SET – 1

Series : BVM/1

कोड नं.
Code No. **65/1/1**

रोल नं.
Roll No. | | | | | | | |

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **100***

*Time allowed : **3** hours* *Maximum Marks : **100***

सामान्य निर्देश :

(i) ***सभी** प्रश्न अनिवार्य हैं ।*

(ii) *इस प्रश्न–पत्र में **29** प्रश्न हैं जो चार खण्डों में विभाजित हैं : अ, ब, स तथा द । खण्ड अ में **4** प्रश्न हैं जिनमें से प्रत्येक **एक अंक** का है । खण्ड ब में **8** प्रश्न हैं जिनमें से प्रत्येक **दो अंक** का है । खण्ड स में **11** प्रश्न हैं जिनमें से प्रत्येक **चार अंक** का है । खण्ड द में **6** प्रश्न हैं जिनमें से प्रत्येक **छः अंक** का है ।*

(iii) *खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं ।*

(iv) *पूर्ण प्रश्न–पत्र में विकल्प नहीं हैं । फिर भी खण्ड अ के **1** प्रश्न, खण्ड ब के **3** प्रश्नों में, खण्ड स के **3** प्रश्नों में तथा खण्ड द के **3** प्रश्नों में आंतरिक विकल्प हैं । ऐसे सभी प्रश्नों में, से आपको **एक** ही विकल्प हल करना है ।*

(v) *कैलकुलेटर के प्रयोग की अनुमति **नहीं** है । यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं ।*

General Instructions :

(i) ***All** questions are compulsory.*

(ii) *This question paper contains **29** questions divided into four sections A, B, C and D. Section A comprises of 4 questions of **one mark** each, Section B comprises of 8 questions of **two marks** each, Section C comprises of 11 questions of **four marks** each and Section D comprises of 6 questions of **six marks** each.*

(iii) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only **one** of the alternatives in all such questions.*

(v) *Use of calculators is not permitted. You may ask logarithmic tables, if required.*

खण्ड – अ
SECTION – A

प्रश्न संख्या **1** से **4** तक के प्रत्येक प्रश्न **1** अंक का है ।

Question numbers **1** to **4** carry **1** mark each.

1. यदि A और B एक ही कोटि 3 के वर्ग आव्यूह हैं और $|A| = 2$ तथा $AB = 2I$ है, तो $|B|$ का मान लिखिए ।

   If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.

2. यदि $f(x) = x + 1$ है, तो $\frac{d}{dx}(fof)(x)$ ज्ञात कीजिए।

If $f(x) = x + 1$, find $\frac{d}{dx}(fof)(x)$.

3. अवकल समीकरण $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$ की कोटि व घात ज्ञात कीजिए।

Find the order and the degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$.

4. यदि एक रेखा x-अक्ष, y-अक्ष तथा z-अक्ष से क्रमशः $90°, 135°, 45°$ के कोण बनाती है। इस रेखा के दिक्–कोसाइन ज्ञात कीजिए।

अथवा

उस रेखा का सदिश समीकरण ज्ञात कीजिए जो बिन्दु (3, 4, 5) से गुजरती है तथा सदिश $2\hat{i} + 2\hat{j} - 3\hat{k}$ के समांतर है।

If a line makes angles $90°, 135°, 45°$ with the x, y and z axes respectively, find its direction cosines.

OR

Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

खण्ड – ब

SECTION – B

प्रश्न संख्या **5** से **12** तक के प्रत्येक प्रश्न के **2** अंक हैं।

Question numbers **5** to **12** carry **2** marks each.

5. जाँच कीजिए कि क्या संक्रिया * जो R पर a * b = ab + 1 द्वारा परिभाषित है (i) द्वि–आधारी संक्रिया होगी या नहीं (ii) यदि यह द्वि–आधारी है, तो क्या यह साहचर्य होगी या नहीं ?

Examine whether the operation * defined on R by a * b = ab + 1 is (i) a binary or not. (ii) if a binary operation, is it associative or not ?

6. आव्यूह A ज्ञात कीजिए यदि 2A – 3B + 5C = O, जहाँ $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ तथा $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$ हैं।

Find a matrix A such that 2A – 3B + 5C = O, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

7. ज्ञात कीजिए : $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\, dx$.

