Class XII : Maths
Chapter 7 : INTEGRALS

Questions and Solutions | Exercise 7.5 - NCERT Books

Question 1:

$$\frac{x}{(x+1)(x+2)}$$

Answer

Let $\frac{x}{(x+1)(x+2)}=\frac{A}{(x+1)}+\frac{B}{(x+2)}$

$\Rightarrow x=A(x+2)+B(x+1)$

Equating the coefficients of x and constant term, we obtain

$A+B=1$

$2A+B=0$

On solving, we obtain

$A=-1$ and $B=2$

$$\therefore \frac{x}{(x+1)(x+2)}=\frac{-1}{(x+1)}+\frac{2}{(x+2)}$$

$$\Rightarrow \int\frac{x}{(x+1)(x+2)}dx=\int\frac{-1}{(x+1)}+\frac{2}{(x+2)}dx$$

$$=-\log|x+1|+2\log|x+2|+C$$

$$=\log(x+2)^2-\log|x+1|+C$$

$$=\log\frac{(x+2)^2}{(x+1)}+C$$

Question 2:

$$\frac{1}{x^2-9}$$

Answer

Let $\frac{1}{(x+3)(x-3)}=\frac{A}{(x+3)}+\frac{B}{(x-3)}$

$1=A(x-3)+B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$$A=-\frac{1}{6} \text{ and } B=\frac{1}{6}$$

$$\therefore \frac{1}{(x+3)(x-3)}=\frac{-1}{6(x+3)}+\frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{\left(x^2-9\right)} dx=\int\left(\frac{-1}{6(x+3)}+\frac{1}{6(x-3)}\right) dx$$

$$=-\frac{1}{6} \log |x+3|+\frac{1}{6} \log |x-3|+C$$

$$=\frac{1}{6} \log \left|\frac{(x-3)}{(x+3)}\right|+C$$

Question 3:

$$\frac{3x-1}{(x-1)(x-2)(x-3)}$$

Answer

Let $$\frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$$

$$3x-1=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) \qquad \ldots(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain

A = 1, B = −5, and C = 4

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx=\int\left\{\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}\right\} dx$$

$$=\log |x-1|-5 \log |x-2|+4 \log |x-3|+C$$

Question 4:

$$\frac{x}{(x-1)(x-2)(x-3)}$$

Answer

Let $\frac{x}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$

$$x=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) \qquad ...(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain

$$A=\frac{1}{2},\ B=-2,\ \text{and } C=\frac{3}{2}$$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)}=\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}$$

$$\Rightarrow \int\frac{x}{(x-1)(x-2)(x-3)}dx=\int\left\{\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}\right\}dx$$

$$=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+C$$

Question 5:

$$\frac{2x}{x^2+3x+2}$$

Answer

Let $\frac{2x}{x^2+3x+2}=\frac{A}{(x+1)}+\frac{B}{(x+2)}$

$$2x=A(x+2)+B(x+1) \qquad ...(1)$$

Substituting x = −1 and −2 in equation (1), we obtain

A = −2 and B = 4

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)} dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+1)} \right\} dx$$

$$= 4 \log|x+2| - 2 \log|x+1| + C$$

Question 6:

$$\frac{1-x^2}{x(1-2x)}$$

Answer

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1 - x^2)$ by $x(1 - 2x)$, we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

Let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$

$$\Rightarrow (2-x) = A(1-2x) + Bx \qquad \ldots(1)$$

Substituting $x = 0$ and $\frac{1}{2}$ in equation (1), we obtain

$A = 2$ and $B = 3$

$$\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left\{\frac{2}{x}+\frac{3}{(1-2x)}\right\}$$

$$\Rightarrow \int\frac{1-x^2}{x(1-2x)}dx=\int\left\{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{1-2x}\right)\right\}dx$$

$$=\frac{x}{2}+\log|x|+\frac{3}{2(-2)}\log|1-2x|+C$$

$$=\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+C$$

Question 7:

$$\frac{x}{(x^2+1)(x-1)}$$

Answer

Let $\dfrac{x}{(x^2+1)(x-1)}=\dfrac{Ax+B}{(x^2+1)}+\dfrac{C}{(x-1)}$

$$x=(Ax+B)(x-1)+C(x^2+1)$$

$$x=Ax^2-Ax+Bx-B+Cx^2+C$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$-A + B = 1$

