

**Series HMJ/1**
**SET-1**

 कोड नं. **65/1/1**  
**Code No.**

 रोल नं. 

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 Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.



नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 15 हैं ।	(I) Please check that this question paper contains 15 printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।	(II) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 36 प्रश्न हैं ।	(III) Please check that this question paper contains 36 questions.
(IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।	(IV) Please write down the Serial Number of the question in the answer-book before attempting it.
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

**गणित**

**MATHEMATICS**

निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80

**सामान्य निर्देश :**

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) यह प्रश्न-पत्र चार खण्डों में विभाजित किया गया है – क, ख, ग एवं घ । इस प्रश्न-पत्र में 36 प्रश्न हैं । सभी प्रश्न अनिवार्य हैं ।
- (ii) खण्ड क में प्रश्न संख्या 1 से 20 तक 20 प्रश्न हैं एवं प्रत्येक प्रश्न 1 अंक का है ।
- (iii) खण्ड ख में प्रश्न संख्या 21 से 26 तक 6 प्रश्न हैं एवं प्रत्येक प्रश्न 2 अंकों का है ।
- (iv) खण्ड ग में प्रश्न संख्या 27 से 32 तक 6 प्रश्न हैं एवं प्रत्येक प्रश्न 4 अंकों का है ।
- (v) खण्ड घ में प्रश्न संख्या 33 से 36 तक 4 प्रश्न हैं एवं प्रत्येक प्रश्न 6 अंकों का है ।
- (vi) प्रश्न-पत्र में समग्र पर कोई विकल्प नहीं है । तथापि एक-एक अंक वाले तीन प्रश्नों में, दो-दो अंकों वाले दो प्रश्नों में, चार-चार अंकों वाले दो प्रश्नों में और छः-छः अंकों वाले दो प्रश्नों में आंतरिक विकल्प दिए गए हैं । ऐसे प्रश्नों में से केवल एक ही विकल्प का उत्तर लिखिए ।
- (vii) इसके अतिरिक्त, आवश्यकतानुसार, प्रत्येक खण्ड और प्रश्न के साथ यथोचित निर्देश दिए गए हैं ।
- (viii) केलकुलेटरों के प्रयोग की अनुमति नहीं है ।

**खण्ड क**

प्रश्न संख्या 1 से 20 तक प्रत्येक प्रश्न 1 अंक का है ।

प्रश्न संख्या 1 से 10 तक बहुविकल्पीय प्रश्न हैं । सही विकल्प चुनिए ।

1. यदि A कोटि 3 का एक वर्ग आव्यूह है तथा  $|A| = 5$  है, तो  $|2A'|$  का मान होगा
  - (A) -10
  - (B) 10
  - (C) -40
  - (D) 40
2. यदि A एक वर्ग आव्यूह है तथा  $A^2 = A$  हो, तो  $(I - A)^3 + A$  बराबर है
  - (A) I
  - (B) 0
  - (C)  $I - A$
  - (D)  $I + A$
3.  $\tan^{-1}(\tan \frac{3\pi}{5})$  का मुख्य मान है
  - (A)  $\frac{2\pi}{5}$
  - (B)  $\frac{-2\pi}{5}$
  - (C)  $\frac{3\pi}{5}$
  - (D)  $\frac{-3\pi}{5}$

**General Instructions :**

Read the following instructions very carefully and strictly follow them :

- (i) This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.
- (ii) **Section A** – Questions no. **1 to 20** comprises of **20** questions of **1** mark each.
- (iii) **Section B** – Questions no. **21 to 26** comprises of **6** questions of **2** marks each.
- (iv) **Section C** – Questions no. **27 to 32** comprises of **6** questions of **4** marks each.
- (v) **Section D** – Questions no. **33 to 36** comprises of **4** questions of **6** marks each.
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is **not** permitted.

**SECTION A**

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1. If A is a square matrix of order 3 and  $|A| = 5$ , then the value of  $|2A'|$  is
  - (A)  $-10$
  - (B)  $10$
  - (C)  $-40$
  - (D)  $40$
2. If A is a square matrix such that  $A^2 = A$ , then  $(I - A)^3 + A$  is equal to
  - (A)  $I$
  - (B)  $0$
  - (C)  $I - A$
  - (D)  $I + A$
3. The principal value of  $\tan^{-1}(\tan \frac{3\pi}{5})$  is
  - (A)  $\frac{2\pi}{5}$
  - (B)  $\frac{-2\pi}{5}$
  - (C)  $\frac{3\pi}{5}$
  - (D)  $\frac{-3\pi}{5}$



4. यदि सदिश  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  का सदिश  $\vec{b} = 2\hat{i} + \lambda\hat{k}$  पर प्रक्षेप शून्य है, तो  $\lambda$  का मान होगा
- (A) 0  
 (B) 1  
 (C)  $-\frac{2}{3}$   
 (D)  $-\frac{3}{2}$
5. समतल  $z = 0$  के लंबवत् तथा बिन्दु  $(-1, 5, 4)$  से गुज़रने वाली रेखा का सदिश समीकरण होगा
- (A)  $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$   
 (B)  $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$   
 (C)  $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$   
 (D)  $\vec{r} = \lambda\hat{k}$
6. कोटि 2 वाले अवकल समीकरण के विशेष हल में स्वेच्छ अचरों की संख्या होगी
- (A) 0  
 (B) 1  
 (C) 2  
 (D) 3
7.  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx$  बराबर है
- (A) -1  
 (B) 0  
 (C) 1  
 (D) 2



4. If the projection of  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  on  $\vec{b} = 2\hat{i} + \lambda\hat{k}$  is zero, then the value of  $\lambda$  is
- (A) 0  
(B) 1  
(C)  $-\frac{2}{3}$   
(D)  $-\frac{3}{2}$
5. The vector equation of the line passing through the point  $(-1, 5, 4)$  and perpendicular to the plane  $z = 0$  is
- (A)  $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$   
(B)  $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$   
(C)  $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$   
(D)  $\vec{r} = \lambda\hat{k}$
6. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)
- (A) 0  
(B) 1  
(C) 2  
(D) 3
7.  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx$  is equal to
- (A) -1  
(B) 0  
(C) 1  
(D) 2

8. बिन्दु  $(4, -7, 3)$  से  $y$ -अक्ष पर डाले गए लम्ब की लम्बाई होगी  
 (A) 3 इकाई  
 (B) 4 इकाई  
 (C) 5 इकाई  
 (D) 7 इकाई
9. यदि  $A$  और  $B$  दो स्वतंत्र घटनाएँ हैं, जहाँ  $P(A) = \frac{1}{3}$  व  $P(B) = \frac{1}{4}$  है, तो  $P(B'|A)$  बराबर है  
 (A)  $\frac{1}{4}$   
 (B)  $\frac{1}{3}$   
 (C)  $\frac{3}{4}$   
 (D) 1
10. रैखिक असमिकाओं के निकाय से नियत सुसंगत क्षेत्र के कोनीय बिंदु  $(0, 0)$ ,  $(4, 0)$ ,  $(2, 4)$  तथा  $(0, 5)$  हैं। यदि  $z = ax + by$ , जहाँ  $a, b > 0$  का अधिकतम मान बिन्दुओं  $(2, 4)$  तथा  $(4, 0)$  दोनों पर हो, तो  
 (A)  $a = 2b$   
 (B)  $2a = b$   
 (C)  $a = b$   
 (D)  $3a = b$

प्रश्न संख्या 11 से 15 तक के सभी प्रश्नों के खाली स्थान भरिए।

11. यदि समस्त  $a_1, a_2 \in A$  के लिए  $(a_1, a_2) \in R$  से  $(a_2, a_1) \in R$  प्राप्त हो, तो समुच्चय  $A$  पर परिभाषित संबंध  $R$  कहलाता है \_\_\_\_\_।
12.  $f(x) = [x]$ ,  $0 < x < 2$  द्वारा परिभाषित महत्तम पूर्णांक फलन  $x =$  \_\_\_\_\_ पर अवकलनीय नहीं होता है।
13. यदि आव्यूह  $A$  की कोटि  $3 \times 2$  है, तो आव्यूह  $A'$  की कोटि होगी \_\_\_\_\_।

अथवा

एक वर्ग आव्यूह  $A$  विषम-सममित आव्यूह होगा, यदि \_\_\_\_\_।



8. The length of the perpendicular drawn from the point  $(4, -7, 3)$  on the  $y$ -axis is
- (A) 3 units  
(B) 4 units  
(C) 5 units  
(D) 7 units
9. If A and B are two independent events with  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{1}{4}$ , then  $P(B' | A)$  is equal to
- (A)  $\frac{1}{4}$   
(B)  $\frac{1}{3}$   
(C)  $\frac{3}{4}$   
(D) 1
10. The corner points of the feasible region determined by the system of linear inequalities are  $(0, 0)$ ,  $(4, 0)$ ,  $(2, 4)$  and  $(0, 5)$ . If the maximum value of  $z = ax + by$ , where  $a, b > 0$  occurs at both  $(2, 4)$  and  $(4, 0)$ , then
- (A)  $a = 2b$   
(B)  $2a = b$   
(C)  $a = b$   
(D)  $3a = b$

*Fill in the blanks in question numbers 11 to 15.*

11. A relation R in a set A is called \_\_\_\_\_, if  $(a_1, a_2) \in R$  implies  $(a_2, a_1) \in R$ , for all  $a_1, a_2 \in A$ .
12. The greatest integer function defined by  $f(x) = [x]$ ,  $0 < x < 2$  is not differentiable at  $x =$  \_\_\_\_\_.
13. If A is a matrix of order  $3 \times 2$ , then the order of the matrix  $A'$  is \_\_\_\_\_.

**OR**

A square matrix A is said to be skew-symmetric, if \_\_\_\_\_.

14. वक्र  $y^2 = 8x$  के मूल-बिन्दु पर अभिलंब का समीकरण है \_\_\_\_\_ ।

अथवा

एक वृत्त की त्रिज्या समान रूप से  $3 \text{ cm/s}$  की दर से बढ़ रही है। उस क्षण पर जबकि वृत्त की त्रिज्या  $2 \text{ cm}$  है, वृत्त के क्षेत्रफल में \_\_\_\_\_  $\text{cm}^2/\text{s}$  की दर से बढ़ोतरी होगी।

15. दो बिन्दुओं A तथा B के स्थिति सदिश क्रमशः  $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$  और  $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$  हैं। बिन्दु P, जो रेखाखण्ड AB को  $2 : 1$  के अनुपात में विभाजित करता है, का स्थिति सदिश है \_\_\_\_\_ ।

प्रश्न संख्या 16 से 20 अति संक्षिप्त उत्तर वाले प्रश्न हैं।

16. यदि  $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$  है, तो  $A (\text{adj } A)$  ज्ञात कीजिए।

17. ज्ञात कीजिए :

$$\int x^4 \log x \, dx$$

अथवा

ज्ञात कीजिए :

$$\int \frac{2x}{\sqrt[3]{x^2 + 1}} \, dx$$

18. मान ज्ञात कीजिए :

$$\int_1^3 |2x - 1| \, dx$$

19. ताश की 52 पत्तों वाली अच्छी प्रकार से फेंटी गई गड्डी में से यादृच्छया तथा बिना प्रतिस्थापना के एक-एक कर के दो पत्ते निकाले गए। एक पत्ता लाल तथा दूसरा काले रंग का आने की प्रायिकता ज्ञात कीजिए।

20. ज्ञात कीजिए :

$$\int \frac{dx}{\sqrt{9 - 4x^2}}$$





14. The equation of the normal to the curve  $y^2 = 8x$  at the origin is \_\_\_\_\_ .

**OR**

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of \_\_\_\_\_  $\text{cm}^2/\text{s}$ .

