## Class XII : Maths

Chapter 4 : Determinants

## Questions and Solutions | Exercise 4.5 - NCERT Books

## Question 1:

Examine the consistency of the system of equations.
$x+2 y=2$
$2 x+3 y=3$
Answer
The given system of equations is:
$x+2 y=2$
$2 x+3 y=3$
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ll}1 & 2 \\ 2 & 3\end{array}\right], X=\left[\begin{array}{l}x \\ y\end{array}\right]$ and $B=\left[\begin{array}{l}2 \\ 3\end{array}\right]$.
Now,
$|A|=1(3)-2(2)=3-4=-1 \neq 0$
$\therefore A$ is non-singular.

Therefore, $A^{-1}$ exists.
Hence, the given system of equations is consistent.

## Question 2:

Examine the consistency of the system of equations.
$2 x-y=5$
$x+y=4$
Answer
The given system of equations is:
$2 x-y=5$
$x+y=4$
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{cc}2 & -1 \\ 1 & 1\end{array}\right], X=\left[\begin{array}{l}x \\ z\end{array}\right]$ and $B=\left[\begin{array}{l}5 \\ 4\end{array}\right]$.
Now,
$|A|=2(1)-(-1)(1)=2+1=3 \neq 0$
$\therefore A$ is non-singular.

Therefore, $A^{-1}$ exists.
Hence, the given system of equations is consistent.

## Question 3:

Examine the consistency of the system of equations.
$x+3 y=5$
$2 x+6 y=8$
Answer
The given system of equations is:
$x+3 y=5$
$2 x+6 y=8$
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ll}1 & 3 \\ 2 & 6\end{array}\right], X=\left[\begin{array}{l}x \\ y\end{array}\right]$ and $B=\left[\begin{array}{l}5 \\ 8\end{array}\right]$.
Now,

$$
|A|=1(6)-3(2)=6-6=0
$$

$\therefore A$ is a singular matrix.

Now,
$(\operatorname{adj} A)=\left[\begin{array}{cc}6 & -3 \\ -2 & 1\end{array}\right]$
$(\operatorname{adj} A) B=\left[\begin{array}{cc}6 & -3 \\ -2 & 1\end{array}\right]\left[\begin{array}{l}5 \\ 8\end{array}\right]=\left[\begin{array}{l}30-24 \\ -10+8\end{array}\right]=\left[\begin{array}{l}6 \\ -2\end{array}\right] \neq O$
Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

## Question 4:

Examine the consistency of the system of equations.
$x+y+z=1$
$2 x+3 y+2 z=2$
$a x+a y+2 a z=4$
Answer
The given system of equations is:
$x+y+z=1$
$2 x+3 y+2 z=2$
$a x+a y+2 a z=4$
This system of equations can be written in the form $A X=B$, where

$$
A=\left[\begin{array}{lll}
1 & 1 & 1 \\
2 & 3 & 2 \\
a & a & 2 a
\end{array}\right], X=\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] \text { and } B=\left[\begin{array}{l}
1 \\
2 \\
4
\end{array}\right] .
$$

Now,

$$
\begin{aligned}
|A| & =1(6 a-2 a)-1(4 a-2 a)+1(2 a-3 a) \\
& =4 a-2 a-a=4 a-3 a=a \neq 0
\end{aligned}
$$

$\therefore A$ is non-singular.
Therefore, $A^{-1}$ exists.
Hence, the given system of equations is consistent.

