FINAL JEE–MAIN EXAMINATION – AUGUST, 2021
Held On Friday 27th August, 2021
TIME: 9:00 AM to 12:00 NOON

SECTION-A

1. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin :-

(1) $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

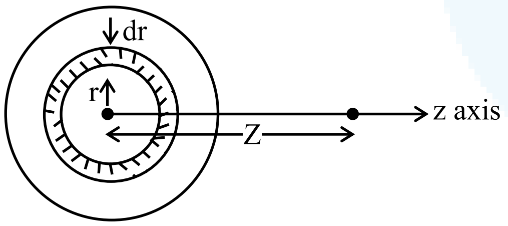
(2) $E = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

(3) $E = \frac{2\varepsilon_0}{\sigma}\left(\frac{1}{(Z^2+R^2)^{1/2}} + Z\right)$

(4) $E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{(Z^2+R^2)} + \frac{1}{Z^2}\right)$

Official Ans. by NTA (1)

Sol. Consider a small ring of radius r and thickness dr on disc.



area of elemental ring on disc

$dA = 2\pi r dr$

charge on this ring $dq = \sigma dA$

$$dE_z = \frac{kdqz}{(z^2+r^2)^{3/2}}$$

$$E = \int_0^R dE_z = \frac{\sigma}{2\in_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]$$

2. There are 10^{10} radioactive nuclei in a given radioactive element, Its half-life time is 1 minute. How many nuclei will remain after 30 seconds ? $\left(\sqrt{2} = 1.414\right)$

(1) 2×10^{10}

(2) 7×10^{9}

(3) 10^{5}

(4) 4×10^{10}

Official Ans. by NTA (2)

Sol. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$

$$\frac{N}{10^{10}} = \left(\frac{1}{2}\right)^{\frac{30}{60}}$$

$$\Rightarrow \quad N = 10^{10} \times \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{10^{10}}{\sqrt{2}} \approx 7 \times 10^{9}$$

3. Which of the following is not a dimensionless quantity ?

(1) Relative magnetic permeability (μ_r)

(2) Power factor

(3) Permeability of free space (μ_0)

(4) Quality factor

Official Ans. by NTA (3)

Sol. $[\mu_r] = 1$ as $\mu_r = \frac{\mu}{\mu_m}$

[power factor ($\cos\phi$)] = 1

$\mu_0 = \frac{B_0}{H}$ (unit = NA^{-2}) : Not dimensionless

$[\mu_0] = [MLT^{-2}A^{-2}]$

quality factor (Q) = $\frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$

So Q is unitless & dimensionless.

4. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be :

(1) ohm (2) mho

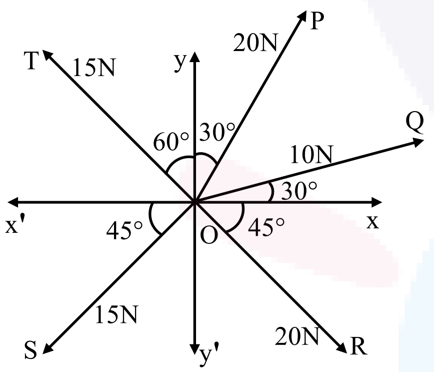
(3) joule (4) newton

Official Ans. by NTA (1)

Sol. Unit of $\frac{E}{H}$ is $\frac{\text{volt / metre}}{\text{Ampere / metre}} = \frac{\text{volt}}{\text{Ampere}} = \text{ohm}$

5. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately ...... N.

[Take $\sqrt{3} = 1.7, \sqrt{2} = 1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis]

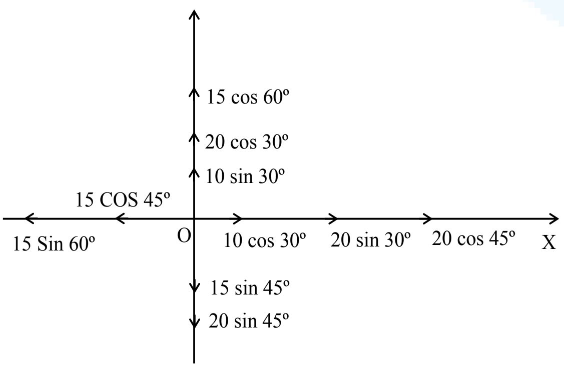


(1) $9.25\hat{i} + 5\hat{j}$ (2) $3\hat{i} + 15\hat{j}$

(3) $2.5\hat{i} - 14.5\hat{j}$ (4) $-1.5\hat{i} - 15.5\hat{j}$

Official Ans. by NTA (1)

