numbers} = 9000$$

$$\text{required numbers} = 9000 - 3857 = 5143$$

4. If  $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx =$

$$\alpha \log_e |1 + \tan x| + \beta \log_e |1 - \tan x + \tan^2 x| + \gamma \tan^{-1} \left( \frac{2 \tan x - 1}{\sqrt{3}} \right) + C,$$

when C is constant of integration, then the value of  $18(\alpha + \beta + \gamma^2)$  is \_\_\_\_\_.

**Official Ans. by NTA (3)**

Sol. 
$$= \int \frac{\frac{\sin x}{\cos^3 x}}{1 + \tan^3 x} dx = \int \frac{\tan x \cdot \sec^2 x}{(\tan x + 1)(1 + \tan^2 x - \tan x)} dx$$

Let  $\tan x = t \Rightarrow \sec^2 x \cdot dx = dt$

$$= \int \frac{t}{(t+1)(t^2-t+1)} dt$$

$$= \int \left( \frac{A}{t+1} + \frac{B(2t-1)}{t^2-t+1} + \frac{C}{t^2-t+1} \right) dx$$

$$\Rightarrow A(t^2 - t + 1) + B(2t - 1)(t^2 - t + 1) + C(t + 1) = t$$

$$\Rightarrow t^2(A + 2B) + t(-A + B + C) + A - B + C = 1$$

$$\therefore A + 2B = 0 \quad \dots(1)$$

$$-A + B + C = 1 \quad \dots(2)$$

$$A - B + C = 0 \quad \dots(3)$$

$$\Rightarrow C = \frac{1}{2} \Rightarrow A - B = -\frac{1}{2} \quad \dots(4)$$

$$A + 2B = 0$$

$$A - B = -\frac{1}{2}$$

$$\Rightarrow 3B = \frac{1}{2} \Rightarrow B = \frac{1}{6}$$

$$A = -\frac{1}{3}$$

$$I = -\frac{1}{3} \int \frac{dt}{1+t} + \frac{1}{6} \int \frac{2t-1}{t^2-t+1} dt + \frac{1}{2} \int \frac{dt}{t^2-t+1}$$

$$= -\frac{1}{3} \ln|1 + \tan x| + \frac{1}{6} \ln|\tan^2 x - \tan x + 1|$$

$$+ \frac{1}{2} \cdot \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{\tan x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right)$$

$$= -\frac{1}{3} \ln|1 + \tan x| + \frac{1}{6} \ln|\tan^2 x - \tan x + 1|$$

$$+ \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{2 \tan x - 1}{\sqrt{3}} \right) + C$$

$$\alpha = -\frac{1}{3}, \beta = \frac{1}{6}, \gamma = \frac{1}{\sqrt{3}}$$

$$18(\alpha + \beta + \gamma^2) = 18 \left( -\frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right) = 3$$

5. A tangent line L is drawn at the point (2, -4) on the parabola  $y^2 = 8x$ . If the line L is also tangent to the circle  $x^2 + y^2 = a$ , then 'a' is equal to \_\_\_\_\_.

**Official Ans. by NTA (2)**

Sol. tangent of  $y^2 = 8x$  is  $y = mx + \frac{2}{m}$

$$P(2, -4) \Rightarrow -4 = 2m + \frac{2}{m}$$

$$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$$

$$\therefore \text{tangent is } y = -x - 2$$

$$\Rightarrow x + y + 2 = 0 \quad \dots(1)$$



(1) is also tangent to  $x^2 + y^2 = a$

$$\text{So } \frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$$

$$\Rightarrow a = 2$$

6. If  $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots$ , then  $160S$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (305)**

**Sol.**

$$S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots$$

$$\frac{1}{5}S = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \dots$$

On subtracting

$$\frac{4}{5}S = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \dots$$

$$S = \frac{7}{4} + \frac{1}{10} \left( 1 + \frac{2}{5} + \frac{3}{5^2} + \dots \right)$$

$$S = \frac{7}{4} + \frac{1}{10} \left( 1 - \frac{1}{5} \right)^{-2}$$

$$= \frac{7}{4} + \frac{1}{10} \times \frac{25}{16} = \frac{61}{32}$$

$$\Rightarrow 160S = 5 \times 61 = 305$$

7. The number of elements in the set

$$\left\{ A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \{-1, 0, 1\} \text{ and } (I - A)^3 = I - A^3 \right\},$$

where  $I$  is  $2 \times 2$  identity matrix, is :

