

Class XII : Maths
Chapter 7 : INTEGRALS

Questions and Solutions | Exercise 7.8 - NCERT E

Question 3:

$$\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

Answer

$$\text{Let } I = \int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

$$\begin{aligned} \int (4x^3 - 5x^2 + 6x + 9) dx &= 4 \left(\frac{x^4}{4} \right) - 5 \left(\frac{x^3}{3} \right) + 6 \left(\frac{x^2}{2} \right) + 9(x) \\ &= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(1)$$

$$\begin{aligned} I &= \left\{ 2^4 - \frac{5 \cdot (2)^3}{3} + 3(2)^2 + 9(2) \right\} - \left\{ (1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1) \right\} \\ &= \left(16 - \frac{40}{3} + 12 + 18 \right) - \left(1 - \frac{5}{3} + 3 + 9 \right) \\ &= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9 \\ &= 33 - \frac{35}{3} \\ &= \frac{99 - 35}{3} \\ &= \frac{64}{3} \end{aligned}$$

Question 4:

$$\int_0^{\frac{\pi}{4}} \sin 2x dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \sin 2x dx$$

$$\int \sin 2x dx = \left(\frac{-\cos 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
I &= F\left(\frac{\pi}{4}\right) - F(0) \\
&= -\frac{1}{2} \left[\cos 2\left(\frac{\pi}{4}\right) - \cos 0 \right] \\
&= -\frac{1}{2} \left[\cos\left(\frac{\pi}{2}\right) - \cos 0 \right] \\
&= -\frac{1}{2} [0 - 1] \\
&= \frac{1}{2}
\end{aligned}$$

Question 5:

$$\int_0^{\frac{\pi}{2}} \cos 2x \, dx$$

Answer

$$\begin{aligned}
\text{Let } I &= \int_0^{\frac{\pi}{2}} \cos 2x \, dx \\
\int \cos 2x \, dx &= \left(\frac{\sin 2x}{2} \right) = F(x)
\end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
I &= F\left(\frac{\pi}{2}\right) - F(0) \\
&= \frac{1}{2} \left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0 \right] \\
&= \frac{1}{2} [\sin \pi - \sin 0] \\
&= \frac{1}{2} [0 - 0] = 0
\end{aligned}$$

Question 6:

$$\int_1^5 e^x \, dx$$

Answer

$$\text{Let } I = \int_4^5 e^x dx$$

$$\int e^x dx = e^x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(5) - F(4)$$

$$= e^5 - e^4$$

$$= e^4(e-1)$$

Question 7:

$$\int_0^{\frac{\pi}{4}} \tan x dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \tan x dx$$

$$\int \tan x dx = -\log|\cos x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$$

$$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$$

$$= -\log(2)^{-\frac{1}{2}}$$

$$= \frac{1}{2} \log 2$$

Question 8:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x dx$$

Answer

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right) \\ &= \log \left| \operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right| - \log \left| \operatorname{cosec} \frac{\pi}{6} - \cot \frac{\pi}{6} \right| \\ &= \log |\sqrt{2} - 1| - \log |2 - \sqrt{3}| \\ &= \log \left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}} \right) \end{aligned}$$

Question 9:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Answer

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(1) - F(0) \\ &= \sin^{-1}(1) - \sin^{-1}(0) \\ &= \frac{\pi}{2} - 0 \\ &= \frac{\pi}{2} \end{aligned}$$

Question 10:

$$\int_0^1 \frac{dx}{1+x^2}$$

Answer

$$\text{Let } I = \int_0^1 \frac{dx}{1+x^2}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(1) - F(0) \\ &= \tan^{-1}(1) - \tan^{-1}(0) \\ &= \frac{\pi}{4} \end{aligned}$$

Question 11:

$$\int_2^3 \frac{dx}{x^2-1}$$

Answer

$$\text{Let } I = \int_2^3 \frac{dx}{x^2-1}$$

$$\int \frac{dx}{x^2-1} = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(3) - F(2) \\ &= \frac{1}{2} \left[\log \left| \frac{3-1}{3+1} \right| - \log \left| \frac{2-1}{2+1} \right| \right] \\ &= \frac{1}{2} \left[\log \left| \frac{2}{4} \right| - \log \left| \frac{1}{3} \right| \right] \\ &= \frac{1}{2} \left[\log \frac{1}{2} - \log \frac{1}{3} \right] \\ &= \frac{1}{2} \left[\log \frac{3}{2} \right] \end{aligned}$$

Question 12:

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$\int \cos^2 x \, dx = \int \left(\frac{1 + \cos 2x}{2} \right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \left[F\left(\frac{\pi}{2}\right) - F(0) \right] \\ &= \frac{1}{2} \left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left(0 + \frac{\sin 0}{2} \right) \right] \\ &= \frac{1}{2} \left[\frac{\pi}{2} + 0 - 0 - 0 \right] \\ &= \frac{\pi}{4} \end{aligned}$$

