

Class XII : Maths
Chapter 10 : Vector Algebra

Questions and Solutions | Exercise 10.1 - NCERT E

Question 3:

Write two different vectors having same direction.

Answer

Consider $\vec{p} = (\hat{i} + \hat{j} + \hat{k})$ and $\vec{q} = (2\hat{i} + 2\hat{j} + 2\hat{k})$.

The direction cosines of \vec{p} are given by,

$$l = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}, \quad m = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}, \quad \text{and } n = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of \vec{q} are given by

$$l = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \quad m = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\text{and } n = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of \vec{p} and \vec{q} are the same. Hence, the two vectors have the same direction.

Question 4:

Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal

Answer

The two vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ will be equal if their corresponding components are equal.

Hence, the required values of x and y are 2 and 3 respectively.

Question 5:

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

Answer

The vector with the initial point P (2, 1) and terminal point Q (-5, 7) can be given by,

$$\vec{PQ} = (-5-2)\hat{i} + (7-1)\hat{j}$$

$$\Rightarrow \vec{PQ} = -7\hat{i} + 6\hat{j}$$

Hence, the required scalar components are -7 and 6 while the vector components are

$$-7\hat{i} \text{ and } 6\hat{j}.$$

Question 6:

Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Answer

The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

$$\begin{aligned} \therefore \vec{a} + \vec{b} + \vec{c} &= (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k} \\ &= 0\hat{i} - 4\hat{j} - 1\hat{k} \\ &= -4\hat{j} - \hat{k} \end{aligned}$$

Question 7:

Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Answer

The unit vector \hat{a} in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is given by $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

Question 8:

Find the unit vector in the direction of vector \overline{PQ} , where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Answer

The given points are P (1, 2, 3) and Q (4, 5, 6).

$$\therefore \overline{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$|\overline{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Hence, the unit vector in the direction of \overline{PQ} is

$$\frac{\overline{PQ}}{|\overline{PQ}|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Question 9:

For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$

Answer

The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$.

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

$$\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$$

$$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Hence, the unit vector in the direction of $(\vec{a} + \vec{b})$ is

$$\frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

Question 10:

Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

Answer

$$\text{Let } \vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}.$$

$$\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a} = 8 \left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}} \right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

$$= 8 \left(\frac{5\vec{i} - \vec{j} + 2\vec{k}}{\sqrt{30}} \right)$$

$$= \frac{40}{\sqrt{30}}\vec{i} - \frac{8}{\sqrt{30}}\vec{j} + \frac{16}{\sqrt{30}}\vec{k}$$

Question 11:

Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Answer

Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$.

It is observed that $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$

$$\therefore \vec{b} = \lambda\vec{a}$$

where,

$$\lambda = -2$$

Hence, the given vectors are collinear.

Question 12:

Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$

Answer

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$.

$$\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

Hence, the direction cosines of \vec{a} are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)$.

Question 13:

Find the direction cosines of the vector joining the points A (1, 2, -3) and B (-1, -2, 1) directed from A to B.

Answer

The given points are A (1, 2, -3) and B (-1, -2, 1).

$$\therefore \overline{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$$

$$\Rightarrow \overline{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\therefore |\overline{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

Hence, the direction cosines of \overline{AB} are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$.

Question 14:

Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ.

Answer

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Then,

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\vec{a} \text{ are } \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right).$$

Therefore, the direction cosines of

Now, let α , β , and γ be the angles formed by \vec{a} with the positive directions of x , y , and z axes.

$$\cos \alpha = \frac{1}{\sqrt{3}}, \cos \beta = \frac{1}{\sqrt{3}}, \cos \gamma = \frac{1}{\sqrt{3}}.$$

Then, we have

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

Question 15:

Find the position vector of a point R which divides the line joining two points P and Q

whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2:1

(i) internally

(ii) externally

Answer

The position vector of point R dividing the line segment joining two points

P and Q in the ratio $m : n$ is given by:

i. Internally:

$$\frac{m\vec{b} + n\vec{a}}{m+n}$$

ii. Externally:

$$\frac{m\vec{b} - n\vec{a}}{m-n}$$

Position vectors of P and Q are given as:

$$\overrightarrow{OP} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \overrightarrow{OQ} = -\hat{i} + \hat{j} + \hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\begin{aligned} \overrightarrow{OR} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2+1} = \frac{(-2\hat{i} + 2\hat{j} + 2\hat{k}) + (\hat{i} + 2\hat{j} - \hat{k})}{3} \\ &= \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k} \end{aligned}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$\begin{aligned} \overrightarrow{OR} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) - 1(\hat{i} + 2\hat{j} - \hat{k})}{2-1} = \frac{(-2\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})}{1} \\ &= -3\hat{i} + 3\hat{k} \end{aligned}$$

Question 16:

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, -2).

