Class XII : Maths
Chapter 1 : Relations And Functions

Questions and Solutions | Exercise 1.2 - NCERT Books

Question 1:

Show that the function f: $\mathbf{R}_*$ → $\mathbf{R}_*$ defined by $f(x)=\frac{1}{x}$ is one-one and onto, where $\mathbf{R}_*$ is the set of all non-zero real numbers. Is the result true, if the domain $\mathbf{R}_*$ is replaced by **N** with co-domain being same as $\mathbf{R}_*$? Answer

It is given that f: $\mathbf{R}_*$ → $\mathbf{R}_*$ is defined by $f(x)=\frac{1}{x}$. One-one:

$f(x)=f(y)$

$\Rightarrow \frac{1}{x}=\frac{1}{y}$

$\Rightarrow x=y$

∴f is one-one.

Onto:

It is clear that for $y \in \mathbf{R}_*$, there exists $x=\frac{1}{y} \in \mathrm{R}_*$ (Exists as $y \neq 0$) such that

$f(x)=\frac{1}{\left(\frac{1}{y}\right)}=y.$ ∴f

is onto.

Thus, the given function (f) is one-one and onto. Now,

consider function g: **N** → $\mathbf{R}_*$defined by

$g(x)=\frac{1}{x}.$

We have,

$g(x_1)=g(x_2) \Rightarrow \frac{1}{x_1}=\frac{1}{x_2} \Rightarrow x_1=x_2$

∴g is one-one.

Further, it is clear that g is not onto as for $1.2 \in \mathbf{R}_*$ there does not exit any x in **N** such that $g(x) = \frac{1}{1.2}$.

Hence, function g is one-one but not onto.

Question 2:

Check the injectivity and surjectivity of the following functions:

(i) f: $\mathbf{N} \rightarrow \mathbf{N}$ given by $f(x) = x^2$

(ii) f: $\mathbf{Z} \rightarrow \mathbf{Z}$ given by $f(x) = x^2$

(iii) f: $\mathbf{R} \rightarrow \mathbf{R}$ given by $f(x) = x^2$

(iv) f: $\mathbf{N} \rightarrow \mathbf{N}$ given by $f(x) = x^3$

(v) f: $\mathbf{Z} \rightarrow \mathbf{Z}$ given by $f(x) = x^3$

Answer

(i) f: $\mathbf{N} \rightarrow \mathbf{N}$ is given by,

$f(x) = x^2$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any x in **N** such that $f(x) = x^2 = 2$.

$\therefore$ f is not surjective.

Hence, function f is injective but not surjective.

(ii) f: $\mathbf{Z} \rightarrow \mathbf{Z}$ is given by,

$f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore$ f is not injective.

Now,$-2 \in \mathbf{Z}$. But, there does not exist any element $x \in \mathbf{Z}$ such that $f(x) = x^2 = -2$.

$\therefore$ f is not surjective.

Hence, function f is neither injective nor surjective.

(iii) f: $\mathbf{R} \rightarrow \mathbf{R}$ is given by, $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore$ f is not injective.

Now, $-2 \in \mathbf{R}$. But, there does not exist any element $x \in \mathbf{R}$ such that $f(x) = x^2 = -2$.

$\therefore$ f is not surjective.

Hence, function f is neither injective nor surjective.

(iv) f: $\mathbf{N} \to \mathbf{N}$ given by,

$f(x) = x^3$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any element x in domain $\mathbf{N}$ such that $f(x) = x^3 = 2$.

$\therefore$ f is not surjective

Hence, function f is injective but not surjective.

(v) f: $\mathbf{Z} \to \mathbf{Z}$ is given by,

$f(x) = x^3$

It is seen that for $x, y \in \mathbf{Z}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore$ f is injective.

Now, $2 \in \mathbf{Z}$. But, there does not exist any element x in domain $\mathbf{Z}$ such that $f(x) = x^3 = 2$.