Sol.



$$\vec{F}_x = \left(10 \times \frac{\sqrt{3}}{2} + 20\left(\frac{1}{2}\right) + 20\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{\sqrt{3}}{2}\right)\right)\hat{i}$$

$= 9.25\ \hat{i}$

$$\vec{F}_y = \left(15\left(\frac{1}{2}\right) + 20\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 20\left(\frac{1}{\sqrt{2}}\right)\right)\hat{j}$$

$= 5\ \hat{j}$

6. A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is –7°C. Assuming the volume constant ?

(1) 181.46 kg (2) 214.15 kg.

(3) 219.07 kg (4) 123.54 kg

Official Ans. by NTA (4)

Sol. $P_m = \rho RT$

$$\therefore \frac{P_1}{P_2} = \frac{\rho_1 T_1}{\rho_2 T_2}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{P_1 T_2}{P_2 T_1} = \left(\frac{76}{45}\right) \times \frac{266}{300}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{M_1}{M_2} = \frac{76 \times 266}{45 \times 300}$$

$$\therefore M_2 \Rightarrow \frac{45 \times 300 \times 185}{76 \times 266} = 123.54 \text{ kg}$$

7. An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C, the radius of curvature of this mirror is :

(1) $\frac{2d_1d_2}{d_1 - d_2}$

(2) $\frac{2d_1d_2}{d_1 + d_2}$

(3) $\frac{d_1d_2}{d_1 + d_2}$

(4) $\frac{d_1d_2}{d_1 - d_2}$

Official Ans. by NTA (1)

Sol. Using Newton"s formula

$$(f + d_1)(f - d_2) = f^2$$

$$f^2 + fd_1 - fd_2 - d_1d_2 = f^2$$

$$f = \frac{d_1d_2}{d_1 - d_2}$$

$$\therefore R = \frac{2d_1d_2}{d_1 - d_2}$$

8. A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second ?
Given : 1 ly = 9.46×10^{15} m
1 AU = 1.5×10^{11} m

(1) 4.1×10^{8} s (2) 4.5×10^{10} s
(3) 3.5×10^{6} s (4) 7.2×10^{8} s

Official Ans. by NTA (2)

Sol. $R = \frac{\ell}{\theta}$

$$\text{Time} = \frac{4 \times 2\pi R}{v} = \frac{4 \times 2\pi}{v}\left(\frac{\ell}{\theta}\right)$$

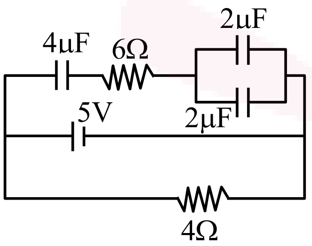
put $\ell = 4.4 \times 9.46 \times 10^{15}$

$v = 8 \times 1.5 \times 10^{11}$

$$\theta = \frac{4}{3600} \times \frac{\pi}{180} \text{ rad.}$$

we get time = 4.5×10^{10} sec

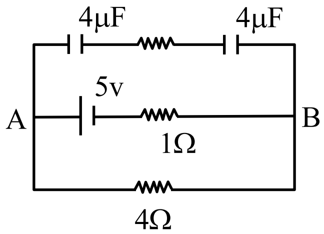
9. Calculate the amount of charge on capacitor of 4 μF. The internal resistance of battery is 1Ω :



(1) 8 μC (2) zero
(3) 16 μC (4) 4 μC

Official Ans. by NTA (1)

Sol. On simplifying circuit we get



No current in upper wire.

$$\therefore \; V_{AB} = \frac{5}{4+1} \times 4 = 4 \text{ v.}$$

$\therefore \; \theta = (C_{eq})v$

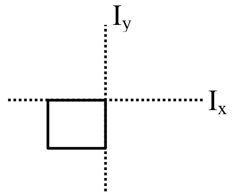
$\Rightarrow 2 \times 4 = 8\mu C$

10. Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by :

(1) $\frac{Ml^2}{6}$ (2) Ml^2 (3) $\frac{Ml^2}{12}$ (4) $\frac{2}{3}Ml^2$

Official Ans. by NTA (4)

Sol. According to perpendicular Axis theorem.