15. The position vectors of two points A and B are  $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$  and  $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$ , respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is \_\_\_\_\_ .

*Question numbers 16 to 20 are very short answer type questions.*

16. If  $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$ , then find  $A(\text{adj } A)$ .

17. Find :

$$\int x^4 \log x \, dx$$

**OR**

- Find :

$$\int \frac{2x}{\sqrt[3]{x^2 + 1}} \, dx$$

18. Evaluate :

$$\int_1^3 |2x - 1| \, dx$$

19. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

20. Find :

$$\int \frac{dx}{\sqrt{9 - 4x^2}}$$

### खण्ड ख

प्रश्न संख्या 21 से 26 तक प्रत्येक प्रश्न 2 अंकों का है।

21. सिद्ध कीजिए कि :

$$\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \leq x \leq 1.$$

अथवा

$f: \mathbb{R}_+ \rightarrow (7, \infty)$ ,  $f(x) = 16x^2 + 24x + 7$  द्वारा परिभाषित एकैकी और आच्छादक फलन पर विचार कीजिए, जहाँ  $\mathbb{R}_+$  सभी धनात्मक वास्तविक संख्याओं का समुच्चय है। फलन  $f$  का प्रतिलोम फलन ज्ञात कीजिए।

22. यदि  $x = at^2$ ,  $y = 2at$  हो, तो  $\frac{d^2y}{dx^2}$  ज्ञात कीजिए।

23. वक्र  $y = x^3 - 3x^2 - 4x$  के वे बिन्दु ज्ञात कीजिए जिन पर स्पर्श-रेखाएँ, रेखा  $4x + y - 3 = 0$  के समांतर हैं।

24. एक मात्रक सदिश ज्ञात कीजिए जो प्रत्येक सदिश  $\vec{a}$  और  $\vec{b}$  के लम्बवत् हो, जहाँ  $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$  और  $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$ .

अथवा

उस समांतर षट्फलक का आयतन ज्ञात कीजिए जिसकी संलग्न भुजाएँ  $2\vec{a}$ ,  $-\vec{b}$  तथा  $3\vec{c}$  द्वारा निरूपित हैं, जहाँ

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k},$$

$$\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k} \text{ तथा}$$

$$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ हैं।}$$

25.  $k$  का वह मान ज्ञात कीजिए जिसके लिए रेखाएँ  $x = -y = kz$  तथा  $x - 2 = 2y + 1 = -z + 1$  एक-दूसरे पर लम्ब हैं।

26. एक व्यस्त चौराहे X पर, हरी बत्ती मिलने की प्रायिकता 30% है। इस चौराहे X पर तीन में से लगातार दो दिन हरी बत्ती के मिलने की प्रायिकता क्या है ?



### SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Prove that  $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$ ,  $\frac{1}{\sqrt{2}} \leq x \leq 1$ .

OR

Consider a bijective function  $f: \mathbb{R}_+ \rightarrow (7, \infty)$  given by  $f(x) = 16x^2 + 24x + 7$ , where  $\mathbb{R}_+$  is the set of all positive real numbers. Find the inverse function of  $f$ .

22. If  $x = at^2$ ,  $y = 2at$ , then find  $\frac{d^2y}{dx^2}$ .

23. Find the points on the curve  $y = x^3 - 3x^2 - 4x$  at which the tangent lines are parallel to the line  $4x + y - 3 = 0$ .

24. Find a unit vector perpendicular to each of the vectors  $\vec{a}$  and  $\vec{b}$  where  $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$  and  $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$ .

OR

Find the volume of the parallelepiped whose adjacent edges are represented by  $2\vec{a}$ ,  $-\vec{b}$  and  $3\vec{c}$ , where

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k},$$

$$\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}, \text{ and}$$

$$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}.$$

25. Find the value of  $k$  so that the lines  $x = -y = kz$  and  $x - 2 = 2y + 1 = -z + 1$  are perpendicular to each other.

26. The probability of finding a green signal on a busy crossing  $X$  is 30%. What is the probability of finding a green signal on  $X$  on two consecutive days out of three ?

### खण्ड ग

प्रश्न संख्या 27 से 32 तक प्रत्येक प्रश्न 4 अंकों का है।

27. माना  $N$  प्राकृत संख्याओं का समुच्चय है। संबंध  $R$ ,  $N \times N$  पर “ $(a, b) R (c, d)$  यदि और केवल यदि  $ad = bc$ , सभी  $a, b, c, d \in N$  के लिए” द्वारा परिभाषित है। दिखाइए कि संबंध  $R$  एक तुल्यता संबंध है।

28. यदि  $y = e^{x^2 \cos x} + (\cos x)^x$  है, तो  $\frac{dy}{dx}$  ज्ञात कीजिए।

29. ज्ञात कीजिए :

$$\int \sec^3 x \, dx$$

30. अवकल समीकरण  $y e^y \, dx = (y^3 + 2x e^y) \, dy$  का व्यापक हल ज्ञात कीजिए।

अथवा

अवकल समीकरण  $x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$ , जहाँ  $x = 1$  पर  $y = \frac{\pi}{4}$  है, का विशिष्ट हल ज्ञात कीजिए।

31. एक फर्नीचर व्यापारी अपनी धनराशि को मेजों या कुर्सियों या दोनों के संयोजनों में निवेश करता है। निवेश के लिए उसके पास ₹ 50,000 हैं और उसके पास अधिकतम 35 वस्तुओं को रखने के लिए स्थान उपलब्ध है। एक कुर्सी का क्रय मूल्य ₹ 1,000 व एक मेज का क्रय मूल्य ₹ 2,000 है। इस व्यापारी को एक कुर्सी बेचकर ₹ 150 व एक मेज को बेचकर ₹ 250 का लाभ अर्जित होता है। उपर्युक्त समस्या के लिए अधिकतम लाभ अर्जित करने के लिए एक रैखिक प्रोग्रामन समस्या बनाइए और आलेखीय विधि से समस्या को हल कीजिए।

32. दो थैले I और II दिए गए हैं। थैले I में 3 लाल तथा 5 काली गेंदें हैं जबकि थैले II में 4 लाल तथा 3 काली गेंदें हैं। थैले I से थैले II में एक गेंद यादृच्छया स्थानान्तरित की जाती है और तत्पश्चात् थैले II में से एक गेंद यादृच्छया निकाली जाती है। यदि यह निकाली गई गेंद काली गेंद है, तो स्थानान्तरित की गई गेंद के काले रंग के होने की प्रायिकता ज्ञात कीजिए।

अथवा

एक कलश में 5 लाल, 2 सफेद तथा 3 काली गेंदें हैं। एक-एक करके, बिना प्रतिस्थापना के, इस कलश से 3 गेंदें यादृच्छया निकाली जाती हैं। सफेद गेंदों की संख्या का प्रायिकता बंटन ज्ञात कीजिए। निकाली गई सफेद गेंदों की संख्या का माध्य व प्रसरण भी ज्ञात कीजिए।

**SECTION C**

Question numbers 27 to 32 carry 4 marks each.

**27.** Let  $N$  be the set of natural numbers and  $R$  be the relation on  $N \times N$  defined by  $(a, b) R (c, d)$  iff  $ad = bc$  for all  $a, b, c, d \in N$ . Show that  $R$  is an equivalence relation.

**28.** If  $y = e^{x^2 \cos x} + (\cos x)^x$ , then find  $\frac{dy}{dx}$ .

**29.** Find :

$$\int \sec^3 x \, dx$$

**30.** Find the general solution of the differential equation

$$y e^y \, dx = (y^3 + 2x e^y) \, dy.$$

**OR**

Find the particular solution of the differential equation

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right), \text{ given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

**31.** A furniture trader deals in only two items — chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1,000 and a table costs him ₹ 2,000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

**32.** There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

**OR**

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

## खण्ड घ

प्रश्न संख्या 33 से 36 तक प्रत्येक प्रश्न 6 अंकों का है।

33. यदि  $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$  है, तो  $A^{-1}$  ज्ञात कीजिए और इसका प्रयोग करके

निम्नलिखित समीकरण निकाय का हल ज्ञात कीजिए :

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$

अथवा

सारणिकों के गुणधर्मों का प्रयोग करके, सिद्ध कीजिए कि

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

34. समाकलन विधि से, त्रिभुज जिसके शीर्ष  $(2, -2)$ ,  $(4, 5)$  तथा  $(6, 2)$  हैं, से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।
35. दिखाइए कि त्रिज्या  $r$  व ऊँचाई  $h$  के लम्ब-वृत्तीय शंकु के अन्तर्गत अधिकतम आयतन के लम्ब-वृत्तीय बेलन की ऊँचाई, शंकु की ऊँचाई की एक-तिहाई है और बेलन का अधिकतम आयतन, शंकु के आयतन का  $\frac{4}{9}$  वाँ भाग है।
36. उस समतल का समीकरण ज्ञात कीजिए, जिसमें बिन्दु  $A(2, 1, -1)$  स्थित है तथा जो समतलों  $2x + y - z = 3$  और  $x + 2y + z = 2$  की प्रतिच्छेदन रेखा के लम्बवत् है। प्राप्त समतल व  $y$ -अक्ष के बीच का कोण भी ज्ञात कीजिए।

अथवा

रेखा  $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$  तथा समतल  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$  के प्रतिच्छेदन बिन्दु  $Q$  की बिन्दु  $P(-2, -4, 7)$  से दूरी ज्ञात कीजिए। रेखा  $PQ$  का सदिश समीकरण भी लिखिए।



### SECTION D

Question numbers 33 to 36 carry 6 marks each.

33. If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ , then find  $A^{-1}$  and use it to solve the following

system of the equations :

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$

**OR**

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

34. Using integration, find the area of the region bounded by the triangle whose vertices are  $(2, -2)$ ,  $(4, 5)$  and  $(6, 2)$ .
35. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height  $h$  and radius  $r$  is one-third of the height of the cone, and the greatest volume of the cylinder is  $\frac{4}{9}$  times the volume of the cone.
36. Find the equation of the plane that contains the point  $A(2, 1, -1)$  and is perpendicular to the line of intersection of the planes  $2x + y - z = 3$  and  $x + 2y + z = 2$ . Also find the angle between the plane thus obtained and the  $y$ -axis.

**OR**

Find the distance of the point  $P(-2, -4, 7)$  from the point of intersection  $Q$  of the line  $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$ . Also write the vector equation of the line  $PQ$ .

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**Senior School Certificate Examination-2020**

**Marking Scheme - Mathematics 65/1/1**

**General instructions:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks \_\_\_\_\_ (example 0-100 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
  - Leaving answer or part thereof unassessed in an answer book.
  - Giving more marks for an answer than assigned to it.
  - Wrong totaling of marks awarded on a reply
  - Wrong transfer of marks from the inside pages of the answer book to the title page.
  - Wrong question wise totaling on the title page.
  - Wrong totaling of marks of the two columns on the title page.
  - Wrong grand total.
  - Marks in words and figures not tallying.
  - Wrong transfer of marks from the answer book to online award list.
  - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
  - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.