Question 13:

$$\int_2^3 \frac{x \, dx}{x^2 + 1}$$

Answer

$$\text{Let } I = \int_2^3 \frac{x}{x^2 + 1} \, dx$$

$$\int \frac{x}{x^2 + 1} \, dx = \frac{1}{2} \int \frac{2x}{x^2 + 1} \, dx = \frac{1}{2} \log(1 + x^2) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(3) - F(2) \\ &= \frac{1}{2} \left[\log(1 + (3)^2) - \log(1 + (2)^2) \right] \\ &= \frac{1}{2} \left[\log(10) - \log(5) \right] \\ &= \frac{1}{2} \log\left(\frac{10}{5}\right) = \frac{1}{2} \log 2 \end{aligned}$$

Question 14:

$$\int_0^1 \frac{2x+3}{5x^2+1} dx$$

Answer

$$\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1} dx$$

$$\begin{aligned} \int \frac{2x+3}{5x^2+1} dx &= \frac{1}{5} \int \frac{5(2x+3)}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x+15}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5x^2+1} dx \\ &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5\left(x^2+\frac{1}{5}\right)} dx \\ &= \frac{1}{5} \log(5x^2+1) + \frac{3}{5} \cdot \frac{1}{\frac{1}{\sqrt{5}}} \tan^{-1} \frac{x}{\frac{1}{\sqrt{5}}} \\ &= \frac{1}{5} \log(5x^2+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}x) \\ &= F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(1) - F(0) \\ &= \left\{ \frac{1}{5} \log(5+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}) \right\} - \left\{ \frac{1}{5} \log(1) + \frac{3}{\sqrt{5}} \tan^{-1}(0) \right\} \\ &= \frac{1}{5} \log 6 + \frac{3}{\sqrt{5}} \tan^{-1} \sqrt{5} \end{aligned}$$

Question 15:

$$\int_0^1 x e^{x^2} dx$$

Answer

$$\text{Let } I = \int_0^1 x e^{x^2} dx$$

$$\text{Put } x^2 = t \Rightarrow 2x dx = dt$$

As $x \rightarrow 0, t \rightarrow 0$ and as $x \rightarrow 1, t \rightarrow 1$,

$$\therefore I = \frac{1}{2} \int_0^1 e^t dt$$

$$\frac{1}{2} \int e^t dt = \frac{1}{2} e^t = F(t)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \frac{1}{2} e - \frac{1}{2} e^0$$

$$= \frac{1}{2} (e - 1)$$

Question 16:

$$\int_0^1 \frac{5x^2}{x^2 + 4x + 3}$$

Answer

$$\text{Let } I = \int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$$

Dividing $5x^2$ by $x^2 + 4x + 3$, we obtain

$$I = \int_1^2 \left\{ 5 - \frac{20x + 15}{x^2 + 4x + 3} \right\} dx$$

$$= \int_1^2 5 dx - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$$

$$= [5x]_1^2 - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$$

$$I = 5 - I_1, \text{ where } I_1 = \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx \quad \dots(1)$$

Consider $I_1 = \int_1^2 \frac{20x+15}{x^2+4x+8} dx$

Let $20x+15 = A \frac{d}{dx}(x^2+4x+3) + B$
 $= 2Ax + (4A+B)$

Equating the coefficients of x and constant term, we obtain

$A = 10$ and $B = -25$

$\Rightarrow I_1 = 10 \int_1^2 \frac{2x+4}{x^2+4x+3} dx - 25 \int_1^2 \frac{dx}{x^2+4x+3}$

Let $x^2+4x+3 = t$

$\Rightarrow (2x+4)dx = dt$

$\Rightarrow I_1 = 10 \int \frac{dt}{t} - 25 \int \frac{dx}{(x+2)^2 - 1^2}$
 $= 10 \log t - 25 \left[\frac{1}{2} \log \left(\frac{x+2-1}{x+2+1} \right) \right]$
 $= \left[10 \log(x^2+4x+3) \right]_1^2 - 25 \left[\frac{1}{2} \log \left(\frac{x+1}{x+3} \right) \right]_1^2$
 $= [10 \log 15 - 10 \log 8] - 25 \left[\frac{1}{2} \log \frac{3}{5} - \frac{1}{2} \log \frac{2}{4} \right]$
 $= [10 \log(5 \times 3) - 10 \log(4 \times 2)] - \frac{25}{2} [\log 3 - \log 5 - \log 2 + \log 4]$
 $= [10 \log 5 + 10 \log 3 - 10 \log 4 - 10 \log 2] - \frac{25}{2} [\log 3 - \log 5 - \log 2 + \log 4]$
 $= \left[10 + \frac{25}{2} \right] \log 5 + \left[-10 - \frac{25}{2} \right] \log 4 + \left[10 - \frac{25}{2} \right] \log 3 + \left[-10 + \frac{25}{2} \right] \log 2$
 $= \frac{45}{2} \log 5 - \frac{45}{2} \log 4 - \frac{5}{2} \log 3 + \frac{5}{2} \log 2$
 $= \frac{45}{2} \log \frac{5}{4} - \frac{5}{2} \log \frac{3}{2}$

Substituting the value of I_1 in (1), we obtain

$$\begin{aligned}
 I &= 5 - \left[\frac{45}{2} \log \frac{5}{4} - \frac{5}{2} \log \frac{3}{2} \right] \\
 &= 5 - \frac{5}{2} \left[9 \log \frac{5}{4} - \log \frac{3}{2} \right]
 \end{aligned}$$

