Class XI : Maths
Chapter 12 : Limits and Derivatives

Questions and Solutions | Exercise 12.1 - NCERT Books

Question 1:

Evaluate the Given limit: $\lim\limits_{x\to 3} x+3$

Answer

$$\lim_{x\to 3} x+3=3+3=6$$

Question 2:

Evaluate the Given limit: $\lim\limits_{x\to \pi}\left(x-\frac{22}{7}\right)$

Answer

$$\lim_{x\to \pi}\left(x-\frac{22}{7}\right)=\left(\pi-\frac{22}{7}\right)$$

Question 3:

Evaluate the Given limit: $\lim\limits_{r\to 1}\pi r^2$

Answer

$$\lim_{r\to 1}\pi r^2=\pi(1)^2=\pi$$

Question 4:

Evaluate the Given limit: $\lim\limits_{x\to 4}\frac{4x+3}{x-2}$

Answer

$$\lim_{x\to 4}\frac{4x+3}{x-2}=\frac{4(4)+3}{4-2}=\frac{16+3}{2}=\frac{19}{2}$$

Question 5:

Evaluate the Given limit: $\lim\limits_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

Answer

$$\lim_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1} = \frac{(-1)^{10} + (-1)^5 + 1}{-1 - 1} = \frac{1 - 1 + 1}{-2} = -\frac{1}{2}$$

Question 6:

Evaluate the Given limit: $\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$

Answer

$$\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$$

Put $x + 1 = y$ so that $y \to 1$ as $x \to 0$.

$$\text{Accordingly, } \lim_{x \to 0} \frac{(x+1)^5 - 1}{x} = \lim_{y \to 1} \frac{y^5 - 1}{y - 1}$$

$$= \lim_{y \to 1} \frac{y^5 - 1^5}{y - 1}$$

$$= 5.1^{5-1} \qquad \left[\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= 5$$

$$\therefore \lim_{x \to 0} \frac{(x+5)^5 - 1}{x} = 5$$

Question 7:

Evaluate the Given limit: $\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

Answer

At $x = 2$, the value of the given rational function takes the form $\frac{0}{0}$.

$$\therefore \lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(3x+5)}{(x-2)(x+2)}$$
$$=\lim_{x\to 2}\frac{3x+5}{x+2}$$
$$=\frac{3(2)+5}{2+2}$$
$$=\frac{11}{4}$$

Question 8:

Evaluate the Given limit: $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

Answer

At x = 2, the value of the given rational function takes the form $\frac{0}{0}$.

$$\therefore \lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}=\lim_{x\to 3}\frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)}$$
$$=\lim_{x\to 3}\frac{(x+3)(x^2+9)}{2x+1}$$
$$=\frac{(3+3)(3^2+9)}{2(3)+1}$$
$$=\frac{6\times 18}{7}$$
$$=\frac{108}{7}$$

Question 9:

Evaluate the Given limit: $\lim_{x\to 0}\frac{ax+b}{cx+1}$

Answer

$$\lim_{x\to 0}\frac{ax+b}{cx+1}=\frac{a(0)+b}{c(0)+1}=b$$

Question 10:

Evaluate the Given limit: $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

Answer

$$\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$$

At z = 1, the value of the given function takes the form $\frac{0}{0}$.

Put $z^{\frac{1}{6}}=x$ so that $z\to 1$ as $x\to 1$.

$$\text{Accordingly, } \lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=\lim_{x\to 1}\frac{x^2-1}{x-1}$$

$$=\lim_{x\to 1}\frac{x^2-1^2}{x-1}$$

$$=2.1^{2-1} \qquad \left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=2$$

$$\therefore \lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=2$$

Question 11:

Evaluate the Given limit: $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

Answer

$$\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}=\frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a}$$
$$=\frac{a+b+c}{a+b+c}$$
$$=1 \qquad [a+b+c\neq 0]$$

Question 12:

Evaluate the Given limit: $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

Answer

$$\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$$

At $x = -2$, the value of the given function takes the form $\frac{0}{0}$.

$$\text{Now, } \lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim_{x\to -2}\frac{\left(\frac{2+x}{2x}\right)}{x+2}$$
$$=\lim_{x\to -2}\frac{1}{2x}$$
$$=\frac{1}{2(-2)}=\frac{-1}{4}$$