$-B + C = 0$

On solving these equations, we obtain

$$A=-\frac{1}{2}, B=\frac{1}{2}, \text{ and } C=\frac{1}{2}$$

From equation (1), we obtain

$$\therefore \frac{x}{(x^2+1)(x-1)} = \frac{\left(-\frac{1}{2}x+\frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{(x-1)}$$

$$\Rightarrow \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{2}\int \frac{x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x-1}dx$$

$$= -\frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

Consider $\int \frac{2x}{x^2+1}dx$, let $(x^2+1) = t \Rightarrow 2x\,dx = dt$

$$\Rightarrow \int \frac{2x}{x^2+1}dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$$

$$\therefore \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

Question 8:

$$\frac{x}{(x-1)^2(x+2)}$$

Answer

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Substituting x = 1, we obtain

$$B = \frac{1}{3}$$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$A=\frac{2}{9}$ and $C=\frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)}=\frac{2}{9(x-1)}+\frac{1}{3(x-1)^2}-\frac{2}{9(x+2)}$$

$$\Rightarrow \int\frac{x}{(x-1)^2(x+2)}dx=\frac{2}{9}\int\frac{1}{(x-1)}dx+\frac{1}{3}\int\frac{1}{(x-1)^2}dx-\frac{2}{9}\int\frac{1}{(x+2)}dx$$

$$=\frac{2}{9}\log|x-1|+\frac{1}{3}\left(\frac{-1}{x-1}\right)-\frac{2}{9}\log|x+2|+C$$

$$=\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+C$$

Question 9:

$$\frac{3x+5}{x^3-x^2-x+1}$$

Answer

$$\frac{3x+5}{x^3-x^2-x+1}=\frac{3x+5}{(x-1)^2(x+1)}$$

Let $\frac{3x+5}{(x-1)^2(x+1)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+1)}$

$$3x+5=A(x-1)(x+1)+B(x+1)+C(x-1)^2$$

$$3x+5=A(x^2-1)+B(x+1)+C(x^2+1-2x) \quad ...(1)$$

Substituting x = 1 in equation (1), we obtain

B = 4

Equating the coefficients of x^2 and x, we obtain

A + C = 0

B − 2C = 3

On solving, we obtain

$A=-\frac{1}{2}$ and $C=\frac{1}{2}$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$$

$$\Rightarrow \int \frac{3x+5}{(x-1)^2(x+1)}dx = -\frac{1}{2}\int \frac{1}{x-1}dx + 4\int \frac{1}{(x-1)^2}dx + \frac{1}{2}\int \frac{1}{(x+1)}dx$$

$$= -\frac{1}{2}\log|x-1| + 4\left(\frac{-1}{x-1}\right) + \frac{1}{2}\log|x+1| + C$$

$$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + C$$

Question 10:

$$\frac{2x-3}{(x^2-1)(2x+3)}$$

Answer

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let $\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$

$$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$$

$$\Rightarrow (2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$$

$$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$$

Equating the coefficients of x^2 and x, we obtain

$$B = -\frac{1}{10},\ A = \frac{5}{2},\ \text{and } C = -\frac{24}{5}$$

$$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)}=\frac{5}{2(x+1)}-\frac{1}{10(x-1)}-\frac{24}{5(2x+3)}$$

$$\Rightarrow \int\frac{2x-3}{(x^2-1)(2x+3)}dx=\frac{5}{2}\int\frac{1}{(x+1)}dx-\frac{1}{10}\int\frac{1}{x-1}dx-\frac{24}{5}\int\frac{1}{(2x+3)}dx$$

$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{24}{5\times 2}\log|2x+3|$$

