Revision Notes

Class - 10 Maths

Chapter 1 - Real Numbers

- **Real numbers:**
  - All rational and irrational numbers taken together make the real numbers. On the number line, any real number can be plotted.

- **Euclid's Division Lemma:**
  - A lemma is a verified statement that is utilised to prove another. Euclid's Division Lemma states that for any two integers a and b, there exists a unique pair of integers q and r such that $a = b \times q + r$ where $0 \leq r < b$.
  - The lemma can be simply stated as :
    $\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$
  - For any pair of dividend and divisor, the quotient and remainder obtained are going to be unique.

- **Euclid's Division Algorithm:**
  - An algorithm is a set of well-defined steps that describe how to solve a certain problem. The Highest Common Factor (HCF) of two positive integers is computed using Euclid's division algorithm.
  - Follow the steps below to find the HCF of two positive integers, say c and d, with $c > d$:
    Step 1: We apply Euclid's Division Lemma to find two integers q and r such that $c = d \times q + r$ where $0 \leq r < d$.
    Step 2: If $r = 0$, the H.C.F is d, else, we apply Euclid's division Lemma to d (the divisor) and r (the remainder) to get another pair of quotient and remainder.
    Step 3: Repeat Steps 1–3 until the remainder is zero. The needed HCF will be the divisor at the last step.

- **The Fundamental Theorem of Arithmetic:**
  The process of expressing a natural number as a product of prime numbers is known as prime factorization.
  Apart from the sequence in which the prime components occur, the prime factorisation for a given number is unique.

Example: $12 = 2 \times 2 \times 3$, here 12 is represented as a product of its prime factors 2 and 3.

- **Finding LCM and HCF:**
  - HCF is the product of the smallest power of each common prime factor in the given numbers.
  - LCM is the product of the greatest power of each prime factor, involved in the given numbers.
  - For any two positive integers a and b, $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$
  - L.C.M can be used to find common occurrence sites. For instance, the time when two people running at different speeds meet, or the ringing of bells with various frequencies.

- **Rational and Irrational numbers:**
  - If a number can be expressed in the form p/q where p and q are integers and $q \neq 0$, then it is called a rational number.
  - If a number cannot be expressed in the form p/q where p and q are integers and $q \neq 0$, then it is called an irrational number.

- **Number Theory:**
  - If p (a prime number) divides a^2, then p divides a as well. For example, 3 divides 6^2, resulting in 36, implying that 3 divides 6.
  - The sum or difference of a rational and an irrational number is irrational
  - A non-zero rational and irrational number's product and quotient are both irrational.
  - $\sqrt{p}$ is irrational when p is a prime number. For example, 7 is a prime number and $\sqrt{7}$ is irrational. The preceding statement can be proven by the process of "Proof by contradiction".

- **Decimal Expansions of Rational Numbers:**
  - Let $x = \dfrac{p}{q}$ be a rational number with the prime factorization $2^n 5^m$, where n and m are non-negative integers. The decimal expansion of x then comes to an end. Then x has a non-terminating repeated decimal expansion (recurring).

- If $\frac{a}{b}$ is a rational number, then its decimal expansion would terminate if both of the following conditions are satisfied :
  a) The H.C.F of a and b is 1.
  b) b can be expressed as a prime factorisation of 2 and 5 i.e in the form 2^n5^m where either m or n, or both can be zero.
- If the prime factorisation of b contains any number other than 2 or 5, then the decimal expansion of that number will be recurring