**QUESTION PAPER CODE 65/1/1**  
**EXPECTED ANSWER/VALUE POINTS**

**SECTION – A**

**Question Numbers 1 to 20 carry 1 mark each.**

**Question Numbers 1 to 10 are multiple choice type questions.**

**Select the correct option.**

<b>Q.No.</b>		<b>Marks</b>
1.	If A is a square matrix of order 3 and $ A  = 5$ , then the value of $ 2A $ is (A) $-10$ (B) $10$ (C) $-40$ (D) $40$ <span style="border: 1px solid black; padding: 2px; display: inline-block;"><b>Ans:</b> (D) <math>40</math></span>	<b>1</b>
2.	If A is a square matrix such that $A^2 = A$ , then $(I - A)^3 + A$ is equal to (A) $I$ (B) $0$ (C) $I - A$ (D) $I + A$ <span style="border: 1px solid black; padding: 2px; display: inline-block;"><b>Ans:</b> (A) <math>I</math></span>	<b>1</b>
3.	The principal value of $\tan^{-1}\left(\tan \frac{3\pi}{5}\right)$  (A) $\frac{2\pi}{5}$ (B) $-\frac{2\pi}{5}$ (C) $\frac{3\pi}{5}$ (D) $-\frac{3\pi}{5}$ <span style="border: 1px solid black; padding: 2px; display: inline-block;"><b>Ans:</b> (B) <math>-\frac{2\pi}{5}</math></span>	<b>1</b>
4.	If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ , is zero, then the value of $\lambda$ is  (A) $0$ (B) $1$ (C) $-\frac{2}{3}$ (D) $\frac{-3}{2}$ <span style="border: 1px solid black; padding: 2px; display: inline-block;"><b>Ans:</b> (C) <math>-\frac{2}{3}</math></span>	<b>1</b>
5.	The vector equation of the line passing through the point $(-1, 5, 4)$ and perpendicular to the plane $z = 0$ is (A) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$ (B) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$ (C) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$ (D) $\vec{r} = \lambda\hat{k}$ <span style="border: 1px solid black; padding: 2px; display: inline-block;"><b>Ans:</b> (B) <math>\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}</math></span>	<b>1</b>
6.	The number of arbitrary constants in the particular solution of a differential equation of second order is (are) (A) $0$ (B) $1$ (C) $2$ (D) $3$ <span style="border: 1px solid black; padding: 2px; display: inline-block;"><b>Ans:</b> (A) <math>0</math></span>	<b>1</b>



7.  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx$   
 (A) -1 (B) 0 (C) 1 (D) 2  
**Ans:** (D) 2 1
8. The length of the perpendicular drawn from the point (4, -7, 3) on the y-axis is  
 (A) 3 units (B) 4 units (C) 5 units (D) 7 units  
**Ans:** (C) 5 units 1
9. If A and B are two independent events with  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{1}{4}$ , then  $P(B'|A)$  is equal to  
 (A)  $\frac{1}{4}$  (B)  $\frac{1}{3}$  (C)  $\frac{3}{4}$  (D) 1  
**Ans:** (C)  $\frac{3}{4}$  1
10. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4,0), (2, 4) and (0, 5). If the maximum value of  $z = ax + by$ , where  $a, b > 0$  occurs at both (2, 4) and (4,0), then  
 (A)  $a = 2b$  (B)  $2a = b$  (C)  $a = b$  (D)  $3a = b$   
**Ans:** (A)  $a = 2b$  1

**Fill in the blanks in questions numbers 11 to 15**

11. A relation R in a set A is called \_\_\_\_\_, if  $(a_1, a_2) \in R$  implies  $(a_2, a_1) \in R$ , for all  $a_1, a_2 \in A$ .  
**Ans:** Symmetric 1
12. The greatest integer function defined by  $f(x) = [x], 0 < x < 2$  is not differentiable at  $x =$  \_\_\_\_\_.  
**Ans:** 1 1
13. If A is a matrix of order  $3 \times 2$ , then the order of the matrix  $A'$  is \_\_\_\_\_.  
**Ans:**  $2 \times 3$  1

**OR**

A square matrix A is said to be skew-symmetric, if \_\_\_\_\_

**Ans:**  $A = -A'$  (or,  $A' = -A$ ) 1

14. The equation of the normal to the curve  $y^2 = 8x$  at the origin is \_\_\_\_\_.  
**Ans:**  $y = 0$  1



**OR**

The radius of a circle is increasing at the uniform rate of 3 cm/s. At the instant when the radius of the circle is 2 cm, its area increases at the rate of \_\_\_\_\_ cm<sup>2</sup>/s.

**Ans:**  $12\pi$

1

15. The position vectors of two points A and B are  $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$  and  $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$ , respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is \_\_\_\_\_

**Ans:**  $2\hat{i} - \hat{j} + \hat{k}$

1

**Question numbers 16 to 20 are very short answer type questions**

16. If  $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$ , then find A (adj A).

**Ans:**  $A \cdot \text{adj}(A) = |A| I$

1/2

$$\therefore A \cdot \text{adj}(A) = 2I \text{ or } \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

1/2

17. Find  $\int x^4 \log x dx$

**Ans:**  $\int x^4 \cdot \log x dx = \log x \cdot \frac{x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5} dx$

1/2

$$= \frac{x^5 \cdot \log x}{5} - \frac{x^5}{25} + c$$

1/2

**OR**

Find  $\int \frac{2x}{\sqrt[3]{x^2+1}} dx$

**Ans:** Let,  $x^2 + 1 = t \therefore 2x dx = dt$

1/2

$$\int \frac{2x}{\sqrt[3]{x^2+1}} dx = \int \frac{1}{\sqrt[3]{t}} dt = \int t^{-1/3} dt = \frac{3}{2} t^{2/3} + c$$

$$= \frac{3}{2} (x^2 + 1)^{2/3} + c$$

1/2

18. Evaluate  $\int_1^3 |2x - 1| dx$ .

**Ans:**  $\int_1^3 |2x - 1| dx = \int_1^3 (2x - 1) dx = \left[ \frac{1}{4}(2x - 1)^2 \right]_1^3$  1/2  
 $= 6$  1/2

19. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

**Ans:**  $\frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26}{51}$  1/2+1/2

20. Find  $\int \frac{dx}{\sqrt{9 - 4x^2}}$ .

**Ans:**  $\int \frac{dx}{\sqrt{9 - 4x^2}} = \int \frac{dx}{\sqrt{3^2 - (2x)^2}}$  1/2  
 $= \frac{1}{2} \sin^{-1} \left( \frac{2x}{3} \right) + c$  1/2

**SECTION-B**

Question numbers 21 to 26 carry 2 marks each.

21. Prove that  $\sin^{-1} \left( 2x\sqrt{1-x^2} \right) = 2 \cos^{-1} x, \frac{1}{\sqrt{2}} \leq x \leq 1$

**Ans:** Put  $x = \cos \theta \Leftrightarrow \theta = \cos^{-1} x$  1/2

L.H.S. =  $\sin^{-1} \left( 2x\sqrt{1-x^2} \right)$   
 $= \sin^{-1} (2 \cos \theta \sin \theta) = \sin^{-1} (\sin 2\theta) = 2\theta = 2 \cos^{-1} x = \text{R.H.S.}$  1 1/2

**OR**

Consider a bijective function  $f : \mathbb{R}_+ \rightarrow (7, \infty)$  given by  $f(x) = 16x^2 + 24x + 7$ , where  $\mathbb{R}_+$  is the set of all positive real numbers. Find the inverse function of  $f$ .

**Ans:** Let  $y = f(x) = 16x^2 + 24x + 7 = (4x + 3)^2 - 2$  1

$\Rightarrow f^{-1}(y) = x = \frac{\sqrt{y+2}-3}{4}$  1

22. If  $x = at^2, y = 2at$ , then find  $\frac{d^2y}{dx^2}$ .

**Ans:**  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$  1

$\frac{d^2y}{dx^2} = -\frac{1}{t^2} \cdot \frac{dt}{dx} = -\frac{1}{t^2} \cdot \frac{1}{2at} = -\frac{1}{2a t^3}$  1

23. Find the points on the curve  $y = x^3 - 3x^2 - 4x$  at which the tangent lines are parallel to the line  $4x + y - 3 = 0$ .

**Ans:**  $\frac{dy}{dx} = -4 \Rightarrow 3x^2 - 6x - 4 = -4$  1

$\Rightarrow 3x(x - 2) = 0 \therefore x = 0 ; x = 2$  1/2

Points on the curve are  $(0, 0), (2, -12)$  1/2

24. Find a unit vector perpendicular to each of the vectors  $\vec{a}$  and  $\vec{b}$

where  $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$  and  $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$ .

**Ans:**  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k}$  1

Unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$  is  $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$  1

**OR**

Find the volume of the parallelepiped whose adjacent edges are represented by

$2\vec{a}, -\vec{b}$  and  $3\vec{c}$ , where  $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

**Ans:** Volume of the parallelepiped =  $\begin{vmatrix} 2 & -2 & 4 \\ -3 & -4 & 5 \\ 6 & -3 & 9 \end{vmatrix}$  1

$= |-24| = 24$  1

25. Find the value of  $k$  so that the lines  $x = -y = kz$  and  $x - 2 = 2y + 1 = -z + 1$  are perpendicular to each other.

**Ans:** The lines,  $\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$  and  $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{-1}$  1

are perpendicular  $\therefore 1 - \frac{1}{2} - \frac{1}{k} = 0 \Rightarrow k = 2$  1

26. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

**Ans:** Probability of green signal on crossing  $X = \frac{30}{100} = \frac{3}{10}$  } 1

Probability of not a green signal on crossing  $X = 1 - \frac{3}{10} = \frac{7}{10}$  }

Probability of a green signal on X on two consecutive days out of three

$= \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{63}{500}$  1

### SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Let N be the set of natural numbers and R be the relation on  $N \times N$  defined by  $(a, b) R (c, d)$  iff  $ad = bc$  for all  $a, b, c, d \in N$ . Show that R is an equivalence relation.

**Ans:** Reflexive: For any  $(a, b) \in N \times N$

$$a \cdot b = b \cdot a$$

$\therefore (a, b) R (a, b)$  thus R is reflexive 1

Symmetric: For  $(a, b), (c, d) \in N \times N$

$$(a, b) R (c, d) \Rightarrow a \cdot d = b \cdot c$$

$$\Rightarrow c \cdot b = d \cdot a$$

$\Rightarrow (c, d) R (a, b) \therefore R$  is symmetric  $1\frac{1}{2}$

Transitive : For any  $(a, b), (c, d), (e, f), \in N \times N$

$$(a, b) R (c, d) \text{ and } (c, d) R (e, f)$$

$$\Rightarrow a \cdot d = b \cdot c \text{ and } c \cdot f = d \cdot e$$

$$\Rightarrow a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$$

$\therefore (a, b) R (e, f), \therefore R$  is transitive  $1\frac{1}{2}$

$\therefore R$  is an equivalence Relation

28. If  $y = e^{x^2 \cos x} + (\cos x)^x$ , then find  $\frac{dy}{dx}$ .

**Ans.** Let  $u = (\cos x)^x \Rightarrow y = e^{x^2 \cos x} + u$

$$\therefore \frac{dy}{dx} = e^{x^2 \cos x} (2x \cdot \cos x - x^2 \cdot \sin x) + \frac{du}{dx} \quad 1\frac{1}{2}$$

$$\log u = \log (\cos x)^x \Rightarrow \log u = x \cdot \log (\cos x)$$

Differentiate w.r.t. "x"



$$\frac{1}{u} \frac{du}{dx} = \log(\cos x) - x \tan x \Rightarrow \frac{du}{dx} = (\cos x)^x \{ \log(\cos x) - x \tan x \} \quad 2$$

Therefore,

$$\frac{dy}{dx} = e^{x^2 \cdot \cos x} (2x \cdot \cos x - x^2 \cdot \sin x) + (\cos x)^x \{ \log(\cos x) - x \tan x \} \quad 1/2$$

