Class XII : Maths
Chapter 2 : Inverse Trignometric Functions

Questions and Solutions | Exercise 2.2 - NCERT Books

Question 1:

Prove $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right),\ x\in\left[-\frac{1}{2},\ \frac{1}{2}\right]$

Answer

To prove: $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right),\ x\in\left[-\frac{1}{2},\ \frac{1}{2}\right]$

Let x = sinθ. Then, $\sin^{-1}x=\theta$.

We have,

R.H.S. = $\sin^{-1}\left(3x-4x^3\right)=\sin^{-1}\left(3\sin\theta-4\sin^3\theta\right)$

$=\sin^{-1}\left(\sin 3\theta\right)$

= 3θ

$=3\sin^{-1}x$

= L.H.S.

Question 2:

Prove $3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right),\ x\in\left[\frac{1}{2},\ 1\right]$

Answer

To prove: $3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right),\ x\in\left[\frac{1}{2},\ 1\right]$

Let x = cosθ. Then, $\cos^{-1}x=\theta$.

We have,

$$
\begin{aligned}
\text{R.H.S.} &= \cos^{-1}\left(4x^3-3x\right)\\
&= \cos^{-1}\left(4\cos^3\theta-3\cos\theta\right)\\
&= \cos^{-1}\left(\cos 3\theta\right)\\
&= 3\theta\\
&= 3\cos^{-1}x\\
&= \text{L.H.S.}
\end{aligned}
$$

Question 3:

Write the function in the simplest form:

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x},\ x\neq 0$$

Answer

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$$

Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$$\therefore \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x$$

Question 4:

Write the function in the simplest form:

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ x<\pi$$

Answer

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ x<\pi$$

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)=\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$$

$$=\tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)=\tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$=\frac{x}{2}$$

Question 5:

Write the function in the simplest form:

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right),\frac{-\pi}{4}<x<\frac{3\pi}{4}$$

Answer

The given function is $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Now,

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)=\tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)=\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$=\tan^{-1}\left(\frac{1-\tan x}{1+1.\tan x}\right)=\tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}\right)$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-x\right)\right]=\frac{\pi}{4}-x$$

Question 6:

Write the function in the simplest form:

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}},\ |x|<a$$

Answer

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

$$\text{Put } x=a\sin\theta \Rightarrow \frac{x}{a}=\sin\theta \Rightarrow \theta=\sin^{-1}\left(\frac{x}{a}\right)$$

$$\therefore \tan^{-1}\frac{x}{\sqrt{a^2-x^2}}=\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right)=\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$=\tan^{-1}(\tan\theta)=\theta=\sin^{-1}\frac{x}{a}$$

Question 7:

Write the function in the simplest form:

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right),\ a>0;\ \frac{-a}{\sqrt{3}}\le x\le\frac{a}{\sqrt{3}}$$

Answer

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$$

Put $x=a\tan\theta \Rightarrow \frac{x}{a}=\tan\theta \Rightarrow \theta=\tan^{-1}\frac{x}{a}$

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)=\tan^{-1}\left(\frac{3a^2\cdot a\tan\theta-a^3\tan^3\theta}{a^3-3a\cdot a^2\tan^2\theta}\right)$$

$$=\tan^{-1}\left(\frac{3a^3\tan\theta-a^3\tan^3\theta}{a^3-3a^3\tan^2\theta}\right)$$

$$=\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$$

$$=\tan^{-1}(\tan 3\theta)$$

$$=3\theta$$

$$=3\tan^{-1}\frac{x}{a}$$

Question 8:

Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Answer

Let $\sin^{-1}\frac{1}{2}=x$. Then, $\sin x=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$.

$$\therefore \sin^{-1}\frac{1}{2}=\frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$=\tan^{-1}\left[2\cos\frac{\pi}{3}\right]=\tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$=\tan^{-1}1=\frac{\pi}{4}$$

Find the value of $\cot\left(\tan^{-1} a+\cot^{-1} a\right)$

Answer

$\cot\left(\tan^{-1} a+\cot^{-1} a\right)$

$=\cot\left(\frac{\pi}{2}\right) \qquad \left[\tan^{-1} x+\cot^{-1} x=\frac{\pi}{2}\right]$

$=0$

Question 9:

Find the value of $\tan \frac{1}{2}\left[\sin^{-1} \frac{2x}{1+x^{2}}+\cos^{-1} \frac{1-y^{2}}{1+y^{2}}\right],\ |x|<1,\ y>0$ and $xy<1$

Answer

Let $x = \tan \theta$. Then, $\theta = \tan^{-1} x$.