$\therefore$ f is not surjective.

Hence, function f is injective but not surjective.

Question 3:

Prove that the Greatest Integer Function f: $\mathbf{R} \to \mathbf{R}$ given by $f(x) = [x]$, is neither oneonce nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Answer f: $\mathbf{R} \to \mathbf{R}$ is given by, $f(x) = [x]$

It is seen that $f(1.2) = [1.2] = 1$, $f(1.9) = [1.9] = 1$.

$\therefore$ $f(1.2) = f(1.9)$, but $1.2 \neq 1.9$.

$\therefore$ f is not one-one.

Now, consider 0.7 ∈ **R**.

It is known that $f(x)$ = $[x]$ is always an integer. Thus, there does not exist any element x ∈ **R** such that $f(x)$ = 0.7.

∴ f is not onto.

Hence, the greatest integer function is neither one-one nor onto.

Question 4:

Show that the Modulus Function f: **R** → **R** given by $f(x)=|x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is − x, if x is negative.

Answer f: **R** → **R** is given by,

$$f(x)=|x|=\begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x<0 \end{cases}$$

It is seen that $f(-1)=|-1|=1, f(1)=|1|=1$.

∴$f(-1) = f(1)$, but −1 ≠ 1.

∴ f is not one-one.

Now, consider −1 ∈ **R**.

It is known that $f(x)$ = $|x|$ is always non-negative. Thus, there does not exist any element x in domain **R** such that $f(x)$ = $|x|$ = −1.

∴ f is not onto.

Hence, the modulus function is neither one-one nor onto.

Question 5:

Show that the Signum Function f: **R** → **R**, given by

$$f(x)=\begin{cases} 1, \text{ if } x>0 \\ 0, \text{ if } x=0 \\ -1, \text{ if } x<0 \end{cases}$$

is neither one-one nor onto.

Answer f: **R** → **R** is given

by,

$$f(x)=\begin{cases}1, \text{ if } x>0\\0, \text{ if } x=0\\-1, \text{ if } x<0\end{cases}$$

It is seen that $f(1) = f(2) = 1$, but $1 \neq 2$.

$\therefore f$ is not one-one.

Now, as $f(x)$ takes only 3 values (1, 0, or −1) for the element −2 in co-domain **R**, there does not exist any x in domain **R** such that $f(x) = -2$. $\therefore$ f is not onto.

Hence, the signum function is neither one-one nor onto.

Question 6:

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one. Answer

It is given that $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$. f:

$A \rightarrow B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$.

$\therefore f(1) = 4, f(2) = 5, f(3) = 6$

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one.

Question 7:

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) f: **R** → **R** defined by $f(x) = 3 - 4x$ (ii) f: **R** → **R** defined by $f(x) = 1 + x^2$ Answer (i) f: **R** → **R** is defined as $f(x) = 3 - 4x$.

Let $x_1,\ x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$

$\Rightarrow 3 - 4x_1 = 3 - 4x_2$
$\Rightarrow -4x_1 = -4x_2$
$\Rightarrow x_1 = x_2$

$\therefore$ *f* is one-one.

For any real number (*y*) in **R**, there exists $\frac{3-y}{4}$ in **R** such that

$$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y.$$

$\therefore$*f* is onto.

Hence, *f* is bijective.

(ii) *f*: **R** → **R** is defined as

$f(x) = 1 + x^2$

Let $x_1,\ x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$
$\Rightarrow x_1^2 = x_2^2$
$\Rightarrow x_1 = \pm x_2$

$\therefore$ $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$. For instance, $f(1) = f(-1) = 2$

$\therefore$ *f* is not one-one.

Consider an element −2 in co-domain **R**.

It is seen that $f(x) = 1 + x^2$ is positive for all $x \in \mathbf{R}$.

Thus, there does not exist any *x* in domain **R** such that *f*(*x*) = −2.

$\therefore$ *f* is not onto.

Hence, *f* is neither one-one nor onto.