Question 13:

Evaluate the Given limit: $\lim_{x\to 0}\frac{\sin ax}{bx}$

Answer

$$\lim_{x\to 0}\frac{\sin ax}{bx}$$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

$$\text{Now, } \lim_{x\to 0}\frac{\sin ax}{bx} = \lim_{x\to 0}\frac{\sin ax}{ax}\times\frac{ax}{bx}$$

$$= \lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)\times\left(\frac{a}{b}\right)$$

$$= \frac{a}{b}\lim_{ax\to 0}\left(\frac{\sin ax}{ax}\right) \qquad [x\to 0 \Rightarrow ax\to 0]$$

$$= \frac{a}{b}\times 1 \qquad \left[\lim_{y\to 0}\frac{\sin y}{y} = 1\right]$$

$$= \frac{a}{b}$$

Question 14:

Evaluate the Given limit: $\lim_{x\to 0}\frac{\sin ax}{\sin bx},\ a,\ b\neq 0$

Answer

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx},\ a,\ b\neq 0$$

At x = 0, the value of the given function takes the form $\frac{0}{0}$.

$$\text{Now, } \lim_{x\to 0}\frac{\sin ax}{\sin bx} = \lim_{x\to 0}\frac{\left(\frac{\sin ax}{ax}\right)\times ax}{\left(\frac{\sin bx}{bx}\right)\times bx}$$

$$= \left(\frac{a}{b}\right)\times\frac{\lim_{ax\to 0}\left(\frac{\sin ax}{ax}\right)}{\lim_{bx\to 0}\left(\frac{\sin bx}{bx}\right)} \qquad \left[\begin{array}{l} x\to 0\Rightarrow ax\to 0 \\ \text{and } x\to 0\Rightarrow bx\to 0\end{array}\right]$$

$$= \left(\frac{a}{b}\right)\times\frac{1}{1} \qquad \left[\lim_{y\to 0}\frac{\sin y}{y} = 1\right]$$

$$= \frac{a}{b}$$

Question 15:

Evaluate the Given limit: $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Answer

$$\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

It is seen that $x \to \pi \Rightarrow (\pi - x) \to 0$

$$\therefore \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \frac{1}{\pi} \lim_{(\pi - x) \to 0} \frac{\sin(\pi - x)}{(\pi - x)}$$

$$= \frac{1}{\pi} \times 1 \qquad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1\right]$$

$$= \frac{1}{\pi}$$

Question 16:

Evaluate the given limit: $\lim_{x \to 0} \frac{\cos x}{\pi - x}$

Answer

$$\lim_{x \to 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$$

Question 17:

Evaluate the Given limit: $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

Answer

$$\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$$

At x = 0, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}=\lim_{x\to 0}\frac{1-2\sin^2 x-1}{1-2\sin^2\frac{x}{2}-1} \qquad \left[\cos x=1-2\sin^2\frac{x}{2}\right]$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{\sin^2\frac{x}{2}}=\lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\times x^2}{\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\times\frac{x^2}{4}}$$

$$=4\frac{\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$$

$$=4\frac{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2}{\left(\lim_{\frac{x}{2}\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2} \qquad \left[x\to 0\Rightarrow\frac{x}{2}\to 0\right]$$

$$=4\frac{1^2}{1^2} \qquad \left[\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

$$=4$$

Question 18:

Evaluate the Given limit: $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$

Answer

$$\lim_{x \to 0} \frac{ax + x\cos x}{b \sin x}$$

At x = 0, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\lim_{x \to 0} \frac{ax + x\cos x}{b \sin x} = \frac{1}{b} \lim_{x \to 0} \frac{x(a + \cos x)}{\sin x}$$
$$= \frac{1}{b} \lim_{x \to 0} \left(\frac{x}{\sin x} \right) \times \lim_{x \to 0} (a + \cos x)$$
$$= \frac{1}{b} \times \frac{1}{\left(\lim_{x \to 0} \frac{\sin x}{x} \right)} \times \lim_{x \to 0} (a + \cos x)$$
$$= \frac{1}{b} \times (a + \cos 0) \qquad \left[\lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$
$$= \frac{a+1}{b}$$

Question 19:

Evaluate the Given limit: $\lim_{x \to 0} x \sec x$

Answer

$$\lim_{x \to 0} x \sec x = \lim_{x \to 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$$