Find : $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\, dx$.

8. ज्ञात कीजिए : $\int \sqrt{1 - \sin 2x}\, dx,\ \frac{\pi}{4} < x < \frac{\pi}{2}$

अथवा

ज्ञात कीजिए : $\int \sin^{-1}(2x)\, dx$.

Find : $\int \sqrt{1 - \sin 2x}\, dx,\ \frac{\pi}{4} < x < \frac{\pi}{2}$

OR

Find : $\int \sin^{-1}(2x)\, dx$.

9. वक्रों के कुल $y = e^{2x}(a + bx)$, जिसमें a, b स्वेच्छ अचर हैं, को निरूपित करने वाला अवकल समीकरण ज्ञात कीजिए।

Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where ‘a’ and ‘b’ are arbitrary constants.

10. यदि दो मात्रक सदिशों का योग एक मात्रक सदिश हो, तो सिद्ध कीजिए कि उन दो सदिशों के अन्तर का परिमाण $\sqrt{3}$ होगा ।

अथवा

यदि $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ तथा $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$ है, तो $[\vec{a}\ \vec{b}\ \vec{c}]$ ज्ञात कीजिए ।

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a}\ \vec{b}\ \vec{c}]$.

11. एक पाँसा जिस पर 1, 2, 3 लाल रंग से तथा 4, 5, 6 हरे रंग से लिखा गया है, को उछाला जाता है । "संख्या सम होने" की घटना को A से व "संख्या लाल रंग में लिखी है" की घटना B से परिभाषित है । ज्ञात कीजिए कि क्या ये दो घटनाएँ A तथा B स्वतंत्र हैं या नहीं ।

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.

12. एक पासे को छः बार उछाला जाता है । यदि "पासे पर विषम संख्या प्राप्त होना" एक सफलता है, तो (i) 5 सफलताएँ (ii) अधिकतम 5 सफलताएँ, की प्रायिकताएँ क्या-क्या होंगी ?

अथवा

एक यादृच्छिक चर X का प्रायिकता बंटन P(X) निम्न प्रकार से है, जहाँ 'k' कोई संख्या है :

$$P(X = x) = \begin{cases} k, & \text{यदि } x = 0 \\ 2k, & \text{यदि } x = 1 \\ 3k, & \text{यदि } x = 2 \\ 0, & \text{अन्यथा} \end{cases}$$

'k' का मान ज्ञात कीजिए ।

A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of (i) 5 successes ? (ii) atmost 5 successes ?

OR

The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

खण्ड – स

SECTION – C

प्रश्न संख्या **13** से **23** के प्रत्येक प्रश्न के **4** अंक हैं ।

Question numbers **13** to **23** carry **4** marks each.

13. दिखाइए कि समुच्चय $\mathbb{R}$ में R = {(a, b) : a ≤ b} द्वारा परिभाषित संबंध R स्वतुल्य व संक्रामक है, परन्तु सममित नहीं है ।

अथवा

सिद्ध कीजिए कि फलन $f : N \to N$, $f(x) = x^2 + x + 1$, द्वारा परिभाषित है, एक एकैकी फलन है किंतु आच्छादक नहीं ।

फलन $f : N \to S$, जहाँ S फलन f का परिसर है, का प्रतिलोम भी ज्ञात कीजिए ।

Show that the relation R on $\mathbb{R}$ defined as R = {(a, b) : a ≤ b}, is reflexive, and transitive but not symmetric.

OR

Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f : N \to S$, where S is range of f.

14. हल कीजिए : $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$

Solve : $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$.

15. सारणिकों के गुणधर्मों का प्रयोग करके, सिद्ध कीजिए कि $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$.

Using properties of determinants, prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$.

16. यदि $\log (x^2 + y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$ हो, तो दर्शाइए कि $\frac{dy}{dx} = \frac{x+y}{x-y}$.

अथवा

यदि $x^y - y^x = a^b$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

If $\log (x^2 + y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

17. यदि $y = (\sin^{-1}x)^2$ है, तो सिद्ध कीजिए कि $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$.