Question 8:

Let A and B be sets. Show that f: $A \times B \to B \times A$ such that $(a, b) = (b, a)$ is bijective function. Answer f: $A \times B \to B \times A$ is defined as $f(a, b) = (b, a)$.

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that $f(a_1, b_1) = f(a_2, b_2)$

$$\Rightarrow (b_1, a_1) = (b_2, a_2)$$
$$\Rightarrow b_1 = b_2 \text{ and } a_1 = a_2$$
$$\Rightarrow (a_1, b_1) = (a_2, b_2)$$

∴ f is one-one.

Now, let $(b, a) \in B \times A$ be any element.

Then, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$. [By definition of f]

∴ f is onto.

Hence, f is bijective.

Question 9:

Let f: $\mathbf{N} \to \mathbf{N}$ be defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in \mathbf{N}.$$

State whether the function f is bijective. Justify your answer. Answer

f: $\mathbf{N} \to \mathbf{N}$ is defined as

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in \mathbf{N}.$$

It can be observed that:

$$f(1) = \frac{1+1}{2} = 1 \text{ and } f(2) = \frac{2}{2} = 1 \qquad [\text{By definition of } f]$$

$$\therefore f(1) = f(2), \text{ where } 1 \neq 2.$$

∴ f is not one-one.

Consider a natural number (n) in co-domain **N.**

Case **I:** n is odd

$\therefore n = 2r + 1$ for some $r \in$ **N.** Then, there exists $4r + 1 \in$ **N** such that

$$f(4r+1) = \frac{4r+1+1}{2} = 2r+1$$

Case **II:** n is even

$\therefore n = 2r$ for some $r \in$ **N.** Then,there exists $4r \in$ **N** such that $f(4r) = \frac{4r}{2} = 2r$.

$\therefore$ f is onto.

Hence, f is not a bijective function.

Question 10:

Let A = **R** − {3} and B = **R** − {1}. Consider the function f: A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Answer

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x, y \in$ A such that $f(x) = f(y)$.

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$
$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$
$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$
$$\Rightarrow -3x - 2y = -3y - 2x$$
$$\Rightarrow 3x - 2x = 3y - 2y$$
$$\Rightarrow x = y$$

$\therefore$ f is one-one.

Let $y \in B = \mathbf{R} - \{1\}$. Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$. Now,

$$f(x) = y$$
$$\Rightarrow \frac{x-2}{x-3} = y$$
$$\Rightarrow x - 2 = xy - 3y$$
$$\Rightarrow x(1-y) = -3y + 2$$
$$\Rightarrow x = \frac{2-3y}{1-y} \in A \qquad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Question 11:

Let f: $\mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto Answer

f: $\mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x^4$.

Let $x, y \in \mathbf{R}$ such that $f(x) = f(y)$.

$$\Rightarrow x^4 = y^4$$
$$\Rightarrow x = \pm y$$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$.

For instance,

$f(1) = f(-1) = 1$

∴ *f* is not one-one.

Consider an element 2 in co-domain **R**. It is clear that there does not exist any *x* in domain **R** such that $f(x) = 2$.

∴ *f* is not onto.

Hence, function *f* is neither one-one nor onto.

The correct answer is D.

Question 12:

Let *f*: **R** → **R** be defined as $f(x) = 3x$. Choose the correct answer.

(A) *f* is one-one onto (B) *f* is many-one onto

(C) *f* is one-one but not onto (D) *f* is neither one-one nor onto

Answer

f: **R** → **R** is defined as $f(x) = 3x$. Let

$x, y \in$ **R** such that $f(x) = f(y)$.

$\Rightarrow 3x = 3y$

$\Rightarrow x = y$

∴*f* is one-one.

Also, for any real number (y) in co-domain **R**, there exists $\frac{y}{3}$ in **R** such that

$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$. ∴*f*

is onto.

Hence, function *f* is one-one and onto.

The correct answer is A.