29. Find  $\int \sec^3 x dx$ .

$$\text{Ans. } \int \sec^3 x dx = \int \sec x \cdot \sec^2 x dx = \int \sqrt{1 + \tan^2 x} \cdot \sec^2 x dx \quad 1 \frac{1}{2}$$

$$\text{(Put } \tan x = t ; \sec^2 x dx = dt) \quad 1/2$$

$$= \int \sqrt{1 + t^2} dt$$

$$= \frac{t}{2} \sqrt{1 + t^2} + \frac{1}{2} \log |t + \sqrt{1 + t^2}| + c \quad 1 \frac{1}{2}$$

$$= \frac{\sec x \cdot \tan x}{2} + \frac{1}{2} \log |\tan x + \sec x| + c \quad 1/2$$

30. Find the general solution of the differential equation  $ye^y dx = (y^3 + 2xe^y) dy$ .

$$\text{Ans. } y \cdot e^y dx = (y^3 + 2xe^y) dy \Rightarrow y \cdot e^y \frac{dy}{dx} = y^3 + 2xe^y$$

$$\therefore \frac{dx}{dy} - \frac{2}{y} x = y^2 \cdot e^{-y} \quad 1$$

$$\text{I.F. (Integrating factor)} = e^{-2 \int \frac{1}{y} dy} = e^{-2 \log y} = e^{\log \frac{1}{y^2}} = \frac{1}{y^2} \quad 1$$

$\therefore$  Solution is

$$x \cdot \frac{1}{y^2} = \int y^2 \cdot e^{-y} \cdot \frac{1}{y^2} dy + c = \int e^{-y} dy + c \quad 1$$

$$\Rightarrow \frac{x}{y^2} = -e^{-y} + c \text{ or } x = -y^2 e^{-y} + cy^2 \quad 1$$

**OR**

Find the particular solution of the differential equation

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right), \text{ given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

Ans. The differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan \frac{y}{x}, \text{ let } y = vx \therefore \frac{dy}{dx} = v + x \frac{dv}{dx} \quad 1$$



$$\Rightarrow v + x \frac{dv}{dx} = v - \tan v \Rightarrow \cot v dv = -\frac{1}{x} dx$$

Integrate both sides

$$\log \sin v = -\log |x| + \log c \Rightarrow \log \sin \frac{y}{x} = \log \frac{c}{x} \quad 2$$

$$\Rightarrow x \cdot \sin \frac{y}{x} = c, \text{ Put } y = \frac{\pi}{4} \text{ and } x = 1$$

$$\Rightarrow \sin \frac{\pi}{4} = c \text{ or } c = \frac{1}{\sqrt{2}} \quad 1/2$$

$$\therefore \text{ Particular solution is } x \cdot \sin \left( \frac{y}{x} \right) = \frac{1}{\sqrt{2}} \quad 1/2$$

31. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

Ans.

Let No. of chairs =  $x$ , No. of tables =  $y$

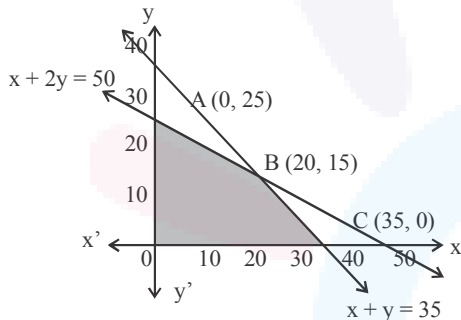
Then L.P. P. is:

$$\text{Maximize (Profit) : } Z = 150x + 250y \quad 1$$

$$\text{Subject to : } x + y \leq 35$$

$$1000x + 2000y \leq 50000 \Rightarrow x + 2y \leq 50 \quad 1$$

$$x, y \geq 0$$



Correct graph

Corner: Value of  $Z$

$$A(0, 25) \quad ₹ 6250$$

$$B(20, 15) \quad ₹ 6750 \text{ (Max)} \quad 1/2$$

$$C(35, 0) \quad ₹ 5250$$

$$\therefore \text{Max } (z) = ₹ 6750$$

$$\text{Number of chairs} = 20, \text{ Tables} = 15$$

32. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

Ans.  $E_1$  = Event that the ball transferred from Bag I is Black

$E_2$  = Event that the ball transferred from Bag I is Red

A = Event that the ball drawn from Bag II is Black

$$P(E_1) = \frac{5}{8}; P(E_2) = \frac{3}{8}; P\left(\frac{A}{E_1}\right) = \frac{4}{8} = \frac{1}{2}; P\left(\frac{A}{E_2}\right) = \frac{3}{8} \quad 1/2$$



Required Probability:

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)} = \frac{\frac{5}{8} \cdot \frac{1}{2}}{\frac{5}{8} \cdot \frac{1}{2} + \frac{3}{8} \cdot \frac{3}{8}} = \frac{20}{29} \quad \frac{1}{2}$$

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

**Ans.** Let X = No. of white balls = 0, 1, 2

$$X : \quad \quad \quad 0 \quad \quad \quad 1 \quad \quad \quad 2 \quad \quad \quad \frac{1}{2}$$

$$P(X) : \quad \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} = \frac{7}{15} \quad 3 \times \frac{8}{10} \times \frac{7}{9} \times \frac{2}{8} = \frac{7}{15} \quad 3 \times \frac{2}{10} \times \frac{1}{9} \times \frac{8}{8} = \frac{1}{15} \quad \frac{1}{2}$$

$$X \cdot P(X) : \quad 0 \quad \quad \quad \frac{7}{15} \quad \quad \quad \frac{2}{15} \quad \quad \quad \frac{1}{2}$$

$$X^2 P(X) : \quad 0 \quad \quad \quad \frac{7}{15} \quad \quad \quad \frac{4}{15}$$

$$\text{Mean} = \sum XP(X) = \frac{9}{15} = \frac{3}{5} \quad \frac{1}{2}$$

$$\text{Variance} = \sum X^2 P(x) - \left[ \sum X P(X)^2 \right] = \frac{11}{15} - \left[ \frac{3}{5} \right]^2 = \frac{28}{75} \quad 1$$

SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ , then find  $A^{-1}$  and use it to solve the

following system of the equations:

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$

$$\text{Ans. } |A| = 7; \text{adj}(A) = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}; A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \quad 1+1\frac{1}{2}+\frac{1}{2}$$



The system of equations in Matrix form can be written as :

$$A \cdot X = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} \quad 1$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix} \quad 1$$

$$\therefore x = 1, y = -5, z = -5 \quad 1$$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

Ans.  $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$$= \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ca \\ a^2+b^2 & c^2 & ab \end{vmatrix} \quad (C_1 \rightarrow C_1 - 2C_3) \quad 1$$

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix} \quad (C_1 \rightarrow C_1 + C_2) \quad 1$$

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix} \quad (R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1) \quad 2$$

$$= (b-a)(c-a) \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix} \quad 1$$

Expand along  $C_1$

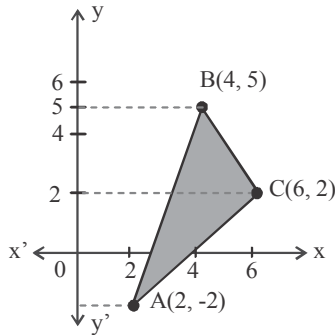
$$= (a^2+b^2+c^2)(b-a)(c-a)(-b^2-ab+c^2+ac)$$

$$= (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2) \quad 1$$



34. Using integration, find the area of the region bounded by the triangle whose vertices are (2, -2), (4,5) and (6,2).

Ans.



Let A(2, -2) ; B(4, 5) ; C(6, 2)

Equations of the lines

$$AB : x = \frac{2}{7}(y+9)$$

$$BC : x = -\frac{2}{3}(y-11)$$

$$AC : x = y + 4$$

1/2

Correct graph

1/2

$$\text{ar}(\Delta ABC) = \int_{-2}^2 (y+4)dy + \left(\frac{-2}{3}\right) \int_2^5 (y-11)dy - \int_{-2}^5 \frac{2}{7}(y+9)dy$$

2

$$= \frac{1}{2} [(y+4)^2]_{-2}^2 - \frac{1}{3} [(y-11)^2]_2^5 - \frac{1}{7} [(y+9)^2]_{-2}^5$$

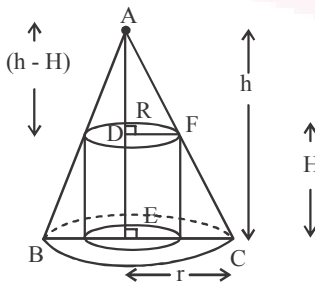
1/2

$$= 16 + 15 - 21 = 10$$

1/2

35. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is  $\frac{4}{9}$  times the volume of the cone.

Ans.



Let H = Height of cylinder

R = Radius of cylinder

$$\text{Volume of cone} = \frac{\pi}{3} r^2 h$$

1/2

$$V = \text{Volume of cylinder} = \pi R^2 H$$

1/2

$$\Delta ADF \sim \Delta AEC \Rightarrow \frac{h-H}{h} = \frac{R}{r} \Rightarrow R = \frac{r}{R} (h-H)$$

1

$$\therefore V = \pi \cdot H \cdot \frac{r^2}{h^2} (h-H)^2 = \frac{\pi r^2}{h^2} (H^3 - 2hH^2 + Hh^2)$$

1

$$V'(H) = \frac{\pi r^2}{h^2} (3H^2 - 4hH + h^2), V'(h) = 0 \Rightarrow H = \frac{h}{3}$$

1+1

$$V''(H) = \frac{\pi r^2}{h^2} (6H - 4h), V''\left(H = \frac{h}{3}\right) = \frac{\pi r^2}{h^2} (-2h) < 0$$

1/2

$$\therefore V \text{ is max iff } H = \frac{h}{3} \text{ and } R = \frac{2r}{3}$$



$$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{3\pi R^2 H}{\pi r^2 h} = 3 \cdot \frac{4r^2}{9} \cdot \frac{h}{3} \cdot \frac{1}{r^2 h} = \frac{4}{9} \quad 1/2$$

36. Find the equation of the plane that contains the point A(2,1,-1) and is perpendicular to the line of intersection of the planes  $2x + y - z = 3$  and  $x + 2y + z = 2$ . Also find the angle between the plane thus obtained and the y-axis.

**Ans.** Let equation of the required plane be:

$$a(x - 2) + b(y - 1) + c(z + 1) = 0 \quad 1 \frac{1}{2}$$

$$\text{Also : } \begin{aligned} 2a + b - c &= 0 \\ a + 2b + c &= 0 \end{aligned}$$

$$\text{Solving: } \frac{a}{3} = \frac{b}{-3} = \frac{c}{3} = k \Rightarrow a = 3k, b = -3k, c = 3k \quad 1 \frac{1}{2}$$

$$\therefore \text{Equation of plane is : } 3k(x - 2) - 3k(y - 1) + 3k(z + 1) = 0$$

$$\Rightarrow x - y + z = 0 \quad 1 \frac{1}{2}$$

Let angle between y-axis and plane =  $\theta$

$$\text{then, } \sin \theta = \left| \frac{0 - 1 + 0}{\sqrt{1 + 1 + 1}} \right| = \left| \frac{-1}{\sqrt{3}} \right| \Rightarrow \theta = \sin^{-1} \left( \frac{1}{\sqrt{3}} \right) \quad 1 \frac{1}{2}$$

**OR**

Find the distance of the point P(-2, -4, 7) from the point of intersection Q of the line  $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$ . Also write the vector equation of the line PQ.

$$\text{Ans. General point on line is: } \vec{r} = (3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k} \quad 1$$

For the point of intersection:

$$\left[ (3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k} \right] \cdot (\hat{i} - \hat{j} + \hat{k}) = 6 \quad 1$$

$$\Rightarrow 3 + 2\lambda + 2 + \lambda + 6 + 2\lambda = 6 \Rightarrow \lambda = -1 \quad 1$$

$$\therefore Q(\hat{i} - \hat{j} + 4\hat{k}) = Q(1, -1, 4) \quad 1$$

$$PQ = 3\sqrt{3}, \text{ equation of the line PQ : } \vec{r} = -2\hat{i} - 4\hat{j} + 7\hat{k} + \mu(3\hat{i} + 3\hat{j} - 3\hat{k}) \quad 1+1$$



QUESTION PAPER CODE 65/1/2  
EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions.