If $y = (\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$.

18. वक्र $y = \sqrt{3x-2}$ की उस स्पर्श–रेखा का समीकरण ज्ञात कीजिए जो रेखा $4x - 2y + 5 = 0$ के समान्तर है। स्पर्श बिन्दु से वक्र पर बने अभिलंब का समीकरण भी ज्ञात कीजिए।

Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

19. ज्ञात कीजिए : $\int \frac{3x+5}{x^2+3x-18}\,dx$.

Find : $\int \frac{3x+5}{x^2+3x-18}\,dx$.

20. सिद्ध कीजिए कि $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, अतः $\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$ का मान ज्ञात कीजिए।

Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, hence evaluate $\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$.

21. अवकल समीकरण : $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$ को हल कीजिए, दिया गया है $y = 0$ यदि $x = 1$.

अथवा

अवकल समीकरण : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ को हल कीजिए, दिया गया है $y(0) = 0$.

Solve the differential equation : $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$, given that $y = 0$ when $x = 1$.

OR

Solve the differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

22. यदि $\hat{i} + \hat{j} + \hat{k},\ 2\hat{i} + 5\hat{j},\ 3\hat{i} + 2\hat{j} - 3\hat{k}$ तथा $\hat{i} - 6\hat{j} - \hat{k}$ क्रमशः बिन्दु A, B, C और D के स्थिति सदिश हों तो सरल रेखाओं AB तथा CD के बीच का कोण ज्ञात कीजिए। ज्ञात कीजिए कि क्या सदिश $\overrightarrow{AB}$ तथा $\overrightarrow{CD}$ संरेख हैं या नहीं।

If $\hat{i} + \hat{j} + \hat{k},\ 2\hat{i} + 5\hat{j},\ 3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

23. λ का वह मान ज्ञात कीजिए जिसके लिए निम्न रेखाएँ लम्बवत हैं :

$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ तथा $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$, यह भी ज्ञात कीजिए कि क्या ये रेखाएँ परस्पर प्रतिच्छेद करती हैं या नहीं।

Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

खण्ड – द

SECTION – D

प्रश्न संख्या **24** से **29** के प्रत्येक प्रश्न के **6** अंक हैं ।

Question numbers **24** to **29** carry **6** marks each.

24. यदि $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ हो, तो A^{-1} ज्ञात कीजिए ।

अत: निम्न समीकरण निकाय को हल कीजिए :

$x + y + z = 6$

$x + 2z = 7$

$3x + y + z = 12$

अथवा

प्रारंभिक संक्रियाओं द्वारा निम्न आव्यूह का प्रतिलोम ज्ञात कीजिए :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations

$x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$.

OR

Find the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

25. आयताकार आधार व आयताकार दीवारों की 2 m गहरी और 8 m^3 आयतन की एक बिना ढक्कन की टंकी का निर्माण करना है । यदि टंकी के निर्माण में आधार के लिए ₹ $70/m^2$ और दीवारों पर ₹ $45/m^2$ व्यय आता है तो न्यूनतम खर्च से बनी टंकी की लागत क्या है ?

A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank ?

26. समाकलन विधि द्वारा, त्रिभुज ABC का क्षेत्रफल ज्ञात कीजिए, जहाँ A(2, 5), B (4, 7) तथा C(6, 2) त्रिभुज ABC के शीर्ष हैं ।

अथवा

समाकलन विधि से, x-अक्ष से ऊपर तथा वृत्त $x^2 + y^2 = 8x$ एवं परवलय $y^2 = 4x$ के अंतः भाग के मध्यवर्ती क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

OR

Find the area of the region lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.

27. बिन्दुओं (2, 2, – 1), (3, 4, 2) तथा (7, 0, 6) से गुजरने वाले समतल के सदिश व कार्तीय समीकरण ज्ञात कीजिए । अत: उस समतल का समीकरण ज्ञात कीजिए जो बिन्दु (4, 3, 1) से गुजरता है और ऊपर प्राप्त समतल के समान्तर है ।

अथवा

उस समतल का सदिश समीकरण ज्ञात कीजिए जो रेखा $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ तथा बिंदु (–1, 3, –4) को अंतर्विष्ट करता है । इस समतल पर बिन्दु (2, 1, 4) से डाले गए लंब की दूरी भी ज्ञात कीजिए ।

Find the vector and Cartesian equations of the plane passing through the points (2, 2 –1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

OR

Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and the point (–1, 3, – 4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane, thus obtained.