$I_x + I_y = I_z$

$$I_z \Rightarrow \frac{m\ell^2}{3} + \frac{m\ell^2}{3}$$

$$= \frac{2m\ell^2}{3}$$

11. For a transistor in CE mode to be used as an amplifier, it must be operated in :
(1) Both cut-off and Saturation
(2) Saturation region only
(3) Cut-off region only
(4) The active region only

Official Ans. by NTA (4)

Sol. Active region of the CE transistor is linear region and is best suited for its use as an amplifier

12. An ideal gas is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is :

(1) $\frac{1}{T}$ (2) $\frac{2}{T}$ (3) $\frac{4}{T}$ (4) $\frac{3}{T}$

Official Ans. by NTA (3)

Sol. PT^3 = constant

$$\left(\frac{nRT}{v}\right)T^3 = \text{constant}$$

$T^4 V^{-1}$ = constant

$T^4 = kV$

$$\Rightarrow 4\frac{\Delta T}{T} = \frac{\Delta V}{V} \text{ ....(1)}$$

$\Delta V = V\gamma\Delta T$.............(2)

comparing (1) and (2)
we get

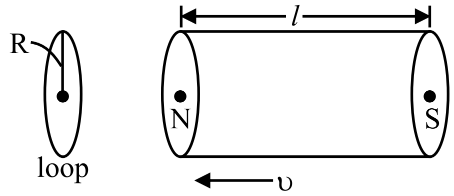
$$\gamma = \frac{4}{T}$$

13. In a photoelectric experiment, increasing the intensity of incident light :
(1) increases the number of photons incident and also increases the K.E. of the ejected electrons
(2) increases the frequency of photons incident and increases the K.E. of the ejected electrons.
(3) increases the frequency of photons incident and the K.E. of the ejected electrons remains unchanged
(4) increases the number of photons incident and the K.E. of the ejected electrons remains unchanged

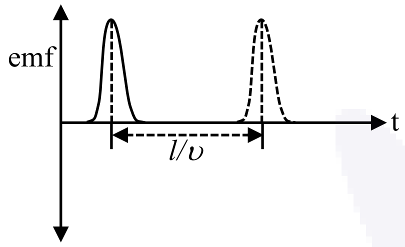
Official Ans. by NTA (4)

Sol. → Increasing intensity means number of incident photons are increased.
→Kinetic energy of ejected electrons depend on the frequency of incident photons, not the intensity.

14. A bar magnet is passing through a conducting loop of radius R with velocity υ. The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve :



(1)



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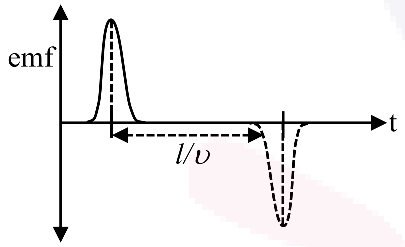

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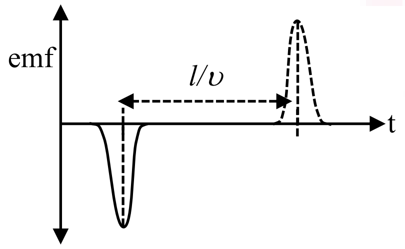

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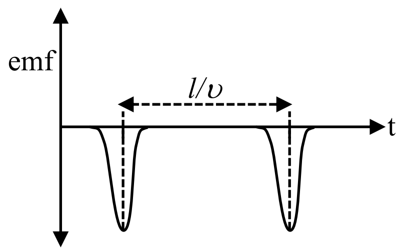
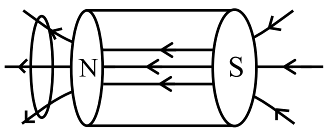


Official Ans. by NTA (3)

Sol.