Select the correct option.

Q.No.		Marks
1.	If A is a $3 \times 3$ matrix and $ A  = -2$ , then value of $ A(\text{adj } A) $ is (A) $-2$ (B) $2$ (C) $-8$ (D) $8$ <b>Ans:</b> (C) $-8$	1
2.	The number of arbitrary constants in the particular solution of a differential equation of second order is (are) (A) $0$ (B) $1$ (C) $2$ (D) $3$ <b>Ans:</b> (A) $0$	1
3.	The principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ (A) $\frac{13\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$ <b>Ans:</b> (D) $\frac{\pi}{6}$	1
4.	The corner points of the feasible region determined by the system of linear inequalities are $(0, 0)$ , $(4, 0)$ , $(2, 4)$ and $(0, 5)$ . If the maximum value of $z = ax + by$ , where $a, b > 0$ occurs at both $(2, 4)$ and $(4, 0)$ , then (A) $a = 2b$ (B) $2a = b$ (C) $a = b$ (D) $3a = b$ <b>Ans:</b> (A) $a = 2b$	1
5.	If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$ , then $P(B'   A)$ is equal to (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $1$ <b>Ans:</b> (C) $\frac{3}{4}$	1
6.	If A is a square matrix such that $A^2 = A$ , then $(I - A)^3 + A$ is equal to (A) $I$ (B) $0$ (C) $I - A$ (D) $I + A$ <b>Ans:</b> (A) $I$	1

7.  $\int_{-\frac{x}{2}}^{\frac{x}{2}} \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$ , where  $x \neq 0$  is equal to

- (A) -2                      (B) 0                      (C) 1                      (D)  $\pi$

**Ans:** (B) 0

1

8. The image of the point (2, -1, 5) in the plane  $\vec{r} \cdot \hat{i} = 0$  is

- (A) (-2, -1, 5)              (B) (2, 1, -5)              (C) (-2, 1, -5)              (D) (2, 0, 0)

**Ans:** (A) (-2, -1, 5)

1

9. If the projection of  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  on  $\vec{b} = 2\hat{i} + \lambda\hat{k}$  is zero, then the value of  $\lambda$  is

- (A) 0                      (B) 1                      (C)  $-\frac{2}{3}$                       (D)  $-\frac{3}{2}$

**Ans:** (C)  $-\frac{2}{3}$

1

10. The vector equation of the line passing through the point (-1, 5, 4) and perpendicular to the plane  $z = 0$  is

- (A)  $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$               (B)  $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$   
 (C)  $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$               (D)  $\vec{r} = \lambda\hat{k}$

**Ans:** (B)  $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

1

**Fill in the blanks in questions numbers 11 to 15**

11. The position vectors of two points A and B are  $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$  and  $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$ , respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is \_\_\_\_\_.

**Ans:**  $2\hat{i} - \hat{j} + \hat{k}$

1

12. The equation of the normal to the curve  $y^2 = 8x$  at the origin is \_\_\_\_\_.

**Ans:**  $y = 0$

1

**OR**

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of \_\_\_\_\_  $\text{cm}^2/\text{s}$ .

**Ans:**  $12\pi$

1



13. On applying elementary column operation  $C_2 \rightarrow C_2 - 3C_1$  in the matrix

equation  $\begin{bmatrix} 4 & -2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ , the RHS (Right Hand Side) of the equation becomes \_\_\_\_\_.

**Ans:**  $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix}$

1

OR

A square matrix A is said to be symmetric if \_\_\_\_\_

**Ans:**  $A = A'$

1

14. A relation R in a set A is called \_\_\_\_\_, if  $(a_1, a_2) \in R$  implies  $(a_2, a_1) \in R$ , for all  $a_1, a_2 \in A$ .

**Ans:** Symmetric

1

15. The greatest integer function defined by  $f(x) = [x]$ ,  $0 < x < 2$  is not differentiable at  $x =$  \_\_\_\_\_.

**Ans:** 1

1

**Question numbers 16 to 20 are very short answer type questions**

16. If A is non-singular square matrix of order 3 and  $A^2 = 2A$ , then find the value of  $|A|$ .

**Ans:**  $|A|^2 = 8|A|$

1/2

$\Rightarrow |A| = 8$

1/2

17. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

**Ans:**  $\frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26}{51}$

1/2+1/2

18. Evaluate  $\int_1^3 |2x - 1| dx$ .

**Ans:**  $\int_1^3 |2x - 1| dx = \int_1^3 (2x - 1) dx = \left[ \frac{1}{4} (2x - 1)^2 \right]_1^3$

1/2

$= 6$

1/2



19. Find :  $\int \frac{dx}{\sqrt{9-4x^2}}$

Ans:  $\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{3^2-(2x)^2}} \quad 1/2$   
 $= \frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + C \quad 1/2$

20. Find:  $\int x^4 \log x \, dx$ .

Ans:  $\int x^4 \cdot \log x \, dx = \log x \cdot \frac{x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5} \, dx \quad 1/2$   
 $= \frac{x^5 \cdot \log x}{5} - \frac{x^5}{25} + c \quad 1/2$

OR

Find:  $\int \frac{2x}{\sqrt[3]{x^2+1}} \, dx$ .

Ans: Let,  $x^2+1=t$   
 $\therefore 2x \, dx = dt \quad 1/2$

$\int \frac{2x}{\sqrt[3]{x^2+1}} \, dx = \int \frac{1}{\sqrt[3]{t}} \, dt = \int t^{-1/3} \, dt = \frac{3}{2} t^{2/3} + c \quad 1/2$   
 $= \frac{3}{2} (x^2+1)^{2/3} + c$

**SECTION-B**

Question numbers 21 to 26 carry 2 marks each.

21. Find a unit vector perpendicular to each of the vectors  $\vec{a}$  and  $\vec{b}$

where  $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$  and  $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$ .

Ans:  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k} \quad 1$

Unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$  is  $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k} \quad 1$

OR

Find the volume of the parallelopiped whose adjacent edges are represented by

$2\vec{a}, -\vec{b}$  and  $3\vec{c}$ , where  $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$



**Ans:** Volume of the parallelopiped =  $\begin{vmatrix} 2 & -2 & 4 \\ -3 & -4 & 5 \\ 6 & -3 & 9 \end{vmatrix}$  1

$$= |-24| = 24$$
 1

22. Examine the applicability of Rolle's theorem for the function  $f(x) = \sin 2x$  in  $[0, \pi]$ . Hence find the points where the tangent is parallel to x-axis.

**Ans:** As, sine function and polynomial function are everywhere continuous and differentiable.

$\therefore$   $\left. \begin{array}{l} \text{(i) } f(x) = \sin 2x \text{ is continuous on } [0, \pi] \\ \text{(ii) } f(x) = \sin 2x \text{ is differentiable on } (0, \pi) \\ \text{(iii) } f(0) = 0 = f(\pi) \end{array} \right\}$  1

$\therefore$  Rolle's Theorem is applicable for  $f(x) = \sin 2x$

Solving,  $f'(x) = 0$  or  $2 \cos 2x = 0 \Rightarrow \cos 2x = 0$

$\therefore$   $\left. 2x = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4} \right\}$  1/2

The points where the tangent is parallel to x-axis are:  $\left(\frac{\pi}{4}, 1\right); \left(\frac{3\pi}{4}, -1\right)$  1/2

23. Find the values of x for which the function  $f(x) = 2 + 3x - x^3$  is decreasing.

**Ans:**  $f(x)$  is decreasing iff  $f'(x) \leq 0$  1

$$\Leftrightarrow 3 - 3x^2 \leq 0 \text{ or } x^2 \geq 1$$

$$\Leftrightarrow x \leq -1 \text{ or } x \geq 1$$
 1

24. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

**Ans:** Probability of green signal on crossing X =  $\frac{30}{100} = \frac{3}{10}$  1

Probability of not a green signal on crossing X =  $1 - \frac{3}{10} = \frac{7}{10}$  1

Probability of a green signal on X on two consecutive days out of three

$$= \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{63}{500}$$
 1

25. Prove that  $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \leq x \leq 1$

**Ans:** Put  $x = \cos \theta \Leftrightarrow \theta = \cos^{-1}x$  1/2

$$\text{L.H.S.} = \sin^{-1}(2x\sqrt{1-x^2})$$

$$= \sin^{-1}(2\cos \theta \sin \theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\cos^{-1}x = \text{R.H.S.} \quad \text{1 1/2}$$

**OR**

Consider a bijective function  $f: R_+ \rightarrow (7, \infty)$  given by  $f(x) = 16x^2 + 24x + 7$ , where  $R_+$  is the set of all positive real numbers. Find the inverse function of  $f$ .

**Ans:** Let  $y = f(x) = 16x^2 + 24x + 7 = (4x + 3)^2 - 2$  1

$$\Rightarrow f^{-1}(y) = x = \frac{\sqrt{y+2}-3}{4} \quad \text{1}$$

26. Find the value of  $k$  so that the lines  $x - y = kz$  and  $x - 2 = 2y + 1 = -z + 1$  are perpendicular to each other.

**Ans:** The lines,  $\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$  and  $\frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$  1

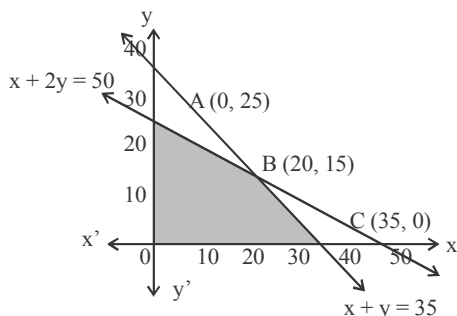
are perpendicular  $\therefore 1 - \frac{1}{2} - \frac{1}{k} = 0 \Rightarrow k = 2$  1

**SECTION-C**

**Question numbers 27 to 32 carry 4 marks each.**

27. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

**Ans.**



Let No. of chairs =  $x$ , No. of tables =  $y$

Then L.P.P. is:

Maximize (Profit) :  $Z = 150x + 250y$  1

Subject to:  $x + y \leq 35$   
 $1000x + 2000y \leq 50000 \Rightarrow x + 2y \leq 50$   
 $x, y \geq 0$  1

Correct graph 1 1/2



Corner:	Value of Z	
A(0, 25)	₹ 6250	}
B(20, 15)	₹ 6750 (Max)	
C(35, 0)	₹ 5250	
∴ Max (Z) = ₹ 6750		
Number of chairs = 20, Tables = 15		1/2

28. If  $x = a \sec^3 \theta$ ,  $y = a \tan^3 \theta$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .

**Ans.**  $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a \tan^2 \theta \sec^2 \theta}{3a \sec^3 \theta \tan \theta} = \sin \theta$  2

$\frac{d^2y}{dx^2} = \cos \theta \cdot \frac{d\theta}{dx} = \frac{1}{3a \sec^4 \theta \cdot \tan \theta}$  1 1/2