28. एक निर्माता के पास तीन कारीगर A, B तथा C हैं । कारीगर A 1% त्रुटिपूर्ण इकाइयों का उत्पादन करता है, कारीगर B 5% तथा कारीगर C 7% त्रुटिपूर्ण इकाइयों का उत्पादन करते हैं । A सिर्फ 50% समय ही उत्पादन करता है, B सिर्फ 30% समय व C सिर्फ 20% समय ही उत्पादन करते हैं । यदि कुल उत्पादन का एक ढेर बना लिया जाता है और उस ढेर से यादृच्छया निकाली गई एक इकाई त्रुटिपूर्ण हो, तो इस इकाई के A द्वारा बनाई गई होने की प्रायिकता ज्ञात कीजिए ।

A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

29. एक निर्माता 5 कुशल व 10 अर्धकुशल कारीगरों को काम पर रखता है और एक उत्पाद के दो नमूने A और B बनाता है । नमूना A के प्रत्येक नग बनाने के लिए कुशल कारीगर को 2 घंटे व अर्धकुशल कारीगर को 2 घंटे काम करना पड़ता है । नमूना B के प्रत्येक नग बनाने के लिए कुशल कारीगर को 1 घंटा व अर्ध–कुशल कारीगर को 3 घंटे काम करना पड़ता है । दोनों ही कारीगरों में प्रत्येक को काम करने के लिए प्रतिदिन अधिकतम 8 घण्टे का समय उपलब्ध है । निर्माता को नमूने A के प्रत्येक नग पर ₹ 15 लाभ व नमूने B के प्रत्येक नग पर ₹ 10 का लाभ होता है । नमूना A और नमूना B के कितने नगों का अधिकतम लाभ कमाने के लिए, प्रतिदिन निर्माण करना चाहिए ? इस प्रश्न को रैखिक प्रोग्रामन समस्या के रूप में लिखिए और ग्राफ द्वारा हल कीजिए । अधिकतम लाभ भी ज्ञात कीजिए ।

A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹ 15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

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Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/1/1, 65/1/2, 65/1/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

65/1/1

QUESTION PAPER CODE 65/1/1
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A| \cdot |B| = 2^3|I|$ $\frac{1}{2}$

$\Rightarrow 2 \times |B| = 8 \Rightarrow |B| = 4$ $\frac{1}{2}$

2. $(fof)\,(x) = f(x + 1) = x + 2$ $\frac{1}{2}$

$\frac{d}{dx}(fof)(x) = 1$ $\frac{1}{2}$

3. order = 2, degree = 1 $\frac{1}{2}+\frac{1}{2}$

4. d.c.'s = $<\cos 90°, \cos 135°, \cos 45°>$ $\frac{1}{2}$

$= <0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$ 1

SECTION B

5. As $a, b \in R \Rightarrow ab \in R \Rightarrow ab + 1 \in R \Rightarrow a*b \in R \Rightarrow *$ is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general $(a*b)*c \neq a*(b*c) \Rightarrow *$ is not associative. 1

6. $2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 1

$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ 1

7. Put tan x = t $\Rightarrow \sec^2 x\, dx = dt$ $\frac{1}{2}$

$$I=\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx=\int \frac{dt}{\sqrt{t^2+4}}=\log|t+\sqrt{t^2+4}|+C$$ 1

$$=\log|\tan x+\sqrt{\tan^2 x+4}|+C$$ $\frac{1}{2}$

8. Let $I=\int \sqrt{1-\sin 2x}\,dx$

$=\int (\sin x-\cos x)dx$ as $\sin x>\cos x$ when $x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ 1

