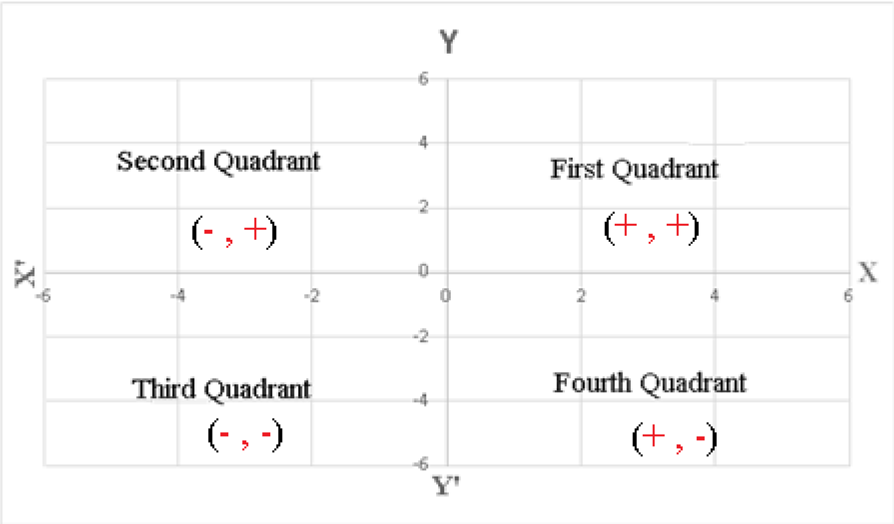
Revision Notes

Class 10 Mathematics

Chapter 7 – Co-ordinate Geometry

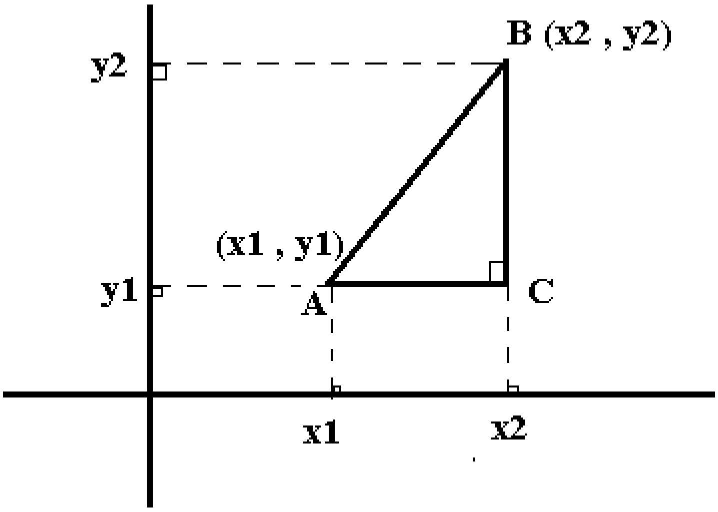
Important Terms and Concepts:



- "The **coordinate axes** are of perpendicular lines " XOX " and " YOY " intersecting at O ."
- The plane is divided into **four quadrants** by the axes.
- The **Cartesian plane** is the plane that contains the axes.
- The lines " XOX " and " YOY " are known as the x -axis and y -axis, respectively, and are commonly drawn **horizontally** and **vertically** as seen in the picture.
- O, is the **Point of Intersection of the axes** which is known as **origin.**
- **Abscissae** are the values of x measured along the x -axis from O. x has positive values along OX, but **negative** values along OX' .
- Similarly, ordinate refers to the values of y measured along the y axis from O.
- y has positive values along OY, but negative values along OY'.
- The **coordinates of a point** are the ordered pair containing the **abscissa** and **ordinate of a point**.