Question 20:

Evaluate the Given limit: $\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx} \quad a, b, a+b \neq 0$

Answer

At x = 0, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\lim_{x\to 0}\frac{\sin ax + bx}{ax + \sin bx}$$

$$= \lim_{x\to 0}\frac{\left(\frac{\sin ax}{ax}\right)ax + bx}{ax + bx\left(\frac{\sin bx}{bx}\right)}$$

$$= \frac{\left(\lim_{ax\to 0}\frac{\sin ax}{ax}\right)\times \lim_{x\to 0}(ax) + \lim_{x\to 0} bx}{\lim_{x\to 0} ax + \lim_{x\to 0} bx\left(\lim_{bx\to 0}\frac{\sin bx}{bx}\right)} \quad [\text{As } x\to 0 \Rightarrow ax\to 0 \text{ and } bx\to 0]$$

$$= \frac{\lim_{x\to 0}(ax) + \lim_{x\to 0} bx}{\lim_{x\to 0} ax + \lim_{x\to 0} bx} \quad \left[\lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= \frac{\lim_{x\to 0}(ax + bx)}{\lim_{x\to 0}(ax + bx)}$$

$$= \lim_{x\to 0}(1)$$

$$= 1$$

Question 21:

Evaluate the Given limit: $\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$

Answer

At x = 0, the value of the given function takes the form $\infty - \infty$.

Now,

$$\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$$

$$= \lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\frac{\left(\frac{1-\cos x}{x}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\lim_{x\to 0}\frac{1-\cos x}{x}}{\lim_{x\to 0}\frac{\sin x}{x}}$$

$$= \frac{0}{1} \qquad \left[\lim_{x\to 0}\frac{1-\cos x}{x} = 0 \text{ and } \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= 0$$

Question 22:

$$\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}$$

Answer

$$\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}$$

At $x = \frac{\pi}{2}$, the value of the given function takes the form $\frac{0}{0}$.

Now, put $x - \frac{\pi}{2} = y$ so that $x \to \frac{\pi}{2},\ y \to 0$.

$$\therefore \lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}} = \lim_{y\to 0} \frac{\tan 2\left(y+\frac{\pi}{2}\right)}{y}$$

$$= \lim_{y\to 0} \frac{\tan(\pi+2y)}{y}$$

$$= \lim_{y\to 0} \frac{\tan 2y}{y} \qquad [\tan(\pi+2y) = \tan 2y]$$

$$= \lim_{y\to 0} \frac{\sin 2y}{y\cos 2y}$$

$$= \lim_{y\to 0} \left(\frac{\sin 2y}{2y} \times \frac{2}{\cos 2y}\right)$$

$$= \left(\lim_{2y\to 0} \frac{\sin 2y}{2y}\right) \times \lim_{y\to 0}\left(\frac{2}{\cos 2y}\right) \qquad [y\to 0 \Rightarrow 2y\to 0]$$

$$= 1\times\frac{2}{\cos 0} \qquad \left[\lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= 1\times\frac{2}{1}$$

$$= 2$$

Question 23:

Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} 2x+3, & x\le 0 \\ 3(x+1), & x>0 \end{cases}$

Answer

The given function is

$$f(x) = \begin{cases} 2x+3, & x\le 0 \\ 3(x+1), & x>0 \end{cases}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}[2x+3] = 2(0)+3 = 3$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} 3(x+1) = 3(0+1) = 3$$

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 3$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 3(x+1) = 3(1+1) = 6$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 3(x+1) = 3(1+1) = 6$$

$$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = 6$$

Question 24:

Find $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^2-1, & x \le 1 \\ -x^2-1, & x > 1 \end{cases}$

Answer

The given function is

$$f(x) = \begin{cases} x^2-1, & x \le 1 \\ -x^2-1, & x > 1 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}\left[x^2-1\right] = 1^2-1 = 1-1 = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}\left[-x^2-1\right] = -1^2-1 = -1-1 = -2$$

It is observed that $\lim_{x\to 1^-} f(x) \ne \lim_{x\to 1^+} f(x)$.

Hence, $\lim_{x\to 1} f(x)$ does not exist.