$= -\cos x - \sin x + C$ 1

OR

$I=\int \sin^{-1}(2x).1\,dx$

$= x.\sin^{-1}(2x)-\int \frac{2x}{\sqrt{1-4x^2}}dx$ 1

$= x.\sin^{-1}(2x)+\frac{1}{4}\int \frac{-8x}{\sqrt{1-4x^2}}dx = x\sin^{-1}(2x)+\frac{1}{2}.\sqrt{1-4x^2}+C$ 1

9. $y'=be^{2x}+2y \Rightarrow b=\frac{y'-2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$$\frac{e^{2x}.(y''-2y')-(y'-2y).2x^{2x}}{(e^{2x})^2}=0$$ 1

$\Rightarrow y''-4y'+4y=0$ or $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$ $\frac{1}{2}$

10. Given $|\hat{a}+\hat{b}|=1$

As $|\hat{a}+\hat{b}|^2+|\hat{a}-\hat{b}|^2=2(|\hat{a}|^2+|\hat{b}|^2)$ 1

$\Rightarrow 1+|\vec{a}-\vec{b}|^2=2(1+1)$

$\Rightarrow |\hat{a}-\hat{b}|^2 = 3 \Rightarrow |\hat{a}-\hat{b}| = \sqrt{3}$ 1

OR

$[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$ 1

$= -30$ 1

11. $A = \{2, 4, 6\},\ B = \{1, 2, 3\},\ A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2},\ P(B) = \frac{1}{2},\ P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

12. Let X: getting an odd number

$p = \frac{1}{2},\ q = \frac{1}{2},\ n = 6$ $\frac{1}{2}$

(i) $P(X=5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X \le 5) = 1 - P(X=6) = 1 - \frac{1}{64} = \frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow k = \frac{1}{6}$ 1

13. Clearly $a \le a\ \forall\ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow$ a $\le$ b and b $\le$ c $\Rightarrow$ a $\le$ c $\Rightarrow$ (a, c) $\in$ R

$\Rightarrow$ R is transitive. $1\frac{1}{2}$

For non-symmetric:

Let a = 1, b = 2. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2,1) \notin R$

$\Rightarrow$ R is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \ne 0$ $\quad (\because x_1, x_2 \in N)$ $1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for y = 1 $\in$ N, there is no x $\in$ N for which f(x) = 1 $1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \frac{\sqrt{4x-3}-1}{2}$ 1

14. $\tan^{-1}\left(\frac{4x+6x}{1-(4x)(6x)}\right) = \frac{\pi}{4}$ 1

$\Rightarrow \frac{10x}{1-24x^2} = 1 \Rightarrow 24x^2 + 10x - 1 = 0$ $1\frac{1}{2}$

$\Rightarrow x = \frac{1}{12}$ or $-\frac{1}{2}$ 1 as $x=-\frac{1}{2}$ does not satisfy the given equation, so $x=\frac{1}{12}$ $\frac{1}{2}$

15. LHS = $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \rightarrow R_1 - R_2,\ R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 2

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding along C_3,

$$= (a-1)^2.(a-1) = (a-1)^3 = \text{RHS.}$$ 1

16. $\log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right) = 2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right)$$ 2

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right) = \frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right)$$ 1

$$\Rightarrow (x+y) = (x-y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx} - \frac{dv}{dx} = 0 \quad ...(1)$$ 1

Now, log u = y.logx

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx} \Rightarrow \frac{du}{dx} = x^y\left(\frac{y}{x} + \log x\cdot\frac{dy}{dx}\right) \quad ...(2)$$ 1

Again, log v = x · log y

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y\cdot 1 \Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \quad ...(3)$$ 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x\cdot\log y - x^{y-1}\cdot y}{x^y\cdot\log x - y^{x-1}\cdot x}$$ $\frac{1}{2}$

17. $y = (\sin^{-1}x)^2$

$$\Rightarrow y' = 2\cdot\sin^{-1}x\cdot\frac{1}{\sqrt{1-x^2}}$$ 1

$$\Rightarrow \sqrt{1-x^2}\cdot y' = 2\sin^{-1}x$$

$$\Rightarrow \sqrt{1-x^2}\cdot y'' + y'\cdot\frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{2}{\sqrt{1-x^2}}$$ 2

$$\Rightarrow (1-x^2)\cdot y'' - xy' = 2 \text{ or } (1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0.$$ 1