## Question 5:

Examine the consistency of the system of equations.
$3 x-y-2 z=2$
$2 y-z=-1$
$3 x-5 y=3$
Answer
The given system of equations is:
$3 x-y-2 z=2$
$2 y-z=-1$
$3 x-5 y=3$
This system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{lll}3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{c}2 \\ -1 \\ 3\end{array}\right]$.
Now,

$$
|A|=3(0-5)-0+3(1+4)=-15+15=0
$$

$\therefore A$ is a singular matrix.
Now,
$(\operatorname{adj} A)=\left[\begin{array}{lll}-5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6\end{array}\right]$
$\therefore(\operatorname{adj} A) B=\left[\begin{array}{lll}-5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6\end{array}\right]\left[\begin{array}{c}2 \\ -1 \\ 3\end{array}\right]=\left[\begin{array}{l}-10-10+15 \\ -6-6+9 \\ -12-12+18\end{array}\right]=\left[\begin{array}{l}-5 \\ -3 \\ -6\end{array}\right] \neq O$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

## Question 6:

Examine the consistency of the system of equations.
$5 x-y+4 z=5$
$2 x+3 y+5 z=2$
$5 x-2 y+6 z=-1$
Answer
The given system of equations is:
$5 x-y+4 z=5$
$2 x+3 y+5 z=2$
$5 x-2 y+6 z=-1$
This system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{lll}5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{r}5 \\ 2 \\ -1\end{array}\right]$.
Now,

$$
\begin{aligned}
|A| & =5(18+10)+1(12-25)+4(-4-15) \\
& =5(28)+1(-13)+4(-19) \\
& =140-13-76 \\
& =51 \neq 0
\end{aligned}
$$

$\therefore A$ is non-singular.

Therefore, $A^{-1}$ exists.
Hence, the given system of equations is consistent.

## Question 7:

Solve system of linear equations, using matrix method.
$5 x+2 y=4$
$7 x+3 y=5$
Answer

The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ll}5 & 2 \\ 7 & 3\end{array}\right], X=\left[\begin{array}{l}x \\ y\end{array}\right]$ and $B=\left[\begin{array}{l}4 \\ 5\end{array}\right]$.
Now, $|A|=15-14=1 \neq 0$.
Thus, $A$ is non-singular. Therefore, its inverse exists.
Now,
$A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)$
$\therefore A^{-1}=\left[\begin{array}{rr}3 & -2 \\ -7 & 5\end{array}\right]$
$\therefore X=A^{-1} B=\left[\begin{array}{rr}3 & -2 \\ -7 & 5\end{array}\right]\left[\begin{array}{l}4 \\ 5\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{l}12-10 \\ -28+25\end{array}\right]=\left[\begin{array}{c}2 \\ -3\end{array}\right]$
Hence, $x=2$ and $y=-3$.

## Question 8:

Solve system of linear equations, using matrix method.
$2 x-y=-2$
$3 x+4 y=3$
Answer
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{cc}2 & -1 \\ 3 & 4\end{array}\right], X=\left[\begin{array}{l}x \\ y\end{array}\right]$ and $B=\left[\begin{array}{c}-2 \\ 3\end{array}\right]$.
Now,
$|A|=8+3=11 \neq 0$
Thus, $A$ is non-singular. Therefore, its inverse exists.

Now,
$A^{-1}=\frac{1}{|A|} \operatorname{adj} A=\frac{1}{11}\left[\begin{array}{cc}4 & 1 \\ -3 & 2\end{array}\right]$
$\therefore X=A^{-1} B=\frac{1}{11}\left[\begin{array}{cc}4 & 1 \\ -3 & 2\end{array}\right]\left[\begin{array}{c}-2 \\ 3\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y\end{array}\right]=\frac{1}{11}\left[\begin{array}{l}-8+3 \\ 6+6\end{array}\right]=\frac{1}{11}\left[\begin{array}{l}-5 \\ 12\end{array}\right]=\left[\begin{array}{c}-\frac{5}{11} \\ \frac{12}{11}\end{array}\right]$
Hence, $x=\frac{-5}{11}$ and $y=\frac{12}{11}$.

## Question 9:

Solve system of linear equations, using matrix method.
$4 x-3 y=3$
$3 x-5 y=7$
Answer
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ll}4 & -3 \\ 3 & -5\end{array}\right], X=\left[\begin{array}{l}x \\ y\end{array}\right]$ and $B=\left[\begin{array}{l}3 \\ 7\end{array}\right]$.
Now,
$|A|=-20+9=-11 \neq 0$
Thus, $A$ is non-singular. Therefore, its inverse exists.