Question 25:

Evaluate $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$

Answer

The given function is

$f(x) = \begin{cases} \frac{|x|}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$

$$\lim_{x\to0^-} f(x) = \lim_{x\to0^-}\left[\frac{|x|}{x}\right]$$
$$= \lim_{x\to0}\left(\frac{-x}{x}\right) \qquad [\text{When } x \text{ is negaitve, } |x| = -x]$$
$$= \lim_{x\to0}(-1)$$
$$= -1$$
$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+}\left[\frac{|x|}{x}\right]$$
$$= \lim_{x\to0}\left[\frac{x}{x}\right] \qquad [\text{When } x \text{ is positive, } |x| = x]$$
$$= \lim_{x\to0}(1)$$
$$= 1$$

It is observed that $\lim_{x\to0^-} f(x) \neq \lim_{x\to0^+} f(x)$.

Hence, $\lim_{x\to0} f(x)$ does not exist.

Question 26:

Find $\lim_{x\to0} f(x)$, where $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Answer

The given function is

$$f(x)=\begin{cases}\dfrac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\left[\frac{x}{|x|}\right]$$

$$=\lim_{x\to 0}\left[\frac{x}{-x}\right] \qquad [\text{When } x<0,\ |x|=-x]$$

$$=\lim_{x\to 0}(-1)$$

$$=-1$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\left[\frac{x}{|x|}\right]$$

$$=\lim_{x\to 0}\left[\frac{x}{x}\right] \qquad [\text{When } x>0,\ |x|=x]$$

$$=\lim_{x\to 0}(1)$$

$$=1$$

It is observed that $\lim_{x\to 0^-} f(x)\neq \lim_{x\to 0^+} f(x)$.

Hence, $\lim_{x\to 0} f(x)$ does not exist.

Question 27:

Find $\lim_{x\to 5}$ $f(x)$, where $f(x)$ = $|x|-5$

Answer

The given function is $f(x)$ = $|x|-5$.

$$\lim_{x\to5^-} f(x) = \lim_{x\to5^-}\left[|x|-5\right]$$
$$= \lim_{x\to5}(x-5) \qquad \left[\text{When } x>0,\ |x|=x\right]$$
$$= 5-5$$
$$= 0$$
$$\lim_{x\to5^+} f(x) = \lim_{x\to5^+}\left(|x|-5\right)$$
$$= \lim_{x\to5}(x-5) \qquad \left[\text{When } x>0,\ |x|=x\right]$$
$$= 5-5$$
$$= 0$$
$$\therefore \lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x) = 0$$
Hence, $\lim_{x\to5} f(x) = 0$

Question 28:

Suppose $f(x) = \begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax & x>1 \end{cases}$ and if $\lim_{x\to1} f(x) = f(1)$ what are possible values of a and b?

Answer

The given function is

$$f(x) = \begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax & x>1 \end{cases}$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1}(a+bx) = a+b$$
$$\lim_{x\to1^+} f(x) = \lim_{x\to1}(b-ax) = b-a$$
$$f(1) = 4$$
It is given that $\lim_{x\to1} f(x) = f(1)$.
$$\therefore \lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = \lim_{x\to1} f(x) = f(1)$$
$\Rightarrow a+b=4$ and $b-a=4$

On solving these two equations, we obtain $a=0$ and $b=4$.

Thus, the respective possible values of a and b are 0 and 4.

Question 29:

Let $a_1,\ a_2,\ \ldots,\ a_n$ be fixed real numbers and define a function

$$f(x)=(x-a_1)(x-a_2)\ldots(x-a_n)$$

What is $\lim_{x\to a_1} f(x)$? For some $a \neq a_1,\ a_2\ldots,\ a_n$, compute $\lim_{x\to a} f(x)$.

Answer

The given function is $f(x)=(x-a_1)(x-a_2)\ldots(x-a_n)$.

$$\lim_{x\to a_1} f(x)=\lim_{x\to a_1}\left[(x-a_1)(x-a_2)\ldots(x-a_n)\right]$$
$$=\left[\lim_{x\to a_1}(x-a_1)\right]\left[\lim_{x\to a_1}(x-a_2)\right]\ldots\left[\lim_{x\to a_1}(x-a_n)\right]$$
$$=(a_1-a_1)(a_1-a_2)\ldots(a_1-a_n)=0$$
$$\therefore \lim_{x\to a_1} f(x)=0$$

$$\text{Now, } \lim_{x\to a} f(x)=\lim_{x\to a}\left[(x-a_1)(x-a_2)\ldots(x-a_n)\right]$$
$$=\left[\lim_{x\to a}(x-a_1)\right]\left[\lim_{x\to a}(x-a_2)\right]\ldots\left[\lim_{x\to a}(x-a_n)\right]$$
$$=(a-a_1)(a-a_2)\ldots\ldots(a-a_n)$$
$$\therefore \lim_{x\to a} f(x)=(a-a_1)(a-a_2)\ldots(a-a_n)$$

Question 30:

If $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$.