18. Let the point of contact be $P(x_1, y_1)$

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}} \quad \text{(slope of tangent)}$$

$$\Rightarrow m_1 = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1-2}}$$ 1

also, slope of given line = 2 = m_2

$m_1 = m_2 \Rightarrow x_1 = \dfrac{41}{48}$ 1

when $x_1 = \dfrac{41}{48}, y_1 = \sqrt{\dfrac{41}{16} - 2} = \dfrac{3}{4}$ $\therefore P\left(\dfrac{41}{48}, \dfrac{3}{4}\right)$ $\frac{1}{2}$

Equation of tangent is: $y - \dfrac{3}{4} = 2\left(x - \dfrac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y - \dfrac{3}{4} = \dfrac{-1}{2}\left(x - \dfrac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

19. $I = \displaystyle\int \frac{3x+5}{x^2+3x-18}\,dx = \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{x^2+3x-18}\,dx$ 1

$= \displaystyle\frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\,dx$ 1

$= \displaystyle\frac{3}{2}\log|x^2+3x-18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C$ 1 + 1

20. Let $I = \displaystyle\int_0^a f(a-x)\,dx$

Put $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

$I = -\displaystyle\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$ $\frac{1}{2}$

II part.

$I = \displaystyle\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

$$\Rightarrow \quad I=\int_0^{\pi}\frac{(\pi-x)\cdot\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \quad 2I=\int_0^{\pi}\frac{\pi\cdot\sin x}{1+\cos^2 x}dx$$ $1\frac{1}{2}$

Put cos x = t ⇒ –sinx dx = dt

$$\Rightarrow I=-\frac{\pi}{2}\cdot\int_1^{-1}\frac{dt}{1+t^2}=\frac{\pi}{2}\times 2\times\int_0^1\frac{dt}{1+t^2}$$

$$=\pi[\tan^{-1}t]_0^1=\frac{\pi^2}{4}$$ $1\frac{1}{2}$

21. Writing $\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}=\frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2}$ $\frac{1}{2}$

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\frac{1}{2}$

Differential equation becomes $v+x\frac{dv}{dx}=v+\sqrt{1+v^2}$

$$\Rightarrow \int\frac{dv}{\sqrt{1+v^2}}=\int\frac{dx}{x}$$ 1

$$\Rightarrow \log\left|v+\sqrt{1+v^2}\right|=\log|x|+\log c$$ 1

$$\Rightarrow v+\sqrt{1+v^2}=cx \Rightarrow y+\sqrt{x^2+y^2}=cx^2$$

when x = 1, y = 0 ⇒ c = 1 $\frac{1}{2}$

$$\therefore\ y+\sqrt{x^2+y^2}=x^2$$ $\frac{1}{2}$

OR

Given equation is $\frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{4x^2}{1+x^2}$ $\frac{1}{2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = 1 + x^2$ 1

Solution is given by,

$$y \cdot (1+x^2) = \int \frac{4x^2}{1+x^2} \cdot (1+x^2)\,dx = \int 4x^2\,dx$$ 1

$$\Rightarrow\ y \cdot (1+x^2) = \frac{4x^3}{3} + c$$ $\frac{1}{2}$

when $x = 0,\ y = 0 \Rightarrow c = 0$ $\frac{1}{2}$

$$y \cdot (1+x^2) = \frac{4x^3}{3} \quad \text{or} \quad y = \frac{4x^3}{3(1+x^2)}$$ $\frac{1}{2}$

22. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

$$\text{Then } \cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \frac{-2-32-2}{\sqrt{18}\sqrt{72}} = -1$$ 1

$\Rightarrow \theta = 180°$ or π $\frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

23. Given lines are: $\frac{x-1}{-3} = \frac{y-2}{\left(\frac{\lambda}{7}\right)} = \frac{z-3}{2}$ and $\frac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5}$ 1

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$$ 1

So, lines are

$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{x-3}{2} \quad \text{and} \quad \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$ $\frac{1}{2}$

Consider $\Delta = \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ 2

$$\therefore \; A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B$ 1

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ 1

$\Rightarrow x = 3, y = 1, z = 2$ $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_2 \to R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 5R_3$

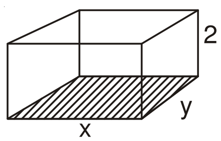
$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{6}{5}R_3, R_2 \to R_2 + \frac{2}{5}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

25.