Distance Formula:

- To find the distance two points $A(x_1, y_1)$ and $B(x_2, y_2)$



- From The Figure,

  $AC = x_2 - x_1$

  $BC = y_2 - y_1$

  Therefore, In ΔABC,

  By using the Pythagoras Theorem, we get

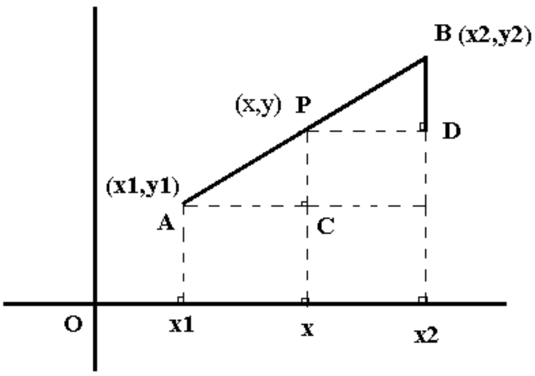
  $AB^2 = AC^2 + BC^2$

  $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

  $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Section Formula:

- To find the coordinates of a point which divides the line segment joining two given points in a given ratio (internally).



- Let $P(x, y)$ divide the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the Ratio $m:n$.

Here,

$\angle PAC = \angle BPD$

$\angle PCA = \angle BDP = 90^0$

Therefore, by using AA similarity method, we get

$AC = x - x_1$

$PD = x_2 - x$

From **Similarity Property,** we get

$$\frac{AC}{PD} = \frac{m}{n}$$

$$= \frac{x-x_1}{x_2-x}$$

$$= \frac{m}{n}$$

Now, make x the subject of the formula,

$nx - nx_1 = mx_2 - mx$

$Mx + nx = mx_2 + nx_1$

$X(m+n) = mx_2 + nx_1$

Therefore,

$$X = \frac{mx_2 + nx_1}{m+n}$$

Similarly, we can show that

$$Y = \frac{my_2 + ny_1}{m+n}$$

Thus, the Co-ordinate of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

Mid−Point Formula:

- If P is the **mid−point** of AB, then $m:n$,

Therefore, the ratio becomes 1:1

And hence,

$$X = \frac{mx_2 + nx_1}{m + n}$$

$$X = \frac{x_2 + x_1}{1 + 1}$$

$$X = \frac{x_2 + x_1}{2}$$

Similarly, we get

$$Y = \frac{y_1 + y_2}{2}$$

Thus, the Co-ordinate of P are $\frac{x_1 + x_2}{}, \frac{y_1 + y_2}{}$

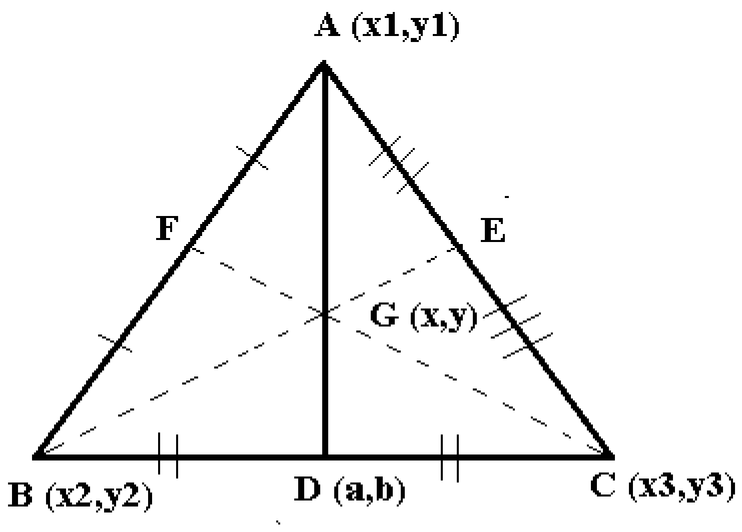
- **Note:**

When the point P divides the line joining AB in the ration $m:n$ externally then,

The Co-ordinate of P are $\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}$

Centroid of a Triangle:

- Centroid is the **point of intersection of three medians**.
- It is the point of intersection of a median
- $AG : GD$ is $2:1$



To find the coordinates of the centroid of a triangle:

Let the coordinates of the vertices of $\triangle ABC$ be $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$

Let $G(x,y)$ be the centroid of the $\triangle ABC$

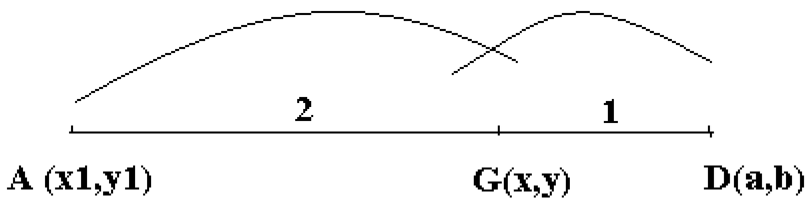
Here, D is the midpoint of BC , and hence

By applying the mid−point formula, we get

$$a = \frac{x_2 + x_3}{2} \text{ and } b = \frac{y_2 + y_3}{2}$$

We know that,

In a $2:1$ ratio, point G splits the median.