$\therefore \sin^{-1} \frac{2x}{1+x^{2}}=\sin^{-1}\left(\frac{2 \tan \theta}{1+\tan^{2} \theta}\right)=\sin^{-1}(\sin 2\theta)=2\theta=2 \tan^{-1} x$

Let $y = \tan \Phi$. Then, $\Phi = \tan^{-1} y$.

$\therefore \cos^{-1} \frac{1-y^{2}}{1+y^{2}}=\cos^{-1}\left(\frac{1-\tan^{2} \phi}{1+\tan^{2} \phi}\right)=\cos^{-1}(\cos 2\phi)=2\phi=2 \tan^{-1} y$

$\therefore \tan \frac{1}{2}\left[\sin^{-1} \frac{2x}{1+x^{2}}+\cos^{-1} \frac{1-y^{2}}{1+y^{2}}\right]$

$=\tan \frac{1}{2}\left[2 \tan^{-1} x+2 \tan^{-1} y\right]$

$=\tan\left[\tan^{-1} x+\tan^{-1} y\right]$

$=\tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$=\frac{x+y}{1-xy}$

Question 10:

Find the values of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Answer

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

We know that $\sin^{-1}(\sin x) = x$ if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, which is the principal value branch of $\sin^{-1}x$.

Here, $\frac{2\pi}{3} \notin \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Now, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ can be written as:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{2\pi}{3}\right)\right] = \sin^{-1}\left(\sin\frac{\pi}{3}\right) \text{ where } \frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Question 11:

Find the values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Answer

$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Now, $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ can be written as:

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[-\tan\left(\frac{-3\pi}{4}\right)\right] = \tan^{-1}\left[-\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left[-\tan\frac{\pi}{4}\right] = \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] \text{ where } -\frac{\pi}{4} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{3\pi}{4}\right)=\tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right]=\frac{-\pi}{4}$$

Question 12:

Find the values of $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Answer

Let $\sin^{-1}\frac{3}{5}=x$. Then, $\sin x=\frac{3}{5}\Rightarrow \cos x=\sqrt{1-\sin^2 x}=\frac{4}{5}\Rightarrow \sec x=\frac{5}{4}$.

$$\therefore \tan x=\sqrt{\sec^2 x-1}=\sqrt{\frac{25}{16}-1}=\frac{3}{4}$$

$$\therefore x=\tan^{-1}\frac{3}{4}$$

$$\therefore \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4} \qquad ...(i)$$

$$\text{Now, } \cot^{-1}\frac{3}{2}=\tan^{-1}\frac{2}{3} \qquad ...(ii) \qquad \left[\tan^{-1}\frac{1}{x}=\cot^{-1}x\right]$$

$$\text{Hence, } \tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right) \qquad [\text{Using (i) and (ii)}]$$

$$=\tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right) \qquad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan\left(\tan^{-1}\frac{9+8}{12-6}\right)$$

$$=\tan\left(\tan^{-1}\frac{17}{6}\right)=\frac{17}{6}$$

Question 13:

Find the values of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$ **(B)** $\frac{5\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{6}$

Answer

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{7\pi}{6} \notin x \in [0, \pi]$.

Now, $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{-7\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{7\pi}{6}\right)\right] \quad [\cos(2\pi + x) = \cos x]$$

$$= \cos^{-1}\left[\cos\frac{5\pi}{6}\right] \text{ where } \frac{5\pi}{6} \in [0, \pi]$$

$$\therefore \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$$

The correct answer is B.

Question 14:

Find the values of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** 1

Answer

Let $\sin^{-1}\left(\frac{-1}{2}\right) = x$. Then, $\sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\frac{-\pi}{6}\right)$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore \sin^{-1}\left(\frac{-1}{2}\right)=\frac{-\pi}{6}$

$$\therefore \sin\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right)=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\left(\frac{3\pi}{6}\right)=\sin\left(\frac{\pi}{2}\right)=1$$

The correct answer is D.

Question 15:

$\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$ is equal to

(A) π (B) $-\frac{\pi}{2}$ (C) 0 (D) $2\sqrt{3}$

Answer

Given that $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\cot^{-1}$ is $(0,\pi)$.

$$\begin{aligned}
&\therefore \tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})\\
&=\tan^{-1}\left(\tan\frac{\pi}{3}\right)-\cot^{-1}\left(-\cot\frac{\pi}{6}\right)\\
&=\frac{\pi}{3}-\cot^{-1}\left[\cot\left(\pi-\frac{\pi}{6}\right)\right]\\
&=\frac{\pi}{3}-\cot^{-1}\left(\cot\frac{5\pi}{6}\right)\\
&=\frac{\pi}{3}-\frac{5\pi}{6}=\frac{2\pi-5\pi}{6}=-\frac{3\pi}{6}=-\frac{\pi}{2}
\end{aligned}$$

Hence, $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})=-\frac{\pi}{2}$