$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{12}{5}\log|2x+3|+C$$

Question 11:

$$\frac{5x}{(x+1)(x^2-4)}$$

Answer

$$\frac{5x}{(x+1)(x^2-4)}=\frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)}=\frac{A}{(x+1)}+\frac{B}{(x+2)}+\frac{C}{(x-2)}$

$$5x=A(x+2)(x-2)+B(x+1)(x-2)+C(x+1)(x+2) \qquad ...(1)$$

Substituting $x = -1, -2$, and 2 respectively in equation (1), we obtain

$$A=\frac{5}{3},\ B=-\frac{5}{2},\text{ and } C=\frac{5}{6}$$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)}=\frac{5}{3(x+1)}-\frac{5}{2(x+2)}+\frac{5}{6(x-2)}$$

$$\Rightarrow \int\frac{5x}{(x+1)(x^2-4)}dx=\frac{5}{3}\int\frac{1}{(x+1)}dx-\frac{5}{2}\int\frac{1}{(x+2)}dx+\frac{5}{6}\int\frac{1}{(x-2)}dx$$

$$=\frac{5}{3}\log|x+1|-\frac{5}{2}\log|x+2|+\frac{5}{6}\log|x-2|+C$$

Question 12:

$$\frac{x^3+x+1}{x^2-1}$$

Answer

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing ($x^3 + x + 1$) by $x^2 - 1$, we obtain

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$$

Let $\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$

$$2x+1 = A(x-1) + B(x+1) \qquad \ldots(1)$$

Substituting $x = 1$ and -1 in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

$$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$$

$$\Rightarrow \int \frac{x^3+x+1}{x^2-1}dx = \int x\,dx + \frac{1}{2}\int \frac{1}{(x+1)}dx + \frac{3}{2}\int \frac{1}{(x-1)}dx$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$$

Question 13:

$$\frac{2}{(1-x)(1+x^2)}$$

Answer

$$\text{Let } \frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$$

$$2 = A(1+x^2) + (Bx+C)(1-x)$$

$$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$$

Equating the coefficient of x^2, x, and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1$, $B = 1$, and $C = 1$

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1} x + C$$

Question 14:

$$\frac{3x-1}{(x+2)^2}$$

Answer

$$\text{Let } \frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

$$\Rightarrow 3x-1 = A(x+2) + B$$

Equating the coefficient of x and constant term, we obtain

$A = 3$

$2A + B = -1 \Rightarrow B = -7$

$$\therefore \frac{3x-1}{(x+2)^2}=\frac{3}{(x+2)}-\frac{7}{(x+2)^2}$$

$$\Rightarrow \int\frac{3x-1}{(x+2)^2}dx=3\int\frac{1}{(x+2)}dx-7\int\frac{x}{(x+2)^2}dx$$

$$=3\log|x+2|-7\left(\frac{-1}{(x+2)}\right)+C$$

$$=3\log|x+2|+\frac{7}{(x+2)}+C$$

Question 15:

$$\frac{1}{x^4-1}$$

Answer

$$\frac{1}{(x^4-1)}=\frac{1}{(x^2-1)(x^2+1)}=\frac{1}{(x+1)(x-1)(1+x^2)}$$

$$\text{Let }\frac{1}{(x+1)(x-1)(1+x^2)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{Cx+D}{(x^2+1)}$$

$$1=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x^2-1)$$

$$1=A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+Cx^3+Dx^2-Cx-D$$

$$1=(A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$$

Equating the coefficient of x^3, x^2, x, and constant term, we obtain

$$A+B+C=0$$

$$-A+B+D=0$$

$$A+B-C=0$$

$$-A+B-D=1$$

On solving these equations, we obtain

$$A=-\frac{1}{4},B=\frac{1}{4},C=0,\text{ and }D=-\frac{1}{2}$$

$$\therefore \frac{1}{x^4-1}=\frac{-1}{4(x+1)}+\frac{1}{4(x-1)}-\frac{1}{2(x^2+1)}$$

$$\Rightarrow \int \frac{1}{x^4-1}dx=-\frac{1}{4}\log|x-1|+\frac{1}{4}\log|x-1|-\frac{1}{2}\tan^{-1}x+C$$

$$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$$

Question 16:

$\frac{1}{x(x^n+1)}$ [Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Answer

$$\frac{1}{x(x^n+1)}$$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x(x^n+1)}=\frac{x^{n-1}}{x^{n-1}x(x^n+1)}=\frac{x^{n-1}}{x^n(x^n+1)}$$

Let $x^n=t \Rightarrow x^{n-1}dx=dt$

$$\therefore \int\frac{1}{x(x^n+1)}dx=\int\frac{x^{n-1}}{x^n(x^n+1)}dx=\frac{1}{n}\int\frac{1}{t(t+1)}dt$$

Let $\frac{1}{t(t+1)}=\frac{A}{t}+\frac{B}{(t+1)}$

$$1=A(1+t)+Bt \qquad \ldots(1)$$

Substituting t = 0, −1 in equation (1), we obtain

A = 1 and B = −1

$$\therefore \frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{(1+t)}$$

$$\Rightarrow \int \frac{1}{x\left(x^n+1\right)} dx = \frac{1}{n} \int \left\{\frac{1}{t} - \frac{1}{(t+1)}\right\} dx$$

$$= \frac{1}{n}\left[\log|t| - \log|t+1|\right] + C$$

$$= -\frac{1}{n}\left[\log\left|x^n\right| - \log\left|x^n+1\right|\right] + C$$

$$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

Question 17:

$\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint: Put sin x = t]

Answer

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$

$$1 = A(2-t) + B(1-t) \quad \text{...(1)}$$

Substituting t = 2 and then t = 1 in equation (1), we obtain

A = 1 and B = −1

$$\therefore \frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$$

$$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \left\{ \frac{1}{1-t} - \frac{1}{(2-t)} \right\} dt$$

$$= -\log|1-t| + \log|2-t| + C$$

$$= \log\left|\frac{2-t}{1-t}\right| + C$$

$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

Question 18:

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

Answer

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

$$\text{Let } \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$$

$$4x^2+10 = (Ax+B)(x^2+4) + (Cx+D)(x^2+3)$$

$$4x^2+10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$$

$$4x^2+10 = (A+C)x^3 + (B+D)x^2 + (4A+3C)x + (4B+3D)$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4A + 3C = 0$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0$, $B = -2$, $C = 0$, and $D = 6$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

$$\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}=1-\left(\frac{-2}{\left(x^2+3\right)}+\frac{6}{\left(x^2+4\right)}\right)$$

$$\Rightarrow \int\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}dx=\int\left\{1+\frac{2}{\left(x^2+3\right)}-\frac{6}{\left(x^2+4\right)}\right\}dx$$

$$=\int\left\{1+\frac{2}{x^2+\left(\sqrt{3}\right)^2}-\frac{6}{x^2+2^2}\right\}$$

$$=x+2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)-6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right)+C$$

$$=x+\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+C$$

Question 19:

$$\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}$$

Answer

$$\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}$$

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore \int\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx=\int\frac{dt}{(t+1)(t+3)} \quad ...(1)$$

$$\text{Let } \frac{1}{(t+1)(t+3)}=\frac{A}{(t+1)}+\frac{B}{(t+3)}$$

$$1=A(t+3)+B(t+1) \quad ...(1)$$

Substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$A=\frac{1}{2}$ and $B=-\frac{1}{2}$

$$\therefore \frac{1}{(t+1)(t+3)}=\frac{1}{2(t+1)}-\frac{1}{2(t+3)}$$

$$\Rightarrow \int\frac{2x}{(x^2+1)(x^2+3)}dx=\int\left\{\frac{1}{2(t+1)}-\frac{1}{2(t+3)}\right\}dt$$

$$=\frac{1}{2}\log\left|(t+1)\right|-\frac{1}{2}\log\left|t+3\right|+C$$

$$=\frac{1}{2}\log\left|\frac{t+1}{t+3}\right|+C$$

$$=\frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right|+C$$

Question 20:

$$\frac{1}{x(x^4-1)}$$

Answer

$$\frac{1}{x(x^4-1)}$$

Multiplying numerator and denominator by x^3, we obtain

$$\frac{1}{x(x^4-1)}=\frac{x^3}{x^4(x^4-1)}$$

$$\therefore \int\frac{1}{x(x^4-1)}dx=\int\frac{x^3}{x^4(x^4-1)}dx$$

Let $x^4=t \Rightarrow 4x^3dx=dt$

$$\therefore \int\frac{1}{x(x^4-1)}dx=\frac{1}{4}\int\frac{dt}{t(t-1)}$$

Let $\frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{(t-1)}$

$1=A(t-1)+Bt$ ...(1)

Substituting t = 0 and 1 in (1), we obtain

A = −1 and B = 1

$$\Rightarrow \frac{1}{t(t+1)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\Rightarrow \int\frac{1}{x\left(x^4-1\right)}dx=\frac{1}{4}\int\left\{\frac{-1}{t}+\frac{1}{t-1}\right\}dt$$

$$=\frac{1}{4}\left[-\log|t|+\log|t-1|\right]+C$$

$$=\frac{1}{4}\log\left|\frac{t-1}{t}\right|+C$$

$$=\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+C$$

Question 21:

$\frac{1}{\left(e^x-1\right)}$ [Hint: Put $e^x = t$]

Answer

$\frac{1}{\left(e^x-1\right)}$

Let $e^x = t \Rightarrow e^x\, dx = dt$

$$\Rightarrow \int\frac{1}{e^x-1}dx=\int\frac{1}{t-1}\times\frac{dt}{t}=\int\frac{1}{t(t-1)}dt$$

$$\text{Let } \frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{t-1}$$

$$1=A(t-1)+Bt \qquad \ldots(1)$$

Substituting t = 1 and t = 0 in equation (1), we obtain

A = −1 and B = 1

$$\therefore \frac{1}{t(t-1)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\Rightarrow \int\frac{1}{t(t-1)}dt=\log\left|\frac{t-1}{t}\right|+C$$

$$=\log\left|\frac{e^x-1}{e^x}\right|+C$$

Question 22:

$\int\frac{xdx}{(x-1)(x-2)}$ equals

A. $\log\left|\frac{(x-1)^2}{x-2}\right|+C$

B. $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

C. $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$

D. $\log\left|(x-1)(x-2)\right|+C$

Answer

$$\text{Let } \frac{x}{(x-1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}$$

$$x=A(x-2)+B(x-1) \qquad \ldots(1)$$

Substituting x = 1 and 2 in (1), we obtain

A = −1 and B = 2

$$\therefore \frac{x}{(x-1)(x-2)} = -\frac{1}{(x-1)} + \frac{2}{(x-2)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)}dx = \int \left\{ \frac{-1}{(x-1)} + \frac{2}{(x-2)} \right\} dx$$

$$= -\log|x-1| + 2\log|x-2| + C$$

$$= \log\left|\frac{(x-2)^2}{x-1}\right| + C$$

Hence, the correct Answer is B.

Question 23:

$\int \frac{dx}{x(x^2+1)}$ equals

A. $\log|x| - \frac{1}{2}\log(x^2+1) + C$

B. $\log|x| + \frac{1}{2}\log(x^2+1) + C$

C. $-\log|x| + \frac{1}{2}\log(x^2+1) + C$

D. $\frac{1}{2}\log|x| + \log(x^2+1) + C$

Answer

Let $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$1 = A(x^2+1) + (Bx+C)x$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$C = 0$

$A = 1$

On solving these equations, we obtain

$A = 1$, $B = -1$, and $C = 0$

$$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$

$$\Rightarrow \int \frac{1}{x(x^2+1)}dx = \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx$$

$$= \log|x| - \frac{1}{2}\log|x^2+1| + C$$

Hence, the correct Answer is A.