Answer

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, -2) is given by,

$$\begin{aligned} \overrightarrow{OR} &= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2} \\ &= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k} \end{aligned}$$

Question 17:

Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

Answer

Position vectors of points A, B, and C are respectively given as:

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\therefore \overline{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\overline{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overline{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore |\overline{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1+9+25 = 35$$

$$|\overline{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1+4+36 = 41$$

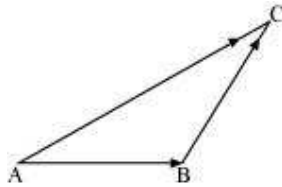
$$|\overline{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4+1+1 = 6$$

$$\therefore |\overline{AB}|^2 + |\overline{CA}|^2 = 35 + 6 = 41 = |\overline{BC}|^2$$

Hence, ABC is a right-angled triangle.

Question 18:

In triangle ABC which of the following is **not** true:



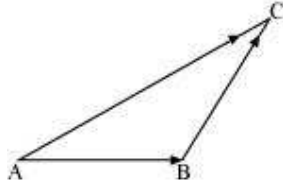
A. $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$

B. $\overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$

C. $\overline{AB} + \overline{BC} - \overline{CA} = \vec{0}$

D. $\overline{AB} - \overline{CB} + \overline{CA} = \vec{0}$

Answer



On applying the triangle law of addition in the given triangle, we have:

$$\overline{AB} + \overline{BC} = \overline{AC} \quad \dots(1)$$

$$\Rightarrow \overline{AB} + \overline{BC} = -\overline{CA}$$

$$\Rightarrow \overline{AB} + \overline{BC} + \overline{CA} = \vec{0} \quad \dots(2)$$

\therefore The equation given in alternative A is true.

$$\overline{AB} + \overline{BC} = \overline{AC}$$

$$\Rightarrow \overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$$

\therefore The equation given in alternative B is true.

From equation (2), we have:

$$\overline{AB} - \overline{CB} + \overline{CA} = \vec{0}$$

\therefore The equation given in alternative D is true.

Now, consider the equation given in alternative C:

$$\overline{AB} + \overline{BC} - \overline{CA} = \vec{0}$$

$$\Rightarrow \overline{AB} + \overline{BC} = \overline{CA} \quad \dots(3)$$

From equations (1) and (3), we have:

$$\overline{AC} = \overline{CA}$$

$$\Rightarrow \overline{AC} = -\overline{AC}$$

$$\Rightarrow \overline{AC} + \overline{AC} = \vec{0}$$

$$\Rightarrow 2\overline{AC} = \vec{0}$$

$$\Rightarrow \overline{AC} = \vec{0}, \text{ which is not true.}$$

Hence, the equation given in alternative C is **incorrect**.

The correct answer is **C**.

Question 19:

If \vec{a} and \vec{b} are two collinear vectors, then which of the following are **incorrect**:

A. $\vec{b} = \lambda \vec{a}$, for some scalar λ

B. $\vec{a} = \pm \vec{b}$

C. the respective components of \vec{a} and \vec{b} are proportional

D. both the vectors \vec{a} and \vec{b} have same direction, but different magnitudes

Answer

If \vec{a} and \vec{b} are two collinear vectors, then they are parallel.

Therefore, we have:

$$\vec{b} = \lambda \vec{a} \text{ (For some scalar } \lambda \text{)}$$

If $\lambda = \pm 1$, then $\vec{a} = \pm \vec{b}$.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{b} = \lambda \vec{a}.$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

Thus, the respective components of \vec{a} and \vec{b} are proportional.

However, vectors \vec{a} and \vec{b} can have different directions.

Hence, the statement given in **D** is **incorrect**.

The correct answer is **D**.