

Class XII : Maths
Chapter 7 : INTEGRALS

Questions and Solutions | Exercise 7.5 - NCERT E

Question 3:

$$x^2 e^x$$

Answer

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$\begin{aligned}
 I &= x^2 \int e^x dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x dx \right\} dx \\
 &= x^2 e^x - \int 2x \cdot e^x dx \\
 &= x^2 e^x - 2 \int x \cdot e^x dx
 \end{aligned}$$

Again integrating by parts, we obtain

$$\begin{aligned}
 &= x^2 e^x - 2 \left[x \cdot \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \cdot \int e^x dx \right\} dx \right] \\
 &= x^2 e^x - 2 \left[x e^x - \int e^x dx \right] \\
 &= x^2 e^x - 2 \left[x e^x - e^x \right] \\
 &= x^2 e^x - 2x e^x + 2e^x + C \\
 &= e^x (x^2 - 2x + 2) + C
 \end{aligned}$$

Question 4:

$$x \log x$$

Answer

Let $I = \int x \log x dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned}
 I &= \log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx \\
 &= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \\
 &= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx \\
 &= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C
 \end{aligned}$$

Question 5:

$x \log 2x$

Answer

Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \log 2x \int x dx - \int \left\{ \left(\frac{d}{dx} 2 \log x \right) \int x dx \right\} dx \\ &= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx \\ &= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C \end{aligned}$$

Question 6:

$x^2 \log x$

Answer

Let $I = \int x^2 \log x dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx \\ &= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \\ &= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx \\ &= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C \end{aligned}$$

Question 7:

$$x \sin^{-1} x$$

Answer

Let $I = \int x \sin^{-1} x \, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \sin^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x \, dx \right\} dx \\ &= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} dx - \int \frac{1}{\sqrt{1-x^2}} dx \right\} \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C \\ &= \frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C \end{aligned}$$

Question 8:

$$x \tan^{-1} x$$

Answer

Let $I = \int x \tan^{-1} x \, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \tan^{-1} x \int x dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x dx \right\} dx \\ &= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} (x - \tan^{-1} x) + C \\ &= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C \end{aligned}$$

Question 9:

$$x \cos^{-1} x$$

Answer

Let $I = \int x \cos^{-1} x dx$

Taking $\cos^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned}
I &= \cos^{-1} x \int x dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int x dx \right\} dx \\
&= \cos^{-1} x \frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx \\
&= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx \\
&= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \left\{ \sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}} \right) \right\} dx \\
&= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \left(\frac{-1}{\sqrt{1-x^2}} \right) dx \\
&= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} I_1 - \frac{1}{2} \cos^{-1} x \quad \dots(1)
\end{aligned}$$

where, $I_1 = \int \sqrt{1-x^2} dx$

$$\begin{aligned}
\Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{d}{dx} \sqrt{1-x^2} \int x dx \\
\Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{-2x}{2\sqrt{1-x^2}} \cdot x dx \\
\Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}} dx \\
\Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx \\
\Rightarrow I_1 &= x\sqrt{1-x^2} - \left\{ \int \sqrt{1-x^2} dx + \int \frac{-dx}{\sqrt{1-x^2}} \right\} \\
\Rightarrow I_1 &= x\sqrt{1-x^2} - \{I_1 + \cos^{-1} x\} \\
\Rightarrow 2I_1 &= x\sqrt{1-x^2} - \cos^{-1} x \\
\therefore I_1 &= \frac{x}{2} \sqrt{1-x^2} - \frac{1}{2} \cos^{-1} x
\end{aligned}$$

Substituting in (1), we obtain

$$\begin{aligned}
I &= \frac{x \cos^{-1} x}{2} - \frac{1}{2} \left(\frac{x}{2} \sqrt{1-x^2} - \frac{1}{2} \cos^{-1} x \right) - \frac{1}{2} \cos^{-1} x \\
&= \frac{(2x^2-1)}{4} \cos^{-1} x - \frac{x}{4} \sqrt{1-x^2} + C
\end{aligned}$$