Therefore, by applying the **section formula**, we get

$$X = \frac{mx_2 + nx_1}{m + n}$$

$$X = \frac{2(a) + 1(x_1)}{2 + 1}$$

$$X = \frac{2\left(\frac{x_2 + x_3}{2}\right) + x_1}{3}$$

$$X = \frac{x_1 + x_2 + x_3}{3}$$

Similarly,

$$Y = \frac{2(b) + 1(y_1)}{2 + 1}$$

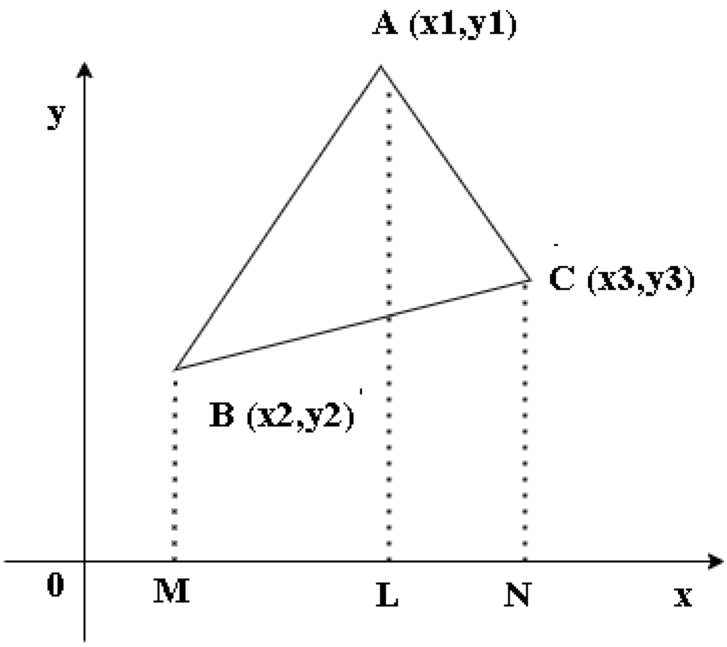
$$Y = \frac{2\left(\frac{y_2 + y_3}{2}\right) + y_1}{3}$$

$$Y = \frac{y_1 + y_3 + y_3}{3}$$

Therefore, **Coordinates of the centroid** are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Area of the Triangle:

To find the area of triangle whose vertices are (x_1,y_1), (x_2,y_2) and (x_3,y_3)



Let, $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ be the vertices of a triangle $\triangle ABC$

Area of $\triangle$ ABC

= Area of Trapezium ABML + Area of Trapezium ALNC

- Area of Trapezium BMNC

$$= \frac{1}{2}ML(MB+LA)+\frac{1}{2}LN(LA+NC)-\frac{1}{2}MN(MB+NC)$$

$$=\frac{1}{2}(x_1-x_2)(y_2+y_1)+\frac{1}{2}(x_3-x_1)(y_1+y_3)-\frac{1}{2}(x_3-x_2)(y_2+y_3)$$

$$=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

Arrow Method:

It is to obtain the formula for the area of the triangle

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

Note:

1. If the points A, B and C we take in the **anticlockwise direction**, then **the area will be positive**.
2. If the points we take in **clockwise direction,** the **area will be negative**.
3. So we always take the **absolute value of the area** calculated.

Therefore,

Area of triangle

$$= \frac{1}{\;} \quad x_1(y_2 \quad y_3) \quad x_2(y_3 \quad y_1) \quad x_3(y_1 \quad y_2) \quad .$$

4. If the area of a triangle is **zero,** then the three points are **collinear.**