Now,
$A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=-\frac{1}{11}\left[\begin{array}{ll}-5 & 3 \\ -3 & 4\end{array}\right]=\frac{1}{11}\left[\begin{array}{ll}5 & -3 \\ 3 & -4\end{array}\right]$
$\therefore X=A^{-1} B=\frac{1}{11}\left[\begin{array}{ll}5 & -3 \\ 3 & -4\end{array}\right]\left[\begin{array}{l}3 \\ 7\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y\end{array}\right]=\frac{1}{11}\left[\begin{array}{ll}5 & -3 \\ 3 & -4\end{array}\right]\left[\begin{array}{l}3 \\ 7\end{array}\right]=\frac{1}{11}\left[\begin{array}{l}15-21 \\ 9-28\end{array}\right]=\frac{1}{11}\left[\begin{array}{l}-6 \\ -19\end{array}\right]=\left[\begin{array}{c}-\frac{6}{11} \\ -\frac{19}{11}\end{array}\right]$
Hence, $x=\frac{-6}{11}$ and $y=\frac{-19}{11}$.

## Question 10:

Solve system of linear equations, using matrix method.
$5 x+2 y=3$
$3 x+2 y=5$

## Answer

The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ll}5 & 2 \\ 3 & 2\end{array}\right], X=\left[\begin{array}{l}x \\ y\end{array}\right]$ and $B=\left[\begin{array}{l}3 \\ 5\end{array}\right]$.
Now,
$|A|=10-6=4 \neq 0$
Thus, $A$ is non-singular. Therefore, its inverse exists.

## Question 11:

Solve system of linear equations, using matrix method.
$2 x+y+z=1$
$x-2 y-z=\frac{3}{2}$
$3 y-5 z=9$
Answer
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ccc}2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5\end{array}\right], X=\left[\begin{array}{c}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{c}1 \\ \frac{3}{2} \\ 9\end{array}\right]$.
Now,
$|A|=2(10+3)-1(-5-3)+0=2(13)-1(-8)=26+8=34 \neq 0$
Thus, $A$ is non-singular. Therefore, its inverse exists.
Now, $A_{11}=13, A_{12}=5, A_{13}=3$
$A_{21}=8, A_{22}=-10, A_{23}=-6$
$A_{31}=1, A_{32}=3, A_{33}=-5$
$\therefore A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{34}\left[\begin{array}{ccc}13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5\end{array}\right]$
$\therefore X=A^{-1} B=\frac{1}{34}\left[\begin{array}{ccc}13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5\end{array}\right]\left[\begin{array}{l}1 \\ \frac{3}{2} \\ 9\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\frac{1}{34}\left[\begin{array}{l}13+12+9 \\ 5-15+27 \\ 3-9-45\end{array}\right]$

$$
=\frac{1}{34}\left[\begin{array}{l}
34 \\
17 \\
-51
\end{array}\right]=\left[\begin{array}{c}
1 \\
\frac{1}{2} \\
-\frac{3}{2}
\end{array}\right]
$$

Hence, $x=1, y=\frac{1}{2}$, and $z=-\frac{3}{2}$.

## Question 12:

Solve system of linear equations, using matrix method.
$x-y+z=4$
$2 x+y-3 z=0$
$x+y+z=2$
Answer
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ccc}1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{l}4 \\ 0 \\ 2\end{array}\right]$.
Now,
$|A|=1(1+3)+1(2+3)+1(2-1)=4+5+1=10 \neq 0$
Thus, $A$ is non-singular. Therefore, its inverse exists.
Now, $A_{11}=4, A_{12}=-5, A_{13}=1$
$A_{21}=2, A_{22}=0, A_{23}=-2$
$A_{31}=2, A_{32}=5, A_{33}=3$
$\therefore A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{10}\left[\begin{array}{ccc}4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3\end{array}\right]$
$\therefore X=A^{-1} B=\frac{1}{10}\left[\begin{array}{ccc}4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3\end{array}\right]\left[\begin{array}{l}4 \\ 0 \\ 2\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\frac{1}{10}\left[\begin{array}{c}16+0+4 \\ -20+0+10 \\ 4+0+6\end{array}\right]$
$=\frac{1}{10}\left[\begin{array}{c}20 \\ -10 \\ 10\end{array}\right]$
$=\left[\begin{array}{c}2 \\ -1 \\ 1\end{array}\right]$
Hence, $x=2, y=-1$, and $z=1$.