Question 10:

$$(\sin^{-1} x)^2$$

Answer

$$\text{Let } I = \int (\sin^{-1} x)^2 \cdot 1 \, dx$$

Taking $(\sin^{-1} x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned} I &= (\sin^{-1} x) \int 1 \, dx - \int \left\{ \frac{d}{dx} (\sin^{-1} x)^2 \cdot \int 1 \cdot dx \right\} dx \\ &= (\sin^{-1} x)^2 \cdot x - \int \frac{2 \sin^{-1} x}{\sqrt{1-x^2}} \cdot x \, dx \\ &= x (\sin^{-1} x)^2 + \int \sin^{-1} x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx \\ &= x (\sin^{-1} x)^2 + \left[\sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right] \\ &= x (\sin^{-1} x)^2 + \left[\sin^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right] \\ &= x (\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - \int 2 \, dx \\ &= x (\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C \end{aligned}$$

Question 11:

$$\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$$

Answer

$$\text{Let } I = \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$$

$$I = \frac{-1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1} x dx$$

Taking $\cos^{-1} x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \frac{-1}{2} \left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right] \\ &= \frac{-1}{2} \left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right] \\ &= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + \int 2 dx \right] \\ &= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + 2x \right] + C \\ &= - \left[\sqrt{1-x^2} \cos^{-1} x + x \right] + C \end{aligned}$$

Question 12:

$$x \sec^2 x$$

Answer

Let $I = \int x \sec^2 x dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$\begin{aligned} I &= x \int \sec^2 x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sec^2 x dx \right\} dx \\ &= x \tan x - \int 1 \cdot \tan x dx \\ &= x \tan x + \log |\cos x| + C \end{aligned}$$

Question 13:

$$\tan^{-1} x$$

Answer

Let $I = \int 1 \cdot \tan^{-1} x dx$

Taking $\tan^{-1} x$ as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned}
 I &= \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx \\
 &= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x dx \\
 &= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx \\
 &= x \tan^{-1} x - \frac{1}{2} \log|1+x^2| + C \\
 &= x \tan^{-1} x - \frac{1}{2} \log(1+x^2) + C
 \end{aligned}$$

Question 14:

$$x(\log x)^2$$

Answer

$$I = \int x(\log x)^2 dx$$

Taking $(\log x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned}
 I &= (\log x)^2 \int x dx - \int \left[\left(\frac{d}{dx} \log x \right)^2 \right] \int x dx dx \\
 &= \frac{x^2}{2} (\log x)^2 - \left[\int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right] \\
 &= \frac{x^2}{2} (\log x)^2 - \int x \log x dx
 \end{aligned}$$

Again integrating by parts, we obtain

$$\begin{aligned}
I &= \frac{x^2}{2}(\log x)^2 - \left[\log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx \right] \\
&= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2} - \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right] \\
&= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx \\
&= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C
\end{aligned}$$

Question 15:

$$(x^2 + 1) \log x$$

Answer

$$\text{Let } I = \int (x^2 + 1) \log x dx = \int x^2 \log x dx + \int \log x dx$$

$$\text{Let } I = I_1 + I_2 \dots (1)$$

$$\text{Where, } I_1 = \int x^2 \log x dx \text{ and } I_2 = \int \log x dx$$

$$I_1 = \int x^2 \log x dx$$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$\begin{aligned}
I_1 &= \log x - \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx \\
&= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \\
&= \frac{x^3}{3} \log x - \frac{1}{3} \left(\int x^2 dx \right) \\
&= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \qquad \dots (2)
\end{aligned}$$

$$I_2 = \int \log x dx$$

Taking $\log x$ as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned}
I_2 &= \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\} \\
&= \log x \cdot x - \int \frac{1}{x} \cdot x dx \\
&= x \log x - \int 1 dx \\
&= x \log x - x + C_2 \qquad \dots (3)
\end{aligned}$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned}
I &= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2 \\
&= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2) \\
&= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C
\end{aligned}$$

Question 16:

$$e^x (\sin x + \cos x)$$

Answer

$$\text{Let } I = \int e^x (\sin x + \cos x) dx$$

$$\text{Let } f(x) = \sin x$$

$$\square f'(x) = \cos x$$

$$\square I = \int e^x \{f(x) + f'(x)\} dx$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore I = e^x \sin x + C$$

Question 17:

$$\frac{xe^x}{(1+x)^2}$$

Answer

$$\text{Let } I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

$$\text{Let } f(x) = \frac{1}{1+x} \quad f'(x) = \frac{-1}{(1+x)^2}$$

$$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{f(x) + f'(x)\} dx$$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