Question 17:

$$\int_0^{\frac{\pi}{4}} (2 \sec^2 x + x^3 + 2) dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{4}} (2 \sec^2 x + x^3 + 2) dx$$

$$\int (2 \sec^2 x + x^3 + 2) dx = 2 \tan x + \frac{x^4}{4} + 2x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
 I &= F\left(\frac{\pi}{4}\right) - F(0) \\
 &= \left\{ \left(2 \tan \frac{\pi}{4} + \frac{1}{4} \left(\frac{\pi}{4} \right)^4 + 2 \left(\frac{\pi}{4} \right) \right) - (2 \tan 0 + 0 + 0) \right\} \\
 &= 2 \tan \frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2} \\
 &= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}
 \end{aligned}$$

Question 18:

$$\int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

Answer

$$\begin{aligned} \text{Let } I &= \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx \\ &= - \int_0^{\pi} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) dx \\ &= - \int_0^{\pi} \cos x \, dx \end{aligned}$$

$$\int \cos x \, dx = \sin x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(\pi) - F(0) \\ &= \sin \pi - \sin 0 \\ &= 0 \end{aligned}$$

Question 19:

$$\int_0^2 \frac{6x+3}{x^2+4} dx$$

Answer

$$\begin{aligned} \text{Let } I &= \int_0^2 \frac{6x+3}{x^2+4} dx \\ \int \frac{6x+3}{x^2+4} dx &= 3 \int \frac{2x+1}{x^2+4} dx \\ &= 3 \int \frac{2x}{x^2+4} dx + 3 \int \frac{1}{x^2+4} dx \\ &= 3 \log(x^2+4) + \frac{3}{2} \tan^{-1} \frac{x}{2} = F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
I &= F(2) - F(0) \\
&= \left\{ 3 \log(2^2 + 4) + \frac{3}{2} \tan^{-1} \left(\frac{2}{2} \right) \right\} - \left\{ 3 \log(0 + 4) + \frac{3}{2} \tan^{-1} \left(\frac{0}{2} \right) \right\} \\
&= 3 \log 8 + \frac{3}{2} \tan^{-1} 1 - 3 \log 4 - \frac{3}{2} \tan^{-1} 0 \\
&= 3 \log 8 + \frac{3}{2} \left(\frac{\pi}{4} \right) - 3 \log 4 - 0 \\
&= 3 \log \left(\frac{8}{4} \right) + \frac{3\pi}{8} \\
&= 3 \log 2 + \frac{3\pi}{8}
\end{aligned}$$

Question 20:

$$\int_0^1 \left(xe^x + \sin \frac{\pi x}{4} \right) dx$$

Answer

$$\text{Let } I = \int_0^1 \left(xe^x + \sin \frac{\pi x}{4} \right) dx$$

$$\begin{aligned}
\int \left(xe^x + \sin \frac{\pi x}{4} \right) dx &= x \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int e^x dx \right\} dx + \left\{ \frac{-\cos \frac{\pi x}{4}}{\frac{\pi}{4}} \right\} \\
&= xe^x - \int e^x dx - \frac{4\pi}{\pi} \cos \frac{x}{4} \\
&= xe^x - e^x - \frac{4\pi}{\pi} \cos \frac{x}{4} \\
&= F(x)
\end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
I &= F(1) - F(0) \\
&= \left(1.e^1 - e^1 - \frac{4}{\pi} \cos \frac{\pi}{4}\right) - \left(0.e^0 - e^0 - \frac{4}{\pi} \cos 0\right) \\
&= e - e - \frac{4}{\pi} \left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi} \\
&= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}
\end{aligned}$$

Question 21:

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} \text{ equals}$$

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
\int_1^{\sqrt{3}} \frac{dx}{1+x^2} &= F(\sqrt{3}) - F(1) \\
&= \tan^{-1} \sqrt{3} - \tan^{-1} 1 \\
&= \frac{\pi}{3} - \frac{\pi}{4} \\
&= \frac{\pi}{12}
\end{aligned}$$

Hence, the correct Answer is D.

Question 22:

$\int_0^2 \frac{dx}{4+9x^2}$ equals

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{4}$

Answer

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\begin{aligned} \therefore \int \frac{dx}{(2)^2 + (3x)^2} &= \frac{1}{3} \int \frac{dt}{(2)^2 + t^2} \\ &= \frac{1}{3} \left[\frac{1}{2} \tan^{-1} \frac{t}{2} \right] \\ &= \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right) \\ &= F(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} &= F\left(\frac{2}{3}\right) - F(0) \\ &= \frac{1}{6} \tan^{-1}\left(\frac{3 \cdot 2}{2 \cdot 3}\right) - \frac{1}{6} \tan^{-1} 0 \\ &= \frac{1}{6} \tan^{-1} 1 - 0 \\ &= \frac{1}{6} \times \frac{\pi}{4} \\ &= \frac{\pi}{24}\end{aligned}$$

Hence, the correct Answer is C.