For what value (s) of a does $\lim_{x\to a} f(x)$ exists?

Answer

The given function is

$$f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$$

When $a=0$,

$$\lim_{x\to0^-} f(x)=\lim_{x\to0^-}(|x|+1)$$
$$=\lim_{x\to0}(-x+1) \qquad [\text{If } x<0,\ |x|=-x]$$
$$=-0+1$$
$$=1$$

$$\lim_{x\to0^+} f(x)=\lim_{x\to0^+}(|x|-1)$$
$$=\lim_{x\to0}(x-1) \qquad [\text{If } x>0,\ |x|=x]$$
$$=0-1$$
$$=-1$$

Here, it is observed that $\lim_{x\to0^-} f(x)\neq \lim_{x\to0^+} f(x)$.

$\therefore \lim_{x\to0} f(x)$ does not exist.

When $a<0$,

$$\lim_{x\to a^-} f(x)=\lim_{x\to a^-}(|x|+1)$$
$$=\lim_{x\to a}(-x+1) \qquad [x<a<0\Rightarrow |x|=-x]$$
$$=-a+1$$

$$\lim_{x\to a^+} f(x)=\lim_{x\to a^+}(|x|+1)$$
$$=\lim_{x\to a}(-x+1) \qquad [a<x<0\Rightarrow |x|=-x]$$
$$=-a+1$$

$\therefore \lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)=-a+1$

Thus, limit of $f(x)$ exists at $x=a$, where $a<0$.

When $a>0$

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^-}(|x|-1)$$
$$= \lim_{x\to a}(x-1) \qquad [0 < x < a \Rightarrow |x| = x]$$
$$= a-1$$
$$\lim_{x\to a^+} f(x) = \lim_{x\to a^+}(|x|-1)$$
$$= \lim_{x\to a}(x-1) \qquad [0 < a < x \Rightarrow |x| = x]$$
$$= a-1$$
$$\therefore \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = a-1$$

Thus, limit of $f(x)$ exists at $x = a$, where $a > 0$.

Thus, $\lim_{x\to a} f(x)$ exists for all $a \neq 0$.

Question 31:

If the function $f(x)$ satisfies $\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$, evaluate $\lim_{x\to 1} f(x)$.

Answer

$$\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$$
$$\Rightarrow \frac{\lim_{x\to 1}(f(x)-2)}{\lim_{x\to 1}(x^2-1)} = \pi$$
$$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi \lim_{x\to 1}(x^2-1)$$
$$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi(1^2-1)$$
$$\Rightarrow \lim_{x\to 1}(f(x)-2) = 0$$
$$\Rightarrow \lim_{x\to 1} f(x) - \lim_{x\to 1} 2 = 0$$
$$\Rightarrow \lim_{x\to 1} f(x) - 2 = 0$$
$$\therefore \lim_{x\to 1} f(x) = 2$$

Question 32:

If $f(x)=\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \\ nx^3+m, & x>1 \end{cases}$. For what integers m and n does $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exist?

Answer

The given function is

$$f(x)=\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \\ nx^3+m, & x>1 \end{cases}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}\left(mx^2+n\right)$$
$$=m(0)^2+n$$
$$=n$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}\left(nx+m\right)$$
$$=n(0)+m$$
$$=m.$$

Thus, $\lim_{x\to 0} f(x)$ exists if $m = n$.

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}\left(nx+m\right)$$
$$=n(1)+m$$
$$=m+n$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}\left(nx^3+m\right)$$
$$=n(1)^3+m$$
$$=m+n$$

$$\therefore \lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=\lim_{x\to 1} f(x).$$

Thus, $\lim_{x\to 1} f(x)$ exists for any integral value of m and n.