## Question 13:

Solve system of linear equations, using matrix method.
$2 x+3 y+3 z=5$
$x-2 y+z=-4$
$3 x-y-2 z=3$

## Answer

The given system of equations can be written in the form $A X=B$, where

$$
A=\left[\begin{array}{ccc}
2 & \begin{array}{c}
3 \\
3 \\
1 \\
3
\end{array} & \begin{array}{c}
-2 \\
-1
\end{array} \\
\hline-2
\end{array}\right], X=\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] \text { and } B=\left[\begin{array}{l}
5 \\
-4 \\
3
\end{array}\right] .
$$

Now,

$$
|A|=2(4+1)-3(-2-3)+3(-1+6)=2(5)-3(-5)+3(5)=10+15+15=40 \neq 0
$$

Thus, $A$ is non-singular. Therefore, its inverse exists.
Now, $A_{11}=5, A_{12}=5, A_{13}=5$

$$
A_{21}=3, A_{22}=-13, A_{23}=11
$$

$$
A_{31}=9, A_{32}=1, A_{33}=-7
$$

$\therefore A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{40}\left[\begin{array}{ccc}5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7\end{array}\right]$
$\therefore X=A^{-1} B=\frac{1}{40}\left[\begin{array}{ccc}5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7\end{array}\right]\left[\begin{array}{l}5 \\ -4 \\ 3\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\frac{1}{40}\left[\begin{array}{l}25-12+27 \\ 25+52+3 \\ 25-44-21\end{array}\right]$

$$
=\frac{1}{40}\left[\begin{array}{l}
40 \\
80 \\
-40
\end{array}\right]
$$

$$
=\left[\begin{array}{c}
1 \\
2 \\
-1
\end{array}\right]
$$

Hence, $x=1, y=2$, and $z=-1$.

## Question 14:

Solve system of linear equations, using matrix method.
$x-y+2 z=7$
$3 x+4 y-5 z=-5$
$2 x-y+3 z=12$
Answer
The given system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{ccc}1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{c}7 \\ -5 \\ 12\end{array}\right]$.
Now,

$$
|A|=1(12-5)+1(9+10)+2(-3-8)=7+19-22=4 \neq 0
$$

Thus, $A$ is non-singular. Therefore, its inverse exists.
Now, $A_{11}=7, A_{12}=-19, A_{13}=-11$
$A_{21}=1, A_{22}=-1, A_{23}=-1$
$A_{31}=-3, A_{32}=11, A_{33}=7$
$\therefore A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{4}\left[\begin{array}{ccc}7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7\end{array}\right]$
$\therefore X=A^{-1} B=\frac{1}{4}\left[\begin{array}{ccc}7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7\end{array}\right]\left[\begin{array}{l}7 \\ -5 \\ 12\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\frac{1}{4}\left[\begin{array}{c}49-5-36 \\ -133+5+132 \\ -77+5+84\end{array}\right]$

$$
=\frac{1}{4}\left[\begin{array}{l}
8 \\
4 \\
12
\end{array}\right]=\left[\begin{array}{l}
2 \\
1 \\
3
\end{array}\right]
$$

Hence, $x=2, y=1$, and $z=3$.