→ When magnet passes through centre region of solenoid , no current / Emf is induced in loop.

→ While entering flux increases so negative induced emf

→ While leaving flux decreases so positive induced emf.

15. Two ions of masses 4 amu and 16 amu have charges +2e and +3e respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same. Then :

(1) lighter ion will be deflected less than heavier ion
(2) lighter ion will be deflected more than heavier ion
(3) both ions will be deflected equally
(4) no ion will be deflected.

Official Ans. by NTA (2)

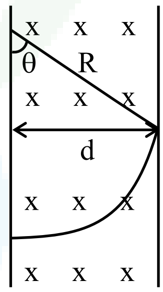
Sol. $r = \dfrac{P}{qB} = \dfrac{\sqrt{2mk}}{qB}$

Given they have same kinetic energy

$r \propto \dfrac{\sqrt{m}}{q}$

$\dfrac{r_1}{r_2} = \dfrac{\sqrt{4}}{2} \times \dfrac{3}{\sqrt{16}} = \dfrac{3}{4}$

$\boxed{r_2 = \dfrac{4r_1}{3}}$ (r_2 is for hearier ion and r_1 is for lighter ion)



$\sin\theta = \dfrac{d}{R}$

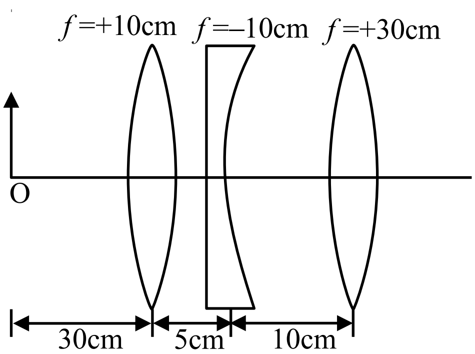
$\theta \rightarrow$ Deflection

$\theta \propto \dfrac{1}{R}$

($R \rightarrow$ Radius of path)

$\because R_2 > R_1 \Rightarrow \theta_2 < \theta_1$

16. Find the distance of the image from object O, formed by the combination of lenses in the figure :



(1) 75 cm (2) 10 cm
(3) 20 cm (4) infinity

Official Ans. by NTA (1)

Sol. $\frac{1}{V_1}+\frac{1}{30}=\frac{1}{10}$

$\frac{1}{V_1}=\frac{2}{30} \Rightarrow V_1 = 15$ cm

$\frac{1}{V_2}-\frac{1}{10}=-\frac{1}{10}$

$\frac{1}{V_2}=0$ $\qquad V_2=\infty$

$V_3 = 30$ cm

$OV_3 = 75$ cm

17. In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^{3} kgm^{-3} ? Take viscosity of liquid = 1.8×10^{-5} Nsm^{-2}. (Neglect buoyancy due to air).

(1) 3.8×10^{-11} N (2) 3.9×10^{-10} N
(3) 1.8×10^{-10} N (4) 5.8×10^{-10} N

Official Ans. by NTA (2)

Sol. Viscous force = Weight

$= \rho \times \left(\frac{4}{3}\pi r^3\right)g$

$= 3.9 \times 10^{-10}$

18. Electric field in a plane electromagnetic wave is given by $E = 50 \sin(500x - 10 \times 10^{10}t)$ V/m
The velocity of electromagnetic wave in this medium is :
(Given C = speed of light in vacuum)

(1) $\frac{3}{2}C$ (2) C (3) $\frac{2}{3}C$ (4) $\frac{C}{2}$

Official Ans. by NTA (3)

Sol. $V = \frac{\omega}{K} = \frac{10\times10^{10}}{500} = 2 \times 10^8$

$V = \frac{2C}{3}$.

19. Five identical cells each of internal resistance 1Ω and emf 5V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same ?

