

Class XII : Maths  
Chapter 4 : Determinants

Questions and Solutions | Exercise 4.2 - NCERT Books

**Question 1:**

Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3) (ii) (2, 7), (1, 1), (10, 8)

(iii) (-2, -3), (3, 2), (-1, -8)

Answer

(i) The area of the triangle with vertices (1, 0), (6, 0), (4, 3) is given by the relation,

$$\begin{aligned}\Delta &= \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix} \\ &= \frac{1}{2} [1(0-3) - 0(6-4) + 1(18-0)] \\ &= \frac{1}{2} [-3+18] = \frac{15}{2} \text{ square units}\end{aligned}$$

(ii) The area of the triangle with vertices (2, 7), (1, 1), (10, 8) is given by the relation,

$$\begin{aligned}\Delta &= \frac{1}{2} \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix} \\ &= \frac{1}{2} [2(1-8) - 7(1-10) + 1(8-10)] \\ &= \frac{1}{2} [2(-7) - 7(-9) + 1(-2)] \\ &= \frac{1}{2} [-14 + 63 - 2] = \frac{1}{2} [-16 + 63] \\ &= \frac{47}{2} \text{ square units}\end{aligned}$$

(iii) The area of the triangle with vertices (-2, -3), (3, 2), (-1, -8)

is given by the relation,

$$\begin{aligned}\Delta &= \frac{1}{2} \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix} \\ &= \frac{1}{2} [-2(2+8) + 3(3+1) + 1(-24+2)] \\ &= \frac{1}{2} [-2(10) + 3(4) + 1(-22)] \\ &= \frac{1}{2} [-20 + 12 - 22] \\ &= -\frac{30}{2} = -15\end{aligned}$$

Hence, the area of the triangle is  $|-15| = 15$  square units .

**Question 2:**

Show that points

$A(a, b+c), B(b, c+a), C(c, a+b)$  are collinear

Answer

Area of  $\Delta ABC$  is given by the relation,

$$\begin{aligned} \Delta &= \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix} \quad (\text{Applying } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1) \\ &= \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} \\ &= \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad (\text{Applying } R_3 \rightarrow R_3 + R_2) \\ &= 0 \quad (\text{All elements of } R_3 \text{ are } 0) \end{aligned}$$

Thus, the area of the triangle formed by points A, B, and C is zero.

Hence, the points A, B, and C are collinear.

**Question 3:**

Find values of  $k$  if area of triangle is 4 square units and vertices are

(i)  $(k, 0), (4, 0), (0, 2)$  (ii)  $(-2, 0), (0, 4), (0, k)$

Answer

We know that the area of a triangle whose vertices are  $(x_1, y_1), (x_2, y_2)$ , and  $(x_3, y_3)$  is the absolute value of the determinant ( $\Delta$ ), where

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

It is given that the area of triangle is 4 square units.

$$\therefore \Delta = \pm 4.$$

(i) The area of the triangle with vertices  $(k, 0), (4, 0), (0, 2)$  is given by the relation,

$$\begin{aligned} \Delta &= \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} \\ &= \frac{1}{2} [k(0-2) - 0(4-0) + 1(8-0)] \\ &= \frac{1}{2} [-2k + 8] = -k + 4 \end{aligned}$$

$$\therefore -k + 4 = \pm 4$$

When  $-k + 4 = -4, k = 8.$

When  $-k + 4 = 4, k = 0.$

Hence,  $k = 0, 8.$

(ii) The area of the triangle with vertices  $(-2, 0)$ ,  $(0, 4)$ ,  $(0, k)$  is given by the relation,

$$\Delta = \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$= \frac{1}{2} [-2(4-k)]$$

$$= k - 4$$

$$\therefore k - 4 = \pm 4$$

When  $k - 4 = -4$ ,  $k = 0$ .

When  $k - 4 = 4$ ,  $k = 8$ .

Hence,  $k = 0, 8$ .

#### Question 4:

(i) Find equation of line joining  $(1, 2)$  and  $(3, 6)$  using determinants

(ii) Find equation of line joining  $(3, 1)$  and  $(9, 3)$  using determinants

Answer

(i) Let  $P(x, y)$  be any point on the line joining points  $A(1, 2)$  and  $B(3, 6)$ . Then, the points  $A, B$ , and  $P$  are collinear. Therefore, the area of triangle  $ABP$  will be zero.

$$\therefore \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [1(6-y) - 2(3-x) + 1(3y-6x)] = 0$$

$$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$$

$$\Rightarrow 2y - 4x = 0$$

$$\Rightarrow y = 2x$$

Hence, the equation of the line joining the given points is  $y = 2x$ .

(ii) Let  $P(x, y)$  be any point on the line joining points  $A(3, 1)$  and

B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2} \begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [3(3-y) - 1(9-x) + 1(9y-3x)] = 0$$

$$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$$

$$\Rightarrow 6y - 2x = 0$$

$$\Rightarrow x - 3y = 0$$

Hence, the equation of the line joining the given points is  $x - 3y = 0$ .

#### Question 5:

If area of triangle is 35 square units with vertices (2, -6), (5, 4), and (k, 4). Then k is

**A.** 12 **B.** -2 **C.** -12, -2 **D.** 12, -2

Answer

**Answer: D**

The area of the triangle with vertices (2, -6), (5, 4), and (k, 4) is given by the relation,

$$\begin{aligned} \Delta &= \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} \\ &= \frac{1}{2} [2(4-4) + 6(5-k) + 1(20-4k)] \\ &= \frac{1}{2} [30 - 6k + 20 - 4k] \\ &= \frac{1}{2} [50 - 10k] \\ &= 25 - 5k \end{aligned}$$

It is given that the area of the triangle is  $\pm 35$ .

Therefore, we have:

$$\Rightarrow 25 - 5k = \pm 35$$

$$\Rightarrow 5(5 - k) = \pm 35$$

$$\Rightarrow 5 - k = \pm 7$$

$$\text{When } 5 - k = -7, k = 5 + 7 = 12.$$

$$\text{When } 5 - k = 7, k = 5 - 7 = -2.$$

Hence,  $k = 12, -2$ .

The correct answer is D.