## Question 15:

$$
\begin{aligned}
& A=\left[\begin{array}{ccc}
2 & -3 & 5 \\
3 & 2 & -4 \\
1 & 1 & -2
\end{array}\right] \text {, find } A^{-1} \text {. Using } A^{-1} \text { solve the system of equations } \\
& \text { If } \begin{aligned}
2 x-3 y+5 z & =11 \\
3 x+2 y-4 z & =-5 \\
x+y-2 z & =-3
\end{aligned}
\end{aligned}
$$

## Answer

$$
A=\left[\begin{array}{ccc}
2 & -3 & 5 \\
3 & 2 & -4 \\
1 & 1 & -2
\end{array}\right]
$$

$$
\therefore|A|=2(-4+4)+3(-6+4)+5(3-2)=0-6+5=-1 \neq 0
$$

Now, $A_{11}=0, A_{12}=2, A_{13}=1$

$$
\begin{align*}
& A_{21}=-1, A_{22}=-9, A_{23}=-5 \\
& A_{31}=2, A_{32}=23, A_{33}=13 \tag{1}
\end{align*}
$$

$\therefore A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=-\left[\begin{array}{lll}0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13\end{array}\right]=\left[\begin{array}{ccc}0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13\end{array}\right]$
Now, the given system of equations can be written in the form of $A X=B$, where

$$
A=\left[\begin{array}{ccc}
2 & -3 & 5 \\
3 & 2 & -4 \\
1 & 1 & -2
\end{array}\right], X=\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] \text { and } B=\left[\begin{array}{c}
11 \\
-5 \\
-3
\end{array}\right]
$$

The solution of the system of equations is given by $X=A^{-1} B$.

$$
\begin{align*}
X & =A^{-1} B \\
\Rightarrow\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] & =\left[\begin{array}{ccc}
0 & 1 & -2 \\
-2 & 9 & -23 \\
-1 & 5 & -13
\end{array}\right]\left[\begin{array}{l}
11 \\
-5 \\
-3
\end{array}\right]  \tag{1}\\
& =\left[\begin{array}{c}
0-5+6 \\
-22-45+69 \\
-11-25+39
\end{array}\right] \\
& =\left[\begin{array}{l}
1 \\
2 \\
3
\end{array}\right]
\end{align*}
$$

Hence, $x=1, y=2$, and $z=3$.

## Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60 . The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90 . The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70 . Find cost of each item per kg by matrix method.
Answer
Let the cost of onions, wheat, and rice per kg be Rs $x$, Rs $y$, and Rs $z$ respectively.
Then, the given situation can be represented by a system of equations as:
$4 x+3 y+2 z=60$
$2 x+4 y+6 z=90$
$6 x+2 y+3 z=70$
This system of equations can be written in the form of $A X=B$, where
$A=\left[\begin{array}{lll}4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{c}60 \\ 90 \\ 70\end{array}\right]$.
$|A|=4(12-12)-3(6-36)+2(4-24)=0+90-40=50 \neq 0$
Now, $\quad A_{11}=0, A_{12}=30, A_{13}=-20$
$A_{21}=-5, A_{22}=0, A_{23}=10$
$A_{31}=10, A_{32}=-20, A_{33}=10$
$\therefore$ adj $A=\left[\begin{array}{ccc}0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10\end{array}\right]$
$\therefore A^{-1}=\frac{1}{|A|} \operatorname{adj} A=\frac{1}{50}\left[\begin{array}{ccc}0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10\end{array}\right]$
Now,
$X=A^{-1} B$
$\Rightarrow X=\frac{1}{50}\left[\begin{array}{ccc}0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10\end{array}\right]\left[\begin{array}{l}60 \\ 90 \\ 70\end{array}\right]$
$\Rightarrow\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\frac{1}{50}\left[\begin{array}{l}0-450+700 \\ 1800+0-1400 \\ -1200+900+700\end{array}\right]$
$=\frac{1}{50}\left[\begin{array}{l}250 \\ 400 \\ 400\end{array}\right]$
$=\left[\begin{array}{l}5 \\ 8 \\ 8\end{array}\right]$
$\therefore x=5, y=8$, and $z=8$.
Hence, the cost of onions is Rs 5 per kg, the cost of wheat is Rs 8 per kg, and the cost of rice is Rs 8 per kg.