$\left. \frac{d^2y}{dx^2} \right|_{\theta=\frac{\pi}{4}} = \frac{1}{12a}$  1/2

29. Find:  $\int \frac{2x+1}{\sqrt{3+2x-x^2}} dx$ .

**Ans.**  $\int \frac{2x+1}{\sqrt{3+2x-x^2}} dx = -\int \frac{2-2x}{\sqrt{3+2x-x^2}} dx + 3 \int \frac{1}{\sqrt{2^2-(x-1)^2}} dx$  2

$= -2\sqrt{3+2x-x^2} + 3 \sin^{-1} \left( \frac{x-1}{2} \right) + c$  2

30. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

**Ans.**  $E_1 =$  Event that the ball transferred from Bag I is Black }  
 $E_2 =$  Event that the ball transferred from Bag I is Red } 1/2  
 $A =$  Event that the ball drawn from Bag II is Black

$P(E_1) = \frac{5}{8}; P(E_2) = \frac{3}{8}; P\left(\frac{A}{E_1}\right) = \frac{4}{8} = \frac{1}{2}; P\left(\frac{A}{E_2}\right) = \frac{3}{8}$  2

Required Probability:

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)} = \frac{\frac{5}{8} \cdot \frac{1}{2}}{\frac{5}{8} \cdot \frac{1}{2} + \frac{3}{8} \cdot \frac{3}{8}} = \frac{20}{29} \quad \frac{1}{2}$$

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

**Ans.** Let  $X =$  No. of white balls = 0, 1, 2

$$X: \quad \quad \quad 0 \quad \quad \quad 1 \quad \quad \quad 2 \quad \quad \quad \frac{1}{2}$$

$$P(X): \quad \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} = \frac{7}{15} \quad 3 \times \frac{8}{10} \times \frac{7}{9} \times \frac{2}{8} = \frac{7}{15} \quad 3 \times \frac{2}{10} \times \frac{1}{9} \times \frac{8}{8} = \frac{1}{15} \quad \frac{1}{2}$$

$$X \cdot P(X): \quad 0 \quad \quad \quad \frac{7}{15} \quad \quad \quad \frac{2}{15} \quad \quad \quad \frac{1}{2}$$

$$X^2 P(X): \quad 0 \quad \quad \quad \frac{7}{15} \quad \quad \quad \frac{4}{15}$$

$$\text{Mean} = \sum XP(X) = \frac{9}{15} = \frac{3}{5} \quad \frac{1}{2}$$

$$\text{Variance} = \sum X^2 P(x) - [\sum XP(X)]^2 = \frac{11}{15} - \left[\frac{3}{5}\right]^2 = \frac{28}{75} \quad 1$$

**31.** Find the general solution of the differential equation  $ye^y dx = (y^3 + 2xe^y)dy$ .

**Ans.**  $y \cdot e^y dx = (y^3 + 2xe^y) dy \Rightarrow y \cdot e^y \frac{dy}{dx} = y^3 + 2xe^y$

$$\therefore \frac{dx}{dy} - \frac{2}{y}x = y^2 \cdot e^{-y} \quad 1$$

$$\text{I.F. (Integrating factor)} = e^{-2 \int \frac{1}{y} dy} = e^{-2 \log y} = e^{\log \frac{1}{y^2}} = \frac{1}{y^2} \quad 1$$

$\therefore$  Solution is

$$x \cdot \frac{1}{y^2} = \int y^2 \cdot e^{-y} \cdot \frac{1}{y^2} dy + c = \int e^{-y} dy + c \quad 1$$

$$\Rightarrow \frac{x}{y^2} = -e^{-y} + c \quad \text{or} \quad x = -y^2 e^{-y} + cy^2 \quad 1$$

**OR**

Find the particular solution of the differential equation

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right), \text{ given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

**Ans.** The differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}, \text{ let } y = vx \therefore \frac{dy}{dx} = v + x \frac{dv}{dx} \quad 1$$

$$\Rightarrow v + x \frac{dv}{dx} = v - \tan v \Rightarrow \cot v \, dv = -\frac{1}{x} dx$$

Integrate both sides

$$\log \sin v = -\log |x| + \log c \Rightarrow \log \sin \frac{y}{x} = \log \frac{c}{x} \quad 2$$

$$\Rightarrow x \cdot \sin \frac{y}{x} = c, \text{ Put } y = \frac{\pi}{4} \text{ and } x = 1$$

$$\Rightarrow \sin \frac{\pi}{4} = c \text{ or } c = \frac{1}{\sqrt{2}} \quad 1/2$$

$$\therefore \text{ Particular solution is } x \cdot \sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}} \quad 1/2$$

- 32.** Let  $N$  be the set of natural numbers and  $R$  be the relation on  $N \times N$  defined by  $(a, b) R (c, d)$  iff  $ad = bc$  for all  $a, b, c, d \in N$ . Show that  $R$  is an equivalence relation.

**Ans:** Reflexive: For any  $(a, b) \in N \times N$

$$a \cdot b = b \cdot a$$

$\therefore (a, b) R (a, b)$  thus  $R$  is reflexive 1

Symmetric: For  $(a, b), (c, d) \in N \times N$

$$(a, b) R (c, d) \Rightarrow a \cdot d = b \cdot c$$

$$\Rightarrow c \cdot b = d \cdot a$$

$\Rightarrow (c, d) R (a, b) \therefore R$  is symmetric 1 1/2

Transitive : For any  $(a, b), (c, d), (e, f) \in N \times N$

$$(a, b) R (c, d) \text{ and } (c, d) R (e, f)$$

$$\Rightarrow a \cdot d = b \cdot c \text{ and } c \cdot f = d \cdot e$$

$$\Rightarrow a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$$

$\therefore (a, b) R (e, f), \therefore R$  is transitive 1 1/2

$\therefore R$  is an equivalence Relation

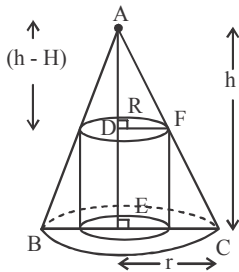


**SECTION-D**

Question numbers 33 to 36 carry 6 marks.

33. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height  $h$  and radius  $r$  is one-third of the height of the cone, and the greatest volume of the cylinder is  $\frac{4}{9}$  times the volume of the cone.

Ans.



Let  $H$  = Height of cylinder

$R$  = Radius of cylinder

Volume of cone =  $\frac{\pi}{3} r^2 h$  1/2

$V$  = Volume of cylinder =  $\pi R^2 H$  1/2

$\Delta ADF \sim \Delta AEC \Rightarrow \frac{h-H}{h} = \frac{R}{r} \Rightarrow R = \frac{r}{h}(h-H)$  1

$\therefore V = \pi \cdot H \cdot \frac{r^2}{h^2} (h-H)^2 = \frac{\pi r^2}{h^2} (H^3 - 2hH^2 + Hh^2)$  1

$V'(H) = \frac{\pi r^2}{h^2} (3H^2 - 4hH + h^2)$ ,  $V'(h) = 0 \Rightarrow H = \frac{h}{3}$  1+1

$V''(H) = \frac{\pi r^2}{h^2} (6H - 4h)$ ,  $V''\left(H = \frac{h}{3}\right) = \frac{\pi r^2}{h^2} (-2h) < 0$  1/2

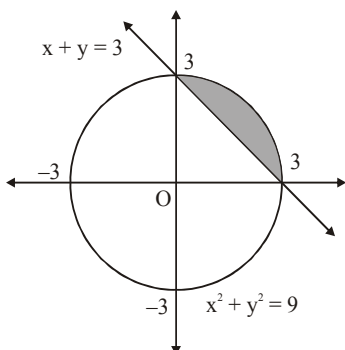
$\therefore V$  is max iff  $H = \frac{h}{3}$  and  $R = \frac{2r}{3}$

$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{3\pi R^2 H}{\pi r^2 h} = 3 \cdot \frac{4r^2}{9} \cdot \frac{h}{3} \cdot \frac{1}{r^2 h} = \frac{4}{9}$  1/2

34. Using integration, find the area of the region  $\{(x, y) : x^2 + y^2 \leq 9, x + y \geq 3\}$

Ans.

Correct graph. 2



Required area

$= \int_0^3 \sqrt{9-x^2} dx - \int_0^3 (3-x) dx$  2

$= \left[ \frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \left( \frac{x}{3} \right) \right]_0^3 - \left[ \frac{1}{2} (3-x)^2 \right]_0^3$  1 1/2

$= \frac{9\pi}{4} - \frac{9}{2}$  or  $\frac{9}{4}(\pi - 2)$  1/2

35. Find the equation of the plane that contains the point  $A(2,1,-1)$  and is perpendicular to the line of intersection of the planes  $2x + y - z = 3$  and  $x + 2y + z = 2$ . Also find the angle between the plane thus obtained and the  $y$ -axis.

**Ans.** Let equation of the required plane be:

$$a(x - 2) + b(y - 1) + c(z + 1) = 0 \quad 1\frac{1}{2}$$

$$\text{Also : } \begin{aligned} 2a + b - c &= 0 \\ a + 2b + c &= 0 \end{aligned}$$

$$\text{Solving: } \frac{a}{3} = \frac{b}{-3} = \frac{c}{3} = k \Rightarrow a = 3k, b = -3k, c = 3k \quad 1\frac{1}{2}$$

$$\therefore \text{Equation of plane is : } 3k(x - 2) - 3k(y - 1) + 3k(z + 1) = 0$$

$$\Rightarrow x - y + z = 0 \quad 1\frac{1}{2}$$

Let angle between  $y$ -axis and plane =  $\theta$

$$\text{then, } \sin \theta = \left| \frac{0 - 1 + 0}{\sqrt{1 + 1 + 1}} \right| = \left| \frac{-1}{\sqrt{3}} \right| \Rightarrow \theta = \sin^{-1} \left( \frac{1}{\sqrt{3}} \right) \quad 1\frac{1}{2}$$

**OR**

Find the distance of the point  $P(-2, -4, 7)$  from the point of intersection  $Q$  of the line  $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$ . Also write the vector equation of the line  $PQ$ .

$$\text{Ans. General point on line is: } \vec{r} = (3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k} \quad 1$$

For the point of intersection:

$$\left[ (3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k} \right] \cdot (\hat{i} - \hat{j} + \hat{k}) = 6 \quad 1$$

$$\Rightarrow 3 + 2\lambda + 2 + \lambda + 6 + 2\lambda = 6 \Rightarrow \lambda = -1 \quad 1$$

$$\therefore Q(\hat{i} - \hat{j} + 4\hat{k}) = Q(1, -1, 4) \quad 1$$

$$PQ = 3\sqrt{3}, \text{ equation of the line } PQ : \vec{r} = -2\hat{i} - 4\hat{j} + 7\hat{k} + \mu(3\hat{i} + 3\hat{j} - 3\hat{k}) \quad 1+1$$

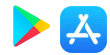
36. If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ , then find  $A^{-1}$  and use it to solve the

following system of the equations:

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$



**Ans.**  $|A| = 7$ ;  $\text{adj}(A) = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$ ;  $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$   $1+1\frac{1}{2}+\frac{1}{2}$

The system of equations in Matrix form can be written as :

$A \cdot X = B$ , where  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ;  $B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$  **1**

$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$  **1**

$\therefore x = 1, y = -5, z = -5$  **1**

**OR**

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

**Ans.**  $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$= \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ca \\ a^2+b^2 & c^2 & ab \end{vmatrix}$   $(C_1 \rightarrow C_1 - 2C_3)$  **1**

$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$   $(C_1 \rightarrow C_1 + C_2)$  **1**

$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$   $(R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1)$  **2**





$$= (b-a)(c-a) \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix} \quad \mathbf{1}$$

Expand along  $C_1$

$$\begin{aligned} &= (a^2 + b^2 + c^2)(b-a)(c-a)(-b^2 - ab + c^2 + ac) \\ &= (a-b)(b-c)(c-a)(a+b+c)(a^2 + b^2 + c^2) \quad \mathbf{1} \end{aligned}$$

**QUESTION PAPER CODE 65/1/3**  
**EXPECTED ANSWER/VALUE POINTS**  
**SECTION – A**

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions.  
 Select the correct option.

