Class XII : Maths
Chapter 3 : Matrices

Questions and Solutions | Exercise 3.2 - NCERT Books

Question 1:

Let $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$

Find each of the following

(i) $A+B$ (ii) $A-B$ (iii) $3A-C$

(iv) AB (v) BA

Answer

(i)

$$A+B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}+\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}=\begin{bmatrix}2+1 & 4+3\\ 3-2 & 2+5\end{bmatrix}=\begin{bmatrix}3 & 7\\ 1 & 7\end{bmatrix}$$

(ii)

$$A-B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}=\begin{bmatrix}2-1 & 4-3\\ 3-(-2) & 2-5\end{bmatrix}=\begin{bmatrix}1 & 1\\ 5 & -3\end{bmatrix}$$

(iii)

$$3A-C=3\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$
$$=\begin{bmatrix}3\times 2 & 3\times 4\\ 3\times 3 & 3\times 2\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$
$$=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$
$$=\begin{bmatrix}6+2 & 12-5\\ 9-3 & 6-4\end{bmatrix}$$
$$=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$$

(iv) Matrix *A* has 2 columns. This number is equal to the number of rows in matrix *B*.

Therefore, *AB* is defined as:

$$AB=\begin{bmatrix}2&4\\3&2\end{bmatrix}\begin{bmatrix}1&3\\-2&5\end{bmatrix}=\begin{bmatrix}2(1)+4(-2)&2(3)+4(5)\\3(1)+2(-2)&3(3)+2(5)\end{bmatrix}$$
$$=\begin{bmatrix}2-8&6+20\\3-4&9+10\end{bmatrix}=\begin{bmatrix}-6&26\\-1&19\end{bmatrix}$$

(v) Matrix *B* has 2 columns. This number is equal to the number of rows in matrix *A*. Therefore, *BA* is defined as:

$$BA=\begin{bmatrix}1&3\\-2&5\end{bmatrix}\begin{bmatrix}2&4\\3&2\end{bmatrix}=\begin{bmatrix}1(2)+3(3)&1(4)+3(2)\\-2(2)+5(3)&-2(4)+5(2)\end{bmatrix}$$
$$=\begin{bmatrix}2+9&4+6\\-4+15&-8+10\end{bmatrix}=\begin{bmatrix}11&10\\11&2\end{bmatrix}$$

Question 2:

Compute the following:

(i) $\begin{bmatrix}a&b\\-b&a\end{bmatrix}+\begin{bmatrix}a&b\\b&a\end{bmatrix}$ (ii) $\begin{bmatrix}a^2+b^2&b^2+c^2\\a^2+c^2&a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab&2bc\\-2ac&-2ab\end{bmatrix}$

(iii) $\begin{bmatrix}-1&4&-6\\8&5&16\\2&8&5\end{bmatrix}+\begin{bmatrix}12&7&6\\8&0&5\\3&2&4\end{bmatrix}$

(v) $\begin{bmatrix}\cos^2 x&\sin^2 x\\\sin^2 x&\cos^2 x\end{bmatrix}+\begin{bmatrix}\sin^2 x&\cos^2 x\\\cos^2 x&\sin^2 x\end{bmatrix}$

Answer

(i)

$$\begin{bmatrix}a&b\\-b&a\end{bmatrix}+\begin{bmatrix}a&b\\b&a\end{bmatrix}=\begin{bmatrix}a+a&b+b\\-b+b&a+a\end{bmatrix}=\begin{bmatrix}2a&2b\\0&2a\end{bmatrix}$$

(ii) $\begin{bmatrix}a^2+b^2&b^2+c^2\\a^2+c^2&a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab&2bc\\-2ac&-2ab\end{bmatrix}$

$$=\begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$=\begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix}+\begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$$=\begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$=\begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix}+\begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$$=\begin{bmatrix} \cos^2 x+\sin^2 x & \sin^2 x+\cos^2 x \\ \sin^2 x+\cos^2 x & \cos^2 x+\sin^2 x \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad (\because \sin^2 x+\cos^2 x=1)$$

Question 3:

Compute the indicated products

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix}$

(iv) $\begin{bmatrix}2 & 3 & 4\\3 & 4 & 5\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & -3 & 5\\0 & 2 & 4\\3 & 0 & 5\end{bmatrix}$