(1) $1\ \Omega$ (2) $25\ \Omega$
(3) $5\ \Omega$ (4) $10\ \Omega$

Official Ans. by NTA (1)

Sol. $i_1 = \frac{25}{5+R}$

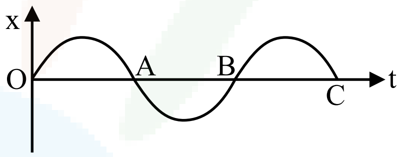
$i_2 = \frac{5}{R+\frac{1}{5}}$

$i_1 = i_2 \Rightarrow 5\left(R+\frac{1}{5}\right)=5+R$

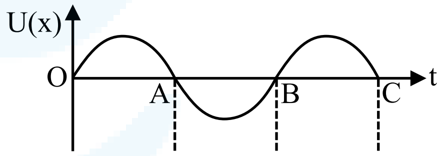
$4R = 4$

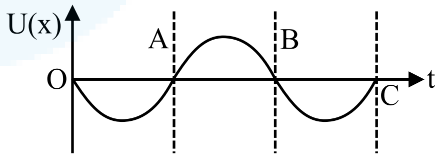
$R = 1\Omega$

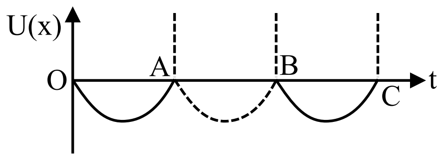
20. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.

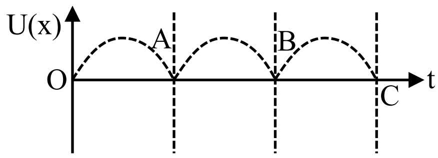


The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure :

(1) 

(2) 

(3) 

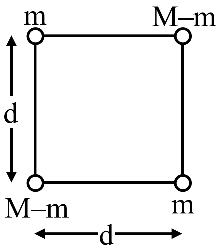
(4) 

Official Ans. by NTA (4)

Sol. Potential energy is maximum at maximum distance from mean.

SECTION-B

1. A body of mass (2M) splits into four masses {m, M – m, m, M – m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x : 1. The value of x is ....



Official Ans. by NTA (2)

Sol. Energy is maximum when mass is split equally so

$$\frac{M}{m}=2$$

2. The alternating current is given by

$$i=\left\{\sqrt{42}\sin\left(\frac{2\pi}{T}t\right)+10\right\}A$$

The r.m.s. value of this current is .... A.

Official Ans. by NTA (11)

Sol. $f_{rms}^2=f_{1rms}^2+f_{2rms}^2$

$$=\left(\frac{\sqrt{42}}{\sqrt{2}}\right)^2+10^2$$

$= 121 \quad \Rightarrow \quad f_{rms} = 11$ A

3. A uniform conducting wire of length is 24a, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0. The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1:\sqrt{y}$ where y is ....

Official Ans. by NTA (3)

Sol. In triangle shape $N_t=\frac{24a}{3a}=8$

In square $N_s=\frac{24a}{4a}=6$

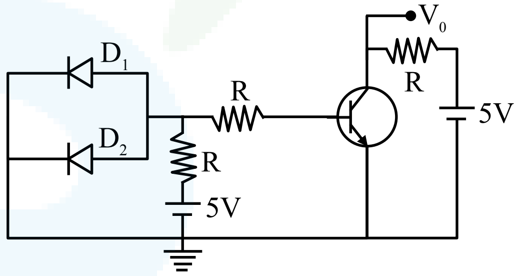
$\frac{M_t}{M_3}=\frac{N_tIA_t}{N_sIA_s}$ [I will be same in both]

$$=\frac{8\times\frac{\sqrt{3}}{4}\times a^2}{6\times a^2}$$

$$\frac{M_t}{M_s}=\frac{1}{\sqrt{3}}$$

$\boxed{y=3}$

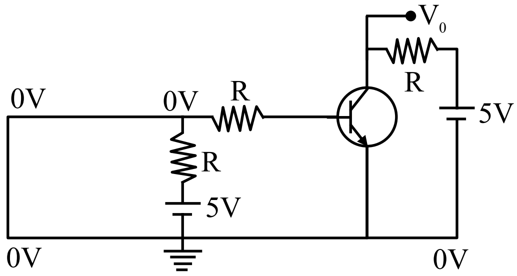
4. A circuit is arranged as shown in figure. The output voltage V_0 is equal to .... V.