Q.No.		Marks
1.	The matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible for (A) $\lambda = -1$ (B) $\lambda = 0$ (C) $\lambda = 1$ (D) $\lambda \in \mathbb{R} - \{1\}$ <b>Ans:</b> (C) $\lambda = 1$	1
2.	The number of arbitrary constants in the particular solution of a differential equation of second order is (are) (A) 0      (B) 1      (C) 2      (D) 3 <b>Ans:</b> (A) 0	1
3.	The value of $\tan^{-1} \left( \tan \frac{7\pi}{6} \right)$ is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{7\pi}{6}$ <b>Ans:</b> (A) $\frac{\pi}{6}$	1
4.	The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$ , where $a, b > 0$ occurs at both (2, 4) and (4, 0), then (A) $a = 2b$ (B) $2a = b$ (C) $a = b$ (D) $3a = b$ <b>Ans:</b> (A) $a = 2b$	1
5.	If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$ , then $P(B'   A)$ is equal to (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) 1 <b>Ans:</b> (C) $\frac{3}{4}$	1
6.	If A is a square matrix such that $A^2 = A$ , then $(I - A)^3 + A$ is equal to (A) I      (B) 0      (C) I - A      (D) I + A <b>Ans:</b> (A) I	1



7.  $\int_1^e \frac{\log x}{x} dx$ , is equal to

- (A)  $\frac{e^2}{2}$                       (B) 1                      (C)  $\frac{1}{2}$                       (D)  $-\infty$

**Ans:** (C)  $\frac{1}{2}$

1

8. A point P lies on the line segment joining the points (-1, 3, 2) and (5, 0, 6). If x-coordinate of P is 2, then its z-coordinate is

- (A) -1                      (B) 4                      (C)  $\frac{3}{2}$                       (D) 8

**Ans:** (B) 4

1

9. If the projection of  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  on  $\vec{b} = 2\hat{i} + \lambda\hat{k}$  is zero, then the value of  $\lambda$  is

- (A) 0                      (B) 1                      (C)  $\frac{-2}{3}$                       (D)  $\frac{-3}{2}$

**Ans:** (C)  $\frac{-2}{3}$

1

10. The vector equation of the line passing through the point (-1, 5, 4) and perpendicular to the plane  $z = 0$  is

- (A)  $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$                       (B)  $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$   
 (C)  $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$                       (D)  $\vec{r} = \lambda\hat{k}$

**Ans:** (B)  $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

1

**Fill in the blanks in question numbers 11 to 15**

11. The position vectors of two points A and B are  $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$  and  $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$ , respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is \_\_\_\_\_.

**Ans:**  $2\hat{i} - \hat{j} + \hat{k}$

1

12. The equation of the normal to the curve  $y^2 = 8x$  at the origin is \_\_\_\_\_.

**Ans:**  $y = 0$

1

**OR**

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of \_\_\_\_\_  $\text{cm}^2/\text{s}$ .

**Ans:**  $12\pi$

1



13. If A is a square matrix of order 3 and  $A_{ij}$  is the cofactor of the element  $a_{ij}$ , then value of  $a_{21} A_{11} + a_{22} A_{12} + a_{23} A_{13}$  is \_\_\_\_\_.

**Ans:** 0

1

OR

If the matrix A is both symmetric and skew symmetric, then A is a \_\_\_\_\_.

**Ans:** Zero matrix

1

14. A relation R in a set A is called \_\_\_\_\_, if  $(a_1, a_2) \in R$  implies  $(a_2, a_1) \in R$ , for all  $a_1, a_2 \in A$ .

**Ans:** Symmetric

1

15. The greatest integer function defined by  $f(x) = [x]$ ,  $0 < x < 2$  is not differentiable at  $x =$  \_\_\_\_\_.

**Ans:** 1

1

**Question numbers 16 to 20 are very short answer type questions**

16. If A is a square matrix of order 3 and  $|A| = 2$ , then find the value of  $|-AA'|$ .

$$\begin{aligned} \text{Ans: } |-AA'| &= -|A|^2 \\ &= -4 \end{aligned}$$

1/2

1/2

17. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

$$\text{Ans: } \frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26}{51}$$

1/2+1/2

18. Evaluate:  $\int_1^3 |2x - 1| dx$ .

$$\begin{aligned} \text{Ans: } \int_1^3 |2x - 1| dx &= \int_1^3 (2x - 1) dx = \left[ \frac{1}{4} (2x - 1)^2 \right]_1^3 \\ &= 6 \end{aligned}$$

1/2

1/2

19. Find :  $\int \frac{dx}{\sqrt{9 - 4x^2}}$

$$\begin{aligned} \text{Ans: } \int \frac{dx}{\sqrt{9 - 4x^2}} &= \int \frac{dx}{\sqrt{3^2 - (2x)^2}} \\ &= \frac{1}{2} \sin^{-1} \left( \frac{2x}{3} \right) + C \end{aligned}$$

1/2

1/2



20. Find:  $\int x^4 \log x \, dx$ .

**Ans:**  $\int x^4 \cdot \log x \, dx = \log x \cdot \frac{x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5} \, dx$  1/2

$$= \frac{x^5 \cdot \log x}{5} - \frac{x^5}{25} + c$$
 1/2

**OR**

Find:  $\int \frac{2x}{\sqrt[3]{x^2+1}} \, dx$ .

**Ans:** Let,  $x^2 + 1 = t$

$\therefore 2x \, dx = dt$  1/2

$$\int \frac{2x}{\sqrt[3]{x^2+1}} \, dx = \int \frac{1}{\sqrt[3]{t}} \, dt = \int t^{-1/3} \, dt = \frac{3}{2} t^{2/3} + c$$

$$= \frac{3}{2} (x^2 + 1)^{2/3} + c$$
 1/2

**SECTION-B**

Question numbers 21 to 26 carry 2 marks each.

21. Find a unit vector perpendicular to each of the vectors  $\vec{a}$  and  $\vec{b}$

where  $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$  and  $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$ .

**Ans:**  $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k}$  1

Unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$  is  $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$  1

**OR**

Find the volume of the parallelepiped whose adjacent edges are represented by

$2\vec{a}$ ,  $-\vec{b}$  and  $3\vec{c}$ , where  $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

**Ans:** Volume of the parallelepiped =  $\begin{vmatrix} 2 & -2 & 4 \\ -3 & -4 & 5 \\ 6 & -3 & 9 \end{vmatrix}$  1

$$= |-24| = 24$$
 1

22. If  $f(x) = \sqrt{\tan \sqrt{x}}$ , then find  $f' \left( \frac{\pi^2}{16} \right)$ .

**Ans:**  $f'(x) = \frac{\sec^2 \sqrt{x}}{4\sqrt{x}\sqrt{\tan(\sqrt{x})}}$  1

$f' \left( \frac{\pi^2}{16} \right) = \frac{2}{4 \cdot \frac{\pi}{4}} = \frac{2}{\pi}$  1

23. Using differentials, find the approximate value of  $\sqrt{25.3}$  up to two places of decimals.

**Ans:** Let  $y = f(x) = \sqrt{x}$ , Let  $x = 25$ ,  $x + \Delta x = 25.3$ ,  $\Delta x = 0.3$  1

$\Delta y \approx \left. \frac{dy}{dx} \right|_{x=25} \cdot \Delta x = \frac{1}{2\sqrt{25}}(0.3) = 0.03$  1/2

$\sqrt{25.3} = f(25) + \Delta y = 5 + 0.03 = 5.03$  (approx.) 1/2

24. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

**Ans:** Probability of green signal on crossing X =  $\frac{30}{100} = \frac{3}{10}$  1  
 Probability of not a green signal on crossing X =  $1 - \frac{3}{10} = \frac{7}{10}$

Probability of a green signal on X on two consecutive days out of three  
 $= \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{63}{500}$  1

25. Prove that  $\sin^{-1} \left( 2x\sqrt{1-x^2} \right) = 2 \cos^{-1} x$ ,  $\frac{1}{\sqrt{2}} \leq x \leq 1$

**Ans:** Put  $x = \cos \theta \Leftrightarrow \theta = \cos^{-1} x$  1/2

L.H.S. =  $\sin^{-1} \left( 2x\sqrt{1-x^2} \right)$   
 $= \sin^{-1} (2 \cos \theta \sin \theta) = \sin^{-1} (\sin 2\theta) = 2\theta = 2 \cos^{-1} x = \text{R.H.S.}$  1 1/2

OR

Consider a bijective function  $f : \mathbb{R}_+ \rightarrow (7, \infty)$  given by  $f(x) = 16x^2 + 24x + 7$ , where  $\mathbb{R}_+$  is the set of all positive real numbers. Find the inverse function of  $f$ .

**Ans:** Let  $y = f(x) = 16x^2 + 24x + 7 = (4x + 3)^2 - 2$  1

$$\Rightarrow f^{-1}(y) = x = \frac{\sqrt{y+2}-3}{4} \quad 1$$

26. Find the value of  $k$  so that the lines  $x - y = kz$  and  $x - 2 = 2y + 1 = -z + 1$  are perpendicular to each other.

**Ans:** The lines,  $\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$  and  $\frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$  1

are perpendicular  $\therefore 1 - \frac{1}{2} - \frac{1}{k} = 0 \Rightarrow k = 2$  1

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

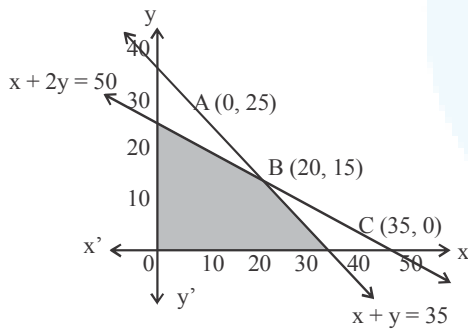
**Ans.**

Let No. of chairs =  $x$ , No. of tables =  $y$

Then L.P. P. is:

Maximize (Profit) :  $Z = 150x + 250y$  1

Subject to :  $x + y \leq 35$   
 $1000x + 2000y \leq 50000 \Rightarrow x + 2y \leq 50$  1  
 $x, y \geq 0$



Correct graph 1 1/2

Corner:	Value of Z	
A(0, 25)	₹ 6250	} <span style="float: right;">1/2</span>
B(20, 15)	₹ 6750 (Max)	
C(35, 0)	₹ 5250	
$\therefore \text{Max}(Z) = ₹ 6750$		

Number of chairs = 20, Tables = 15



28. If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ ,  $a > 0$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{3}$ .