(v) $\begin{bmatrix}2 & 1\\3 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\-1 & 2 & 1\end{bmatrix}$

(vi) $\begin{bmatrix}3 & -1 & 3\\-1 & 0 & 2\end{bmatrix}\begin{bmatrix}2 & -3\\1 & 0\\3 & 1\end{bmatrix}$

Answer

(i) $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$

$$\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$$

$$=\begin{bmatrix}a(a)+b(b) & a(-b)+b(a)\\-b(a)+a(b) & -b(-b)+a(a)\end{bmatrix}$$

$$=\begin{bmatrix}a^2+b^2 & -ab+ab\\-ab+ab & b^2+a^2\end{bmatrix}=\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$$

(ii) $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}=\begin{bmatrix}1(2) & 1(3) & 1(4)\\2(2) & 2(3) & 2(4)\\3(2) & 3(3) & 3(4)\end{bmatrix}=\begin{bmatrix}2 & 3 & 4\\4 & 6 & 8\\6 & 9 & 12\end{bmatrix}$

(iii) $\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix}$

$$=\begin{bmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{bmatrix}$$

$$=\begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}=\begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$$=\begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

$$=\begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}=\begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$$=\begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

$$=\begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

$$=\begin{bmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{bmatrix}$$

$$=\begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}=\begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

Question 4:

If $A=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, and $C=\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$

Answer

$$A+B=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}+\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix}=\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$B-C=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}-\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix}=\begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$A+(B-C)=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+(-1) & 2+(-2) & -3+0\\ 5+4 & 0+(-1) & 2+3\\ 1+1 & -1+2 & 1+0\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4-4 & 1-1 & -1-2\\ 9-0 & 2-3 & 7-2\\ 3-1 & -1-(-2) & 4-3\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix}$$

Hence, we have verified that $A+(B-C)=(A+B)-C$.

Question 5:

If $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$ then compute $3A-5B$.

Answer

$$3A-5B=3\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}-5\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Question 6:

Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Answer

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\qquad\left(\because \cos^2\theta+\sin^2\theta=1\right)$$

Question 7:

Find X and Y, if

(i) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

(ii) $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

Answer

(i)

$$X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} \quad ...(1)$$

$$X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} \quad ...(2)$$

Adding equations (1) and (2), we get:

$$2X=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=\begin{bmatrix}7+3 & 0+0\\ 2+0 & 5+3\end{bmatrix}=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$\text{Now, } X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}7-5 & 0-0\\ 2-1 & 5-4\end{bmatrix}$$

$$\therefore Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

(ii)

$$2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix} \quad ...(3)$$

$$3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix} \quad ...(4)$$

Multiplying equation (3) with (2), we get:

$$2(2X+3Y)=2\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$$

$$\Rightarrow 4X+6Y=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix} \qquad ...(5)$$

Multiplying equation (4) with (3), we get:

$$3(3X+2Y)=3\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$$

$$\Rightarrow 9X+6Y=\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix} \qquad ...(6)$$

From (5) and (6), we have:

$$(4X+6Y)-(9X+6Y)=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix}-\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix}$$

$$\Rightarrow -5X=\begin{bmatrix}4-6 & 6-(-6)\\ 8-(-3) & 0-15\end{bmatrix}=\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}$$

$$\therefore X=-\frac{1}{5}\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & -\frac{12}{5}\\ -\frac{11}{5} & 3\end{bmatrix}$$

Now, $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$

$$\Rightarrow 2\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Question 8:

Find *X*, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Answer

$$2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$
$$\Rightarrow 2X+\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$
$$\Rightarrow 2X=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1-3 & 0-2\\ -3-1 & 2-4\end{bmatrix}$$
$$\Rightarrow 2X=\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

Question 9:

Find x and y, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

Answer

$$2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$
$$\Rightarrow\begin{bmatrix}2 & 6\\ 0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$
$$\Rightarrow\begin{bmatrix}2+y & 6\\ 1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have:

$$2+y=5$$
$$\Rightarrow y=3$$

$$2x+2=8$$
$$\Rightarrow x=3$$

$\therefore x = 3$ and $y = 3$

Question 10:

Solve the equation for *x*, *y*, *z* and *t* if

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Answer

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x+3=9$$
$$\Rightarrow 2x=6$$
$$\Rightarrow x=3$$

$$2y=12$$
$$\Rightarrow y=6$$

$$2z-3=15$$
$$\Rightarrow 2z=18$$
$$\Rightarrow z=9$$

$$2t+6=18$$
$$\Rightarrow 2t=12$$
$$\Rightarrow t=6$$

$$\therefore x=3,\ y=6,\ z=9,\ \text{and } t=6$$

Question 11:

If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find values of *x* and *y*.

Answer

$$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$2x - y = 10$ and $3x + y = 5$

Adding these two equations, we have:

$5x = 15$

$\Rightarrow x = 3$

Now, $3x + y = 5$

$\Rightarrow y = 5 - 3x$

$\Rightarrow y = 5 - 9 = -4$

$\therefore x = 3$ and $y = -4$

Question 12:

Given $3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$, find the values of x, y, z and w.

Answer

$$3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}3x & 3y\\3z & 3w\end{bmatrix}=\begin{bmatrix}x+4 & 6+x+y\\-1+z+w & 2w+3\end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x + 4$$
$$\Rightarrow 2x = 4$$
$$\Rightarrow x = 2$$

$$3y = 6 + x + y$$
$$\Rightarrow 2y = 6 + x = 6 + 2 = 8$$
$$\Rightarrow y = 4$$

$$3w = 2w + 3$$
$$\Rightarrow w = 3$$

$$3z = -1 + z + w$$
$$\Rightarrow 2z = -1 + w = -1 + 3 = 2$$
$$\Rightarrow z = 1$$

$$\therefore x = 2,\ y = 4,\ z = 1, \text{ and } w = 3$$

Question 13:

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$.

Answer

$$F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix},\ F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x)F(y)$$

$$=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y+0 & -\cos x\sin y-\sin x\cos y+0 & 0\\ \sin x\cos y+\cos x\sin y+0 & -\sin x\sin y+\cos x\cos y+0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=F(x+y)$$

$$\therefore F(x)F(y)=F(x+y)$$

Question 14:

Show that

(i) $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\neq\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$

Answer

(i)

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 5(2)-1(3) & 5(1)-1(4) \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix}$$

$$=\begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}=\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix}$$

$$=\begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}=\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0)\\0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0)\\2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -1 & -3\\1 & 0 & 0\\6 & 11 & 6\end{bmatrix}$$

$$\therefore \begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix} \neq \begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$$

Question 15:

Find $A^2-5A+6I$ if $A=\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$

Answer

We have $A^2 = A \times A$

$$A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Question 16:

If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=O$

Answer

$$A^2=AA=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}$$

Now $A^3=A^2\cdot A$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5+0+16 & 0+0+0 & 10+0+24\\ 2+0+10 & 0+8+0 & 4+4+15\\ 8+0+26 & 0+0+0 & 16+0+39\end{bmatrix}$$

$$=\begin{bmatrix}21 & 0 & 34\\ 12 & 8 & 23\\ 34 & 0 & 55\end{bmatrix}$$

$\therefore A^3-6A^2+7A+2I$

$$=\begin{bmatrix}21 & 0 & 34\\ 12 & 8 & 23\\ 34 & 0 & 55\end{bmatrix}-6\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}+7\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}+2\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}21 & 0 & 34\\ 12 & 8 & 23\\ 34 & 0 & 55\end{bmatrix}-\begin{bmatrix}30 & 0 & 48\\ 12 & 24 & 30\\ 48 & 0 & 78\end{bmatrix}+\begin{bmatrix}7 & 0 & 14\\ 0 & 14 & 7\\ 14 & 0 & 21\end{bmatrix}+\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}21+7+2 & 0+0+0 & 34+14+0\\ 12+0+0 & 8+14+2 & 23+7+0\\ 34+14+0 & 0+0+0 & 55+21+2\end{bmatrix}-\begin{bmatrix}30 & 0 & 48\\ 12 & 24 & 30\\ 48 & 0 & 78\end{bmatrix}$$

$$=\begin{bmatrix}30 & 0 & 48\\ 12 & 24 & 30\\ 48 & 0 & 78\end{bmatrix}-\begin{bmatrix}30 & 0 & 48\\ 12 & 24 & 30\\ 48 & 0 & 78\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$$

$$\therefore A^3-6A^2+7A+2I=O$$

Question 17:

If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find k so that $A^2=kA-2I$

Answer

$$A^2=A\cdot A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$$

$$=\begin{bmatrix}3(3)+(-2)(4) & 3(-2)+(-2)(-2)\\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2)\end{bmatrix}=\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}$$

Now $A^2=kA-2I$

$$\Rightarrow\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=k\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=\begin{bmatrix}3k & -2k\\ 4k & -2k\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=\begin{bmatrix}3k-2 & -2k\\ 4k & -2k-2\end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k-2=1$

$\Rightarrow 3k=3$

$\Rightarrow k=1$

Thus, the value of *k* is 1.

Question 18:

If $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity matrix of order 2, show that

$I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$

Answer

On the L.H.S.

$$I+A$$

$$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix} \quad \ldots(1)$$

On the R.H.S.

$$(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}\right)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha+\sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha+\cos\alpha\tan\frac{\alpha}{2}\\ -\cos\alpha\tan\frac{\alpha}{2}+\sin\alpha & \sin\alpha\tan\frac{\alpha}{2}+\cos\alpha\end{bmatrix} \quad \ldots(2)$$

$$=\begin{bmatrix} 1-2\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2} \\ -\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$=\begin{bmatrix} 1-2\sin^2\frac{\alpha}{2}+2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}-\tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Thus, from (1) and (2), we get L.H.S. = R.H.S.

Question 19:

A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) Rs 1,800 (b) Rs 2,000

Answer

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100}\right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$
$$\Rightarrow 5x + 210000 - 7x = 180000$$
$$\Rightarrow 210000 - 2x = 180000$$
$$\Rightarrow 2x = 210000 - 180000$$
$$\Rightarrow 2x = 30000$$
$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$
$$\Rightarrow 5x + 210000 - 7x = 200000$$
$$\Rightarrow 210000 - 2x = 200000$$
$$\Rightarrow 2x = 210000 - 200000$$
$$\Rightarrow 2x = 10000$$
$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond.

Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Answer

The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books.

The selling prices of a chemistry book, a physics book, and an economics book are respectively given as Rs 80, Rs 60, and Rs 40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix}10 & 8 & 10\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix}$$

$$=12[10\times80+8\times60+10\times40]$$
$$=12(800+480+400)$$
$$=12(1680)$$
$$=20160$$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

Question 21:

Assume X, Y, Z, W and P are matrices of order $2\times n, 3\times k, 2\times p,\ n\times 3$, and $p\times k$ respectively. The restriction on n, k and p so that $PY+WY$ will be defined are:

A. $k = 3$, $p = n$

B. k is arbitrary, $p = 2$

C. p is arbitrary, $k = 3$

D. $k = 2$, $p = 3$

Answer

Matrices P and Y are of the orders $p \times k$ and $3 \times k$ respectively.

Therefore, matrix PY will be defined if $k = 3$. Consequently, PY will be of the order $p \times k$.

Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.

Since the number of columns in W is equal to the number of rows in Y, matrix WY is well-defined and is of the order $n \times k$.
Matrices PY and WY can be added only when their orders are the same.
However, PY is of the order $p \times k$ and WY is of the order $n \times k$. Therefore, we must have $p = n$.

Thus, $k = 3$ and $p = n$ are the restrictions on n, k, and p so that $PY + WY$ will be defined.

Question 22:

Assume X, Y, Z, W and P are matrices of order $2\times n, 3\times k, 2\times p, n\times 3$, and $p\times k$ respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is
(A) $p \times 2$ **(B)** $2 \times n$ **(C)** $n \times 3$ **(D)** $p \times n$
Answer
The correct answer is B.
Matrix X is of the order $2 \times n$.
Therefore, matrix $7X$ is also of the same order.
Matrix Z is of the order $2 \times p$, i.e., $2 \times n$ [Since $n = p$]
Therefore, matrix $5Z$ is also of the same order.
Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.
Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.