Official Ans. by NTA (5)

Sol. As diodes D_1 and D_2 are in forward bias, so they acted as neligible resistances

⇒ Input voltage become zero



⇒ Input current is zero

⇒ Output current is zero

⇒ V_0 = 5 volt

5. First, a set of n equal resistors of 10 Ω each are connected in series to a battery of emf 20V and internal resistance 10 Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is ....

Official Ans. by NTA (20)

Sol. In series

$R_{eq} = nR = 10\ n$

$i_s = \dfrac{20}{10+10n} = \dfrac{2}{1+n}$

in parallel

$R_{eq} = \dfrac{10}{n}$

$i_p = \dfrac{20}{\frac{10}{n}+10} = \dfrac{2n}{1+n}$

$\dfrac{i_p}{i_s} = 20$

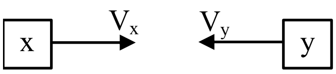
$\dfrac{\left(\frac{2n}{1+n}\right)}{\left(\frac{2}{1+n}\right)} = 20$

$n = 20$

6. Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is .... Hz.

Official Ans. by NTA (1210)

Sol.



$V_x = 36$ km/hr $= 10$ m/s

$V_y = 72$ km/hr $= 20$ m/s

by doppler's effect

$F' = F_0\left(\dfrac{V \pm V_0}{V \pm V_s}\right)$

$1320 = F_0\left(\dfrac{340+20}{340-10}\right) \Rightarrow F_0 = 1210$ Hz

7. If the velocity of a body related to displacement x is given by $\upsilon = \sqrt{5000 + 24x}$ m/s, then the acceleration of the body is ...... m/s^2.

Official Ans. by NTA (12)

Sol. $V = \sqrt{5000+24x}$

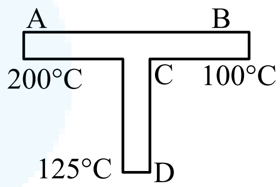
$\dfrac{dV}{dx} = \dfrac{1}{2\sqrt{5000+24x}} \times 24 = \dfrac{12}{\sqrt{5000+24x}}$

now $a = V\dfrac{dV}{dx}$

$= \sqrt{5000+24x} \times \dfrac{12}{\sqrt{5000+24x}}$

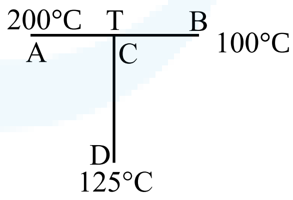
$\boxed{a = 12 m/s^2}$

8. A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure, The end A, B and D are maintained at 200°C, 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is ....



Official Ans. by NTA (2)

Sol.



Rods are identical so

$R_{AB} = R_{CD} = 10$ Kw^{-1}

C is mid-point of AB, so

$R_{AC} = R_{CB} = 5$ Kw^{-1}

at point C

$\dfrac{200-T}{5} = \dfrac{T-125}{10} + \dfrac{T-100}{5}$

$2(200 - T) = T - 125 + 2(T - 100)$

$400 - 2T = T - 125 + 2T - 200$

$T = \dfrac{725}{5} = 145°C$

$I_h = \dfrac{145-125}{10} w = \dfrac{20}{10} w$

$\boxed{I_h = 2w}$

9. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ....

Official Ans. by NTA (2)

Sol. Given $W_A = W_B$

$F_A d\cos 45° = F_B d\cos 60°$

$$F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2}$$

$$\frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\boxed{x = 2}$$

10. A transmitting antenna has a height of 320 m and that of receiving antenna is 2000 m. The maximum distance between them for satisfactory communication in line of sight mode is 'd'. The value of 'd' is .... km.

Official Ans. by NTA (224)

Sol. $d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$d_m = \left(\sqrt{2 \times 6400 \times 10^3 \times 320} + \sqrt{2 \times 6400 \times 10^3 \times 2000}\right) m$$

$$\boxed{d_m = 224 km}$$