Class XII : Maths
Chapter 9 : Differential Equations

Questions and Solutions | Exercise 9.2 - NCERT Books

Question 1:

$y = e^x + 1 \qquad : \quad y'' - y' = 0$

Answer

$y = e^x + 1$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + 1\right)$$

$$\Rightarrow y' = e^x \qquad \ldots(1)$$

Now, differentiating equation (1) with respect to x, we get:

$$\frac{d}{dx}\left(y'\right) = \frac{d}{dx}\left(e^x\right)$$

$$\Rightarrow y'' = e^x$$

Substituting the values of y' and y'' in the given differential equation, we get the L.H.S. as:

$$y'' - y' = e^x - e^x = 0 = \text{R.H.S.}$$

Thus, the given function is the solution of the corresponding differential equation.

Question 2:

$y = x^2 + 2x + C \qquad : \quad y' - 2x - 2 = 0$

Answer

$y = x^2 + 2x + C$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}\left(x^2 + 2x + C\right)$$

$$\Rightarrow y' = 2x + 2$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = $y' - 2x - 2 = 2x + 2 - 2x - 2 = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 3:

$y = \cos x + C \qquad : \ y' + \sin x = 0$

Answer

$y = \cos x + C$

Differentiating both sides of this equation with respect to *x*, we get:

$$y' = \frac{d}{dx}(\cos x + C)$$
$$\Rightarrow y' = -\sin x$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = $y' + \sin x = -\sin x + \sin x = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 4:

$y = \sqrt{1 + x^2} \qquad : \ y' = \frac{xy}{1 + x^2}$

Answer

$y = \sqrt{1 + x^2}$

Differentiating both sides of the equation with respect to *x*, we get:

$$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

$$y' = \frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}\left(1+x^2\right)$$

$$y' = \frac{2x}{2\sqrt{1+x^2}}$$

$$y' = \frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \times \sqrt{1+x^2}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \cdot y$$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 5:

$y = Ax \quad : \quad xy' = y\,(x \neq 0)$

Answer

$y = Ax$

Differentiating both sides with respect to x, we get:

$$y' = \frac{d}{dx}(Ax)$$

$$\Rightarrow y' = A$$

Substituting the value of y' in the given differential equation, we get:

$\text{L.H.S.} = xy' = x \cdot A = Ax = y = \text{R.H.S.}$

Hence, the given function is the solution of the corresponding differential equation.

Question 6:

$y = x\sin x \quad : \quad xy' = y + x\sqrt{x^2 - y^2}\ (x \neq 0 \text{ and } x > y \text{ or } x < -y)$

Answer

$y = x\sin x$

Differentiating both sides of this equation with respect to *x*, we get:

$$y' = \frac{d}{dx}(x\sin x)$$
$$\Rightarrow y' = \sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)$$
$$\Rightarrow y' = \sin x + x\cos x$$

Substituting the value of y' in the given differential equation, we get:

$$\begin{aligned}\text{L.H.S.} = xy' &= x(\sin x + x\cos x) \\ &= x\sin x + x^2\cos x \\ &= y + x^2 \cdot \sqrt{1-\sin^2 x} \\ &= y + x^2\sqrt{1-\left(\frac{y}{x}\right)^2} \\ &= y + x\sqrt{y^2 - x^2} \\ &= \text{R.H.S.}\end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 7:

$xy = \log y + C$: $y' = \dfrac{y^2}{1-xy}\ (xy \neq 1)$

Answer

$xy = \log y + C$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{d}{dx}(xy)=\frac{d}{dx}(\log y)$$
$$\Rightarrow y\cdot\frac{d}{dx}(x)+x\cdot\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx}$$
$$\Rightarrow y+xy'=\frac{1}{y}y'$$
$$\Rightarrow y^2+xy\,y'=y'$$
$$\Rightarrow (xy-1)\,y'=-y^2$$
$$\Rightarrow y'=\frac{y^2}{1-xy}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 8:

$y-\cos y=x$: $(y\sin y+\cos y+x)\,y'=y$

Answer

$$y-\cos y=x \qquad \ldots(1)$$

Differentiating both sides of the equation with respect to *x*, we get:

$$\frac{dy}{dx}-\frac{d}{dx}(\cos y)=\frac{d}{dx}(x)$$
$$\Rightarrow y'+\sin y\cdot y'=1$$
$$\Rightarrow y'(1+\sin y)=1$$
$$\Rightarrow y'=\frac{1}{1+\sin y}$$

Substituting the value of y' in equation (1), we get:

$$\text{L.H.S.} = (y\sin y + \cos y + x)y'$$
$$= (y\sin y + \cos y + y - \cos y) \times \frac{1}{1+\sin y}$$
$$= y(1+\sin y)\cdot\frac{1}{1+\sin y}$$
$$= y$$
$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 9:

$x + y = \tan^{-1} y \qquad : \quad y^2y' + y^2 + 1 = 0$

Answer

$x + y = \tan^{-1} y$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{d}{dx}(x+y) = \frac{d}{dx}\left(\tan^{-1} y\right)$$
$$\Rightarrow 1 + y' = \left[\frac{1}{1+y^2}\right]y'$$
$$\Rightarrow y'\left[\frac{1}{1+y^2} - 1\right] = 1$$
$$\Rightarrow y'\left[\frac{1-\left(1+y^2\right)}{1+y^2}\right] = 1$$
$$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right] = 1$$
$$\Rightarrow y' = \frac{-\left(1+y^2\right)}{y^2}$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = y^2 y' + y^2 + 1 = y^2\left[\frac{-\left(1+y^2\right)}{y^2}\right] + y^2 + 1$$
$$= -1 - y^2 + y^2 + 1$$
$$= 0$$
$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 10:

$y = \sqrt{a^2 - x^2}\, x \in (-a, a) \qquad : \quad x + y\dfrac{dy}{dx} = 0\,(y \neq 0)$

Answer

$$y = \sqrt{a^2 - x^2}$$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{a^2 - x^2}\right)$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}} \cdot \frac{d}{dx}\left(a^2 - x^2\right)$$
$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$$
$$= \frac{-x}{\sqrt{a^2 - x^2}}$$

Substituting the value of $\dfrac{dy}{dx}$ in the given differential equation, we get:

$$\text{L.H.S.} = x + y\frac{dy}{dx} = x + \sqrt{a^2 - x^2} \times \frac{-x}{\sqrt{a^2 - x^2}}$$
$$= x - x$$
$$= 0$$
$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 11:

The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4

Answer

We know that the number of constants in the general solution of a differential equation of order *n* is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Hence, the correct answer is D.

Question 12:

The numbers of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

Answer

In a particular solution of a differential equation, there are no arbitrary constants.

Hence, the correct answer is D.