Ans.  $\frac{dy}{d\theta} = a \sin \theta$ ,  $\frac{dx}{d\theta} = a(1 - \cos \theta)$  1/2+1/2

$$\frac{dy}{dx} = \frac{\sin \theta}{1 - \cos \theta} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = \cot \frac{\theta}{2} \quad 1$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \cdot \operatorname{cosec}^2 \frac{\theta}{2} \cdot \frac{d\theta}{dx} = -\frac{\operatorname{cosec}^2 \frac{\theta}{2}}{2a(1 - \cos \theta)} \quad 1$$

$$\left. \frac{d^2y}{dx^2} \right|_{\theta=\frac{\pi}{3}} = -\frac{1}{2} \times \frac{4}{a \left(1 - \frac{1}{2}\right)} = -\frac{4}{a} \quad 1$$

29. Evaluate  $\int_2^3 e^x dx$  as limit of the sums.

Ans. Let  $f(x) = e^x$ ,  $a = 1$ ,  $b = 3$ ,  $nh = 2$ , .....

$$f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h) = e + e^{1+h} + e^{1+2h} + \dots + e^{1+(n-1)h} = \frac{e(e^{nh} - 1)}{e^h - 1} \quad 2$$

$$\int_1^3 e^x dx = \lim_{h \rightarrow 0} h \cdot \frac{e(e^{nh} - 1)}{e^h - 1} = e(e^2 - 1) \text{ or } e^3 - e \quad 1$$

30. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

Ans.  $E_1 =$  Event that the ball transferred from Bag I is Black }  
 $E_2 =$  Event that the ball transferred from Bag I is Red } 1/2  
 $A =$  Event that the ball drawn from Bag II is Black }

$$P(E_1) = \frac{5}{8}; P(E_2) = \frac{3}{8}; P\left(\frac{A}{E_1}\right) = \frac{4}{8} = \frac{1}{2}; P\left(\frac{A}{E_2}\right) = \frac{3}{8} \quad 2$$





Required Probability:

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)} = \frac{\frac{5}{8} \cdot \frac{1}{2}}{\frac{5}{8} \cdot \frac{1}{2} + \frac{3}{8} \cdot \frac{3}{8}} = \frac{20}{29} \quad \frac{1}{2}$$

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

**Ans.** Let  $X =$  No. of white balls = 0, 1, 2

$X :$	0	1	2	$\frac{1}{2}$
$P(X) :$	$\frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} = \frac{7}{15}$	$3 \times \frac{8}{10} \times \frac{7}{9} \times \frac{2}{8} = \frac{7}{15}$	$3 \times \frac{2}{10} \times \frac{1}{9} \times \frac{8}{8} = \frac{1}{15}$	$\frac{1}{2}$

$X \cdot P(X) :$	0	$\frac{7}{15}$	$\frac{2}{15}$	$\frac{1}{2}$
------------------	---	----------------	----------------	---------------

$X^2 P(X) :$	0	$\frac{7}{15}$	$\frac{4}{15}$	
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Mean =  $\sum XP(X) = \frac{9}{15} = \frac{3}{5} \quad \frac{1}{2}$

Variance =  $\sum X^2 P(x) - [\sum XP(X)]^2 = \frac{11}{15} - \left[\frac{3}{5}\right]^2 = \frac{28}{75} \quad 1$

**31.** Find the general solution of the differential equation  $ye^y dx = (y^3 + 2xe^y) dy$ .

**Ans.**  $y \cdot e^y dx = (y^3 + 2xe^y) dy \Rightarrow y \cdot e^y \frac{dy}{dx} = y^3 + 2xe^y$

$\therefore \frac{dx}{dy} - \frac{2}{y}x = y^2 \cdot e^{-y} \quad 1$

I.F. (Integrating factor) =  $e^{-2 \int \frac{1}{y} dy} = e^{-2 \log y} = e^{\log \frac{1}{y^2}} = \frac{1}{y^2} \quad 1$

$\therefore$  Solution is

$x \cdot \frac{1}{y^2} = \int y^2 \cdot e^{-y} \cdot \frac{1}{y^2} dy + c = \int e^{-y} dy + c \quad 1$

$\Rightarrow \frac{x}{y^2} = -e^{-y} + c$  or  $x = -y^2 e^{-y} + cy^2 \quad 1$



OR

Find the particular solution of the differential equation

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right), \text{ given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

**Ans.** The differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}, \text{ let } y = vx \therefore \frac{dy}{dx} = v + x \frac{dv}{dx} \quad 1$$

$$\Rightarrow v + x \frac{dv}{dx} = v - \tan v \Rightarrow \cot v \, dv = -\frac{1}{x} dx$$

Integrate both sides

$$\log \sin v = -\log |x| + \log c \Rightarrow \log \sin \frac{y}{x} = \log \frac{c}{x} \quad 2$$

$$\Rightarrow x \cdot \sin \frac{y}{x} = c, \text{ Put } y = \frac{\pi}{4} \text{ and } x = 1$$

$$\Rightarrow \sin \frac{\pi}{4} = c \text{ or } c = \frac{1}{\sqrt{2}} \quad 1/2$$

$$\therefore \text{ Particular solution is } x \cdot \sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}} \quad 1/2$$

- 32.** Let  $N$  be the set of natural numbers and  $R$  be the relation on  $N \times N$  defined by  $(a, b) R (c, d)$  iff  $ad = bc$  for all  $a, b, c, d \in N$ . Show that  $R$  is an equivalence relation.

**Ans:** Reflexive: For any  $(a, b) \in N \times N$

$$a \cdot b = b \cdot a$$

$\therefore (a, b) R (a, b)$  thus  $R$  is reflexive 1

Symmetric: For  $(a, b), (c, d) \in N \times N$

$$(a, b) R (c, d) \Rightarrow a \cdot d = b \cdot c$$

$$\Rightarrow c \cdot b = d \cdot a$$

$\Rightarrow (c, d) R (a, b) \therefore R$  is symmetric 1 1/2

Transitive : For any  $(a, b), (c, d), (e, f) \in N \times N$

$$(a, b) R (c, d) \text{ and } (c, d) R (e, f)$$

$$\Rightarrow a \cdot d = b \cdot c \text{ and } c \cdot f = d \cdot e$$

$$\Rightarrow a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$$

$\therefore (a, b) R (e, f), \therefore R$  is transitive 1 1/2

$\therefore R$  is an equivalence Relation

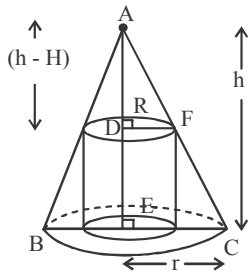


SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height  $h$  and radius  $r$  is one-third of the height of the cone, and the greatest volume of the cylinder is  $\frac{4}{9}$  times the volume of the cone.

Ans.



Let  $H$  = Height of cylinder

$R$  = Radius of cylinder

Volume of cone =  $\frac{\pi}{3} r^2 h$  1/2

$V$  = Volume of cylinder =  $\pi R^2 H$  1/2

$\Delta ADF \sim \Delta AEC \Rightarrow \frac{h-H}{h} = \frac{R}{r} \Rightarrow R = \frac{r}{h}(h-H)$  1

$\therefore V = \pi \cdot H \cdot \frac{r^2}{h^2} (h-H)^2 = \frac{\pi r^2}{h^2} (H^3 - 2hH^2 + Hh^2)$  1

$V'(H) = \frac{\pi r^2}{h^2} (3H^2 - 4hH + h^2), V'(h) = 0 \Rightarrow H = \frac{h}{3}$  1+1

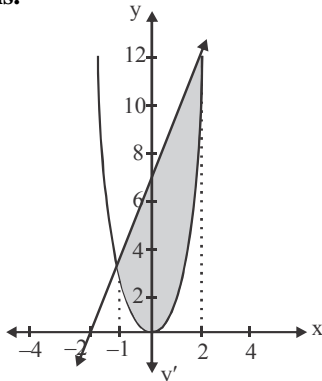
$V''(H) = \frac{\pi r^2}{h^2} (6H - 4h), V''\left(H = \frac{h}{3}\right) = \frac{\pi r^2}{h^2} (-2h) < 0$  1/2

$\therefore V$  is max iff  $H = \frac{h}{3}$  and  $R = \frac{2r}{3}$

$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{3\pi R^2 H}{\pi r^2 h} = 3 \cdot \frac{4r^2}{9} \cdot \frac{h}{3} \cdot \frac{1}{r^2 h} = \frac{4}{9}$  1/2

34. Using integration, find the area of the region enclosed by the parabola  $y = 3x^2$  and the line  $3x - y + 6 = 0$ .

Ans.



Points of intersection  $x = -1, 2$  1

Correct Graph 1

Required area

$= \int_{-1}^2 3(x+2) dx - 3 \int_{-1}^2 x^2 dx$  2

$= \frac{3}{2} [(x+2)^2]_{-1}^2 - [x^3]_{-1}^2$  1 1/2

$= \frac{3}{2} \times 15 - 9 = \frac{27}{2}$  1/2

35. Find the equation of the plane that contains the point A(2,1,-1) and is perpendicular to the line of intersection of the planes  $2x + y - z = 3$  and  $x + 2y + z = 2$ . Also find the angle between the plane thus obtained and the y-axis.

**Ans.** Let equation of the required plane be:

$$a(x - 2) + b(y - 1) + c(z + 1) = 0 \quad \color{red}{1\frac{1}{2}}$$

$$\text{Also : } \begin{aligned} 2a + b - c &= 0 \\ a + 2b + c &= 0 \end{aligned}$$

$$\text{Solving: } \frac{a}{3} = \frac{b}{-3} = \frac{c}{3} = k \Rightarrow a = 3k, b = -3k, c = 3k \quad \color{red}{1\frac{1}{2}}$$

$$\therefore \text{Equation of plane is : } 3k(x - 2) - 3k(y - 1) + 3k(z + 1) = 0$$

$$\Rightarrow x - y + z = 0 \quad \color{red}{1\frac{1}{2}}$$

Let angle between y-axis and plane =  $\theta$

$$\text{then, } \sin \theta = \left| \frac{0 - 1 + 0}{\sqrt{1 + 1 + 1}} \right| = \left| \frac{-1}{\sqrt{3}} \right| \Rightarrow \theta = \sin^{-1} \left( \frac{1}{\sqrt{3}} \right) \quad \color{red}{1\frac{1}{2}}$$

**OR**

Find the distance of the point P(-2, -4, 7) from the point of intersection Q of the line  $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$ . Also write the vector equation of the line PQ.

**Ans.** General point on line is:  $\vec{r} = (3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k}$  **1**

For the point of intersection:

$$[(3 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (6 + 2\lambda)\hat{k}] \cdot (\hat{i} - \hat{j} + \hat{k}) = 6 \quad \color{red}{1}$$

$$\Rightarrow 3 + 2\lambda + 2 + \lambda + 6 + 2\lambda = 6 \Rightarrow \lambda = -1 \quad \color{red}{1}$$

$$\therefore Q(\hat{i} - \hat{j} + 4\hat{k}) = Q(1, -1, 4) \quad \color{red}{1}$$

$$PQ = 3\sqrt{3}, \text{ equation of the line PQ : } \vec{r} = -2\hat{i} - 4\hat{j} + 7\hat{k} + \mu(3\hat{i} + 3\hat{j} - 3\hat{k}) \quad \color{red}{1+1}$$

36. If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ , then find  $A^{-1}$  and use it to solve the

following system of the equations:

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$



**Ans.**  $|A| = 7$ ;  $\text{adj}(A) = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$ ;  $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$   $1 + 1 \frac{1}{2} + \frac{1}{2}$

The system of equations in Matrix form can be written as :

$A \cdot X = B$ , where  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ;  $B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$  1

$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$  1

$\therefore x = 1, y = -5, z = -5$  1

**OR**

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

**Ans.**  $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$= \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ca \\ a^2+b^2 & c^2 & ab \end{vmatrix}$  1  $(C_1 \rightarrow C_1 - 2C_3)$

$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$  1  $(C_1 \rightarrow C_1 + C_2)$

$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$  2  $(R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1)$

$= (b-a)(c-a) \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix}$  1



Expand along  $C_1$

$$= (a^2 + b^2 + c^2)(b-a)(c-a)(-b^2 - ab + c^2 + ac)$$

$$= (a-b)(b-c)(c-a)(a+b+c)(a^2 + b^2 + c^2)$$

1