**Official Ans. by NTA (8)**

**Sol.**  $(I - A)^3 = I^3 - A^3 - 3A(I - A) = I - A^3$

$$\Rightarrow 3A(I - A) = 0 \text{ or } A^2 = A$$

$$\Rightarrow \begin{bmatrix} a^2 & ab + bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$$\Rightarrow a^2 = a, b(a + d - 1) = 0, d^2 = d$$

If  $b \neq 0, a + d = 1 \Rightarrow 4$  ways

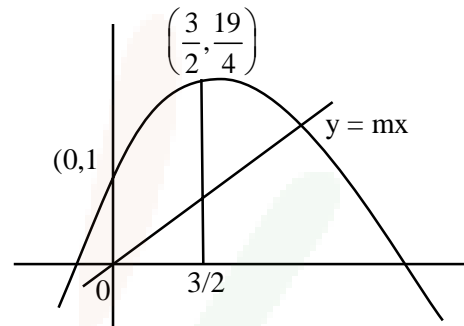
If  $b = 0, a = 0, 1$  &  $d = 0, 1 \Rightarrow 4$  ways

$\Rightarrow$  Total 8 matrices

8. If the line  $y = mx$  bisects the area enclosed by the lines  $x = 0, y = 0, x = \frac{3}{2}$  and the curve  $y = 1 + 4x - x^2$ , then  $12m$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (26)**

**Sol.**



$$\text{Total area} = \int_0^{3/2} (1 + 4x - x^2) dx$$

$$= x + 2x^2 - \frac{x^3}{3} \Big|_0^{3/2} = \frac{39}{8}$$

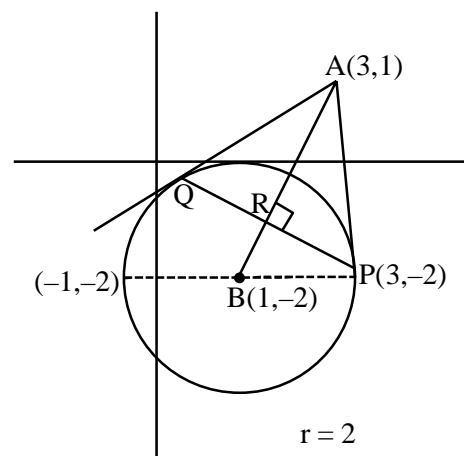
$$\& \frac{39}{16} = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} m$$

$$\Rightarrow 3m = \frac{13}{2} \Rightarrow 12m = 26$$

9. Let  $B$  be the centre of the circle  $x^2 + y^2 - 2x + 4y + 1 = 0$ . Let the tangents at two points  $P$  and  $Q$  on the circle intersect at the point  $A(3, 1)$ . Then  $8 \cdot \left( \frac{\text{area } \Delta APQ}{\text{area } \Delta BPQ} \right)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (18)**

**Sol.**





$$\tan \theta = \frac{3}{2}$$

$$\frac{\text{Area } \triangle APQ}{\text{Area } \triangle BPQ} = \frac{AR}{RB} = \frac{3 \sin \theta}{2 \cos \theta} = \frac{9}{4}$$

$$8 \left( \frac{\text{Area } \triangle APQ}{\text{Area } \triangle BPQ} \right) = 18$$

10. Let  $f(x)$  be a cubic polynomial with  $f(1) = -10$ ,  $f(-1) = 6$ , and has a local minima at  $x = 1$ , and  $f'(x)$  has a local minima at  $x = -1$ . Then  $f(3)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (22)**

**Sol.**  $F'(x) = a(x - 1)(x + 3)$

$$F''(x) = 6a(x + 1)$$

$$F'(x) = 3a(x + 1)^2 + b$$

$$F'(1) = 0 \Rightarrow b = -12a$$

$$F(x) = a(x + 1)^3 - 12ax + c$$

$$= (x + 1)^3 - 12x - 6$$

$$F(3) = 64 - 36 - 6 = 22$$