Question 18:

$$e^x \left(\frac{1 + \sin x}{1 + \cos x} \right)$$

Answer

$$\begin{aligned}
& e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) \\
&= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) \\
&= \frac{e^x \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2 \cos^2 \frac{x}{2}} \\
&= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2 \\
&= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2 \\
&= \frac{1}{2} e^x \left(1 + \tan \frac{x}{2} \right)^2 \\
&= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
&= \frac{1}{2} e^x \left[\sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
&\frac{e^x (1 + \sin x) dx}{(1 + \cos x)} = e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \quad \dots(1)
\end{aligned}$$

Let $\tan \frac{x}{2} = f(x)$ \square $f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

From equation (1), we obtain

$$\int \frac{e^x (1 + \sin x)}{(1 + \cos x)} dx = e^x \tan \frac{x}{2} + C$$

Question 19:

$$e^x \left(\frac{1}{x} - \frac{1}{x^2} \right)$$

Answer

$$\text{Let } I = \int e^x \left[\frac{1}{x} - \frac{1}{x^2} \right] dx$$

$$\text{Also, let } \frac{1}{x} = f(x) \quad \square \quad f'(x) = \frac{-1}{x^2}$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore I = \frac{e^x}{x} + C$$

Question 20:

$$\frac{(x-3)e^x}{(x-1)^3}$$

Answer

$$\begin{aligned} \int e^x \left\{ \frac{x-3}{(x-1)^3} \right\} dx &= \int e^x \left\{ \frac{x-1-2}{(x-1)^3} \right\} dx \\ &= \int e^x \left\{ \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right\} dx \end{aligned}$$

$$\text{Let } f(x) = \frac{1}{(x-1)^2} \quad \square \quad f'(x) = \frac{-2}{(x-1)^3}$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore \int e^x \left\{ \frac{(x-3)}{(x-1)^2} \right\} dx = \frac{e^x}{(x-1)^2} + C$$

Question 21:

$$e^{2x} \sin x$$

Answer

$$\text{Let } I = \int e^{2x} \sin x \, dx \quad \dots(1)$$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} \, dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) \int e^{2x} \, dx \right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} \, dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} \, dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} \, dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} \, dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x \, dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I$$

[From (1)]

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C$$

Question 22:

$$\sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Answer

$$\text{Let } x = \tan \theta \quad \square \quad dx = \sec^2 \theta \, d\theta$$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\square \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \int 2\theta \cdot \sec^2 \theta d\theta = 2 \int \theta \cdot \sec^2 \theta d\theta$$

Integrating by parts, we obtain

$$2 \left[\theta \cdot \int \sec^2 \theta d\theta - \int \left\{ \left(\frac{d}{d\theta} \theta \right) \int \sec^2 \theta d\theta \right\} d\theta \right]$$

$$= 2 \left[\theta \cdot \tan \theta - \int \tan \theta d\theta \right]$$

$$= 2 \left[\theta \tan \theta + \log |\cos \theta| \right] + C$$

$$= 2 \left[x \tan^{-1} x + \log \left| \frac{1}{\sqrt{1+x^2}} \right| \right] + C$$

$$= 2x \tan^{-1} x + 2 \log (1+x^2)^{-\frac{1}{2}} + C$$

$$= 2x \tan^{-1} x + 2 \left[-\frac{1}{2} \log (1+x^2) \right] + C$$

$$= 2x \tan^{-1} x - \log (1+x^2) + C$$

Question 23:

$$\int x^2 e^{x^3} dx \text{ equals}$$

(A) $\frac{1}{3} e^{x^3} + C$

(B) $\frac{1}{3} e^{x^2} + C$

(C) $\frac{1}{2} e^{x^3} + C$

(D) $\frac{1}{3} e^{x^2} + C$

Answer

Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t \square 3x^2 dx = dt$

$$\begin{aligned}\Rightarrow I &= \frac{1}{3} \int e^t dt \\ &= \frac{1}{3} (e^t) + C \\ &= \frac{1}{3} e^{x^3} + C\end{aligned}$$

Hence, the correct Answer is A.

Question 24:

$$\int e^x \sec x (1 + \tan x) dx \text{ equals}$$

- (A) $e^x \cos x + C$ (B) $e^x \sec x + C$
 (C) $e^x \sin x + C$ (D) $e^x \tan x + C$

Answer

$$\int e^x \sec x (1 + \tan x) dx$$

$$\text{Let } I = \int e^x \sec x (1 + \tan x) dx = \int e^x (\sec x + \sec x \tan x) dx$$

$$\text{Also, let } \sec x = f(x) \quad \square \quad \sec x \tan x = f'(x)$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore I = e^x \sec x + C$$

Hence, the correct Answer is B.