Class XII : Maths
Chapter 7 : INTEGRALS

Questions and Solutions | Exercise 7.1 - NCERT Books

Question 1:

sin $2x$

Answer

The anti derivative of sin $2x$ is a function of x whose derivative is sin $2x$.It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti derivative of $\sin 2x \text{ is } -\frac{1}{2}\cos 2x$

Question 2:

Cos $3x$

Answer

The anti derivative of cos $3x$ is a function of x whose derivative is cos $3x$.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti derivative of $\cos 3x \text{ is } \frac{1}{3}\sin 3x$.

Question 3:

e^{2x}

Answer

The anti derivative of e^{2x} is the function of x whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}\left(e^{2x}\right)=2e^{2x}$$
$$\Rightarrow e^{2x}=\frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$
$$\therefore e^{2x}=\frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$.

Question 4:

$(ax+b)^2$

Answer

The anti derivative of $(ax+b)^2$ is the function of *x* whose derivative is $(ax+b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3=3a(ax+b)^2$$
$$\Rightarrow (ax+b)^2=\frac{1}{3a}\frac{d}{dx}(ax+b)^3$$
$$\therefore (ax+b)^2=\frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

Question 5:

$\sin 2x-4e^{3x}$

Answer

The anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is the function of *x* whose derivative is $\left(\sin 2x-4e^{3x}\right)$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)=\sin 2x-4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)$.

Question 6:

$\int\left(4e^{3x}+1\right)dx$

Answer

$\int\left(4e^{3x}+1\right)dx$

$=4\int e^{3x}dx+\int 1dx$

$=4\left(\frac{e^{3x}}{3}\right)+x+C$

$=\frac{4}{3}e^{3x}+x+C$

Question 7:

$\int x^2\left(1-\frac{1}{x^2}\right)dx$

Answer

$\int x^2\left(1-\frac{1}{x^2}\right)dx$

$=\int\left(x^2-1\right)dx$

$=\int x^2dx-\int 1dx$

$=\frac{x^3}{3}-x+C$

Question 8:

$\int\left(ax^2+bx+c\right)dx$

Answer

$$\int\left(ax^2+bx+c\right)dx$$
$$=a\int x^2dx+b\int xdx+c\int 1.dx$$
$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$
$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

Question 9:

$$\int\left(2x^2+e^x\right)dx$$

Answer

$$\int\left(2x^2+e^x\right)dx$$
$$=2\int x^2dx+\int e^xdx$$
$$=2\left(\frac{x^3}{3}\right)+e^x+C$$
$$=\frac{2}{3}x^3+e^x+C$$

Question 10:

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$$

Answer

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$$
$$=\int\left(x+\frac{1}{x}-2\right)dx$$
$$=\int xdx+\int\frac{1}{x}dx-2\int 1.dx$$
$$=\frac{x^2}{2}+\log|x|-2x+C$$

Question 11:

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

Answer

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

$$= \int \left(x + 5 - 4x^{-2}\right) dx$$

$$= \int x dx + 5\int 1.dx - 4\int x^{-2} dx$$

$$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

Question 12:

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

Answer

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

$$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\right) dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}} + C$$

$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$

$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$$

Question 13:

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

Answer

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

On dividing, we obtain

$$= \int (x^2 + 1) dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3} + x + C$$

Question 14:

$$\int (1 - x)\sqrt{x} dx$$

Answer

$$\int (1 - x)\sqrt{x} dx$$

$$= \int \left(\sqrt{x} - x^{\frac{3}{2}} \right) dx$$

$$= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C$$

Question 15:

$$\int \sqrt{x} \left(3x^2 + 2x + 3 \right) dx$$

Answer

$$\int \sqrt{x}\left(3x^2+2x+3\right)dx$$

$$=\int\left(3x^{\frac{5}{2}}+2x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$$

$$=3\int x^{\frac{5}{2}}dx+2\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$$

$$=3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right)+2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)+3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+C$$

$$=\frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+C$$

Question 16:

$$\int\left(2x-3\cos x+e^x\right)dx$$

Answer

$$\int\left(2x-3\cos x+e^x\right)dx$$

$$=2\int xdx-3\int\cos xdx+\int e^x dx$$

$$=\frac{2x^2}{2}-3(\sin x)+e^x+C$$

$$=x^2-3\sin x+e^x+C$$

Question 17:

$$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$$

Answer

$$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$$

$$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}} dx$$

$$= \frac{2x^3}{3} - 3(-\cos x) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$$

Question 18:

$$\int \sec x(\sec x + \tan x) dx$$

Answer

$$\int \sec x(\sec x + \tan x) dx$$

$$= \int(\sec^2 x + \sec x \tan x) dx$$

$$= \int \sec^2 x dx + \int \sec x \tan x dx$$

$$= \tan x + \sec x + C$$

Question 19:

$$\int \frac{\sec^2 x}{\cos ec^2 x} dx$$

Answer

$$\int \frac{\sec^2 x}{\cos ec^2 x} dx$$

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \tan^2 x dx$$

$$= \int \left(\sec^2 x - 1\right) dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + C$$

Question 20:

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx$$

Answer

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx$$

$$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right) dx$$

$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$

$$= 2\tan x - 3\sec x + C$$

Question 21:

The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ **(B)** $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ **(D)** $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

Answer

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$$

$$=\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

Hence, the correct Answer is C.

Question 22:

If $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ such that *f*(2) = 0, then *f*(*x*) is

(A) $x^4+\frac{1}{x^3}-\frac{129}{8}$ **(B)** $x^3+\frac{1}{x^4}+\frac{129}{8}$

(C) $x^4+\frac{1}{x^3}+\frac{129}{8}$ **(D)** $x^3+\frac{1}{x^4}-\frac{129}{8}$

Answer

It is given that,

$$\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$$

∴Anti derivative of $4x^3-\frac{3}{x^4}=f(x)$

$$\therefore f(x)=\int 4x^3-\frac{3}{x^4}dx$$

$$f(x)=4\int x^3dx-3\int(x^{-4})dx$$

$$f(x)=4\left(\frac{x^4}{4}\right)-3\left(\frac{x^{-3}}{-3}\right)+C$$

$$\therefore f(x)=x^4+\frac{1}{x^3}+C$$

Also,

$$f(2)=0$$

$$\therefore f(2)=(2)^4+\frac{1}{(2)^3}+C=0$$

$$\Rightarrow 16+\frac{1}{8}+C=0$$

$$\Rightarrow C=-\left(16+\frac{1}{8}\right)$$

$$\Rightarrow C=\frac{-129}{8}$$

$$\therefore f(x)=x^4+\frac{1}{x^3}-\frac{129}{8}$$

Hence, the correct Answer is A.