$V = 2xy \Rightarrow 2xy = 8$ (given) 1

$\Rightarrow y = \dfrac{4}{x}$

Now, cost, $C = 70xy + 45 \times 2 \times (2x + 2y)$ 1

$= 280 + 180x + \dfrac{720}{x}$ 1

$\dfrac{dC}{dx} = 180 - \dfrac{720}{x^2}$ 1

$\dfrac{dC}{dx} = 0 \Rightarrow x = 2m$ $\frac{1}{2}$

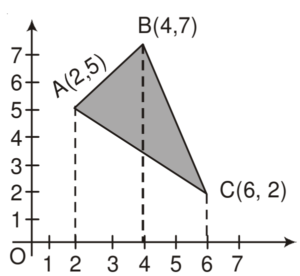
$\dfrac{d^2C}{dx^2} = \dfrac{1440}{x^3} = 180 > 0$ at $x = 2$ $\frac{1}{2}$

$\Rightarrow$ C is minimum at $x = 2m$. $\frac{1}{2}$

Minimum cost $= 280 + 180(2) + \dfrac{720}{2} =$ ₹ 1,000 $\frac{1}{2}$

26.

Correct Figure 1



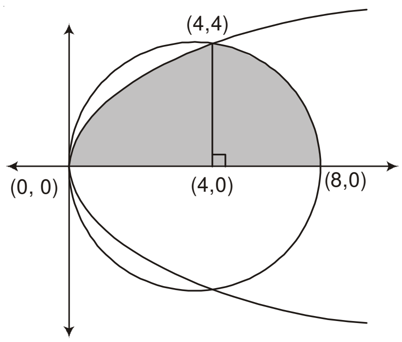
$$\left.\begin{aligned} &\text{Equation of AB} : y = x + 3 \\ &\text{Equation of BC} : y = \frac{-5x}{2} + 17 \\ &\text{Equation of AC} : y = \frac{-3x}{4} + \frac{13}{2} \end{aligned}\right\}$$ $1\frac{1}{2}$

$$\text{Required Area} = \int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2} + 17\right) dx - \int_2^6 \left(\frac{-3x}{4} + \frac{13}{2}\right) dx$$ $1\frac{1}{2}$

$$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4} + 17x\right]_4^6 - \left[\frac{-3x^2}{8} + \frac{13x}{2}\right]_2^6$$ $1\frac{1}{2}$

$= 7$ $\frac{1}{2}$

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

Required Area $= \int_0^4 2\sqrt{x}\, dx + \int_4^8 \sqrt{4^2 - (x-4)^2}\, dx$ $1\frac{1}{2}$

$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$ $1\frac{1}{2}$

$= \left(4\pi + \frac{32}{3}\right)$ 1

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r}\cdot(5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k})\cdot(5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r}\cdot(5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

$\therefore$ Required plane is –3(x + 1) + 3(y – 3) + 3(z + 4) = 0

$\therefore$ –x + y + z = 0

Also vector equation is: $\vec{r}.(-\hat{i}+\hat{j}+\hat{k})=0$ 1

Length of perpendicular from (2, 1, 4) = $\dfrac{|-2+1+4|}{\sqrt{(-1)^2+1^2+1^2}} = \sqrt{3}$ 1

28.

$$\left.\begin{aligned} \text{Let } E_1 &: \text{item is produced by A} \\ E_2 &: \text{item is produced by B} \\ E_3 &: \text{item is produced by C} \\ A &: \text{defective item is found.} \end{aligned}\right\}$$ 1

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$$ 1

$$P(A/E_1) = \frac{1}{100}, P(A/E_2) = \frac{5}{100}, P(A|E_3) = \frac{7}{100}$$ 1

$$P(E_1|A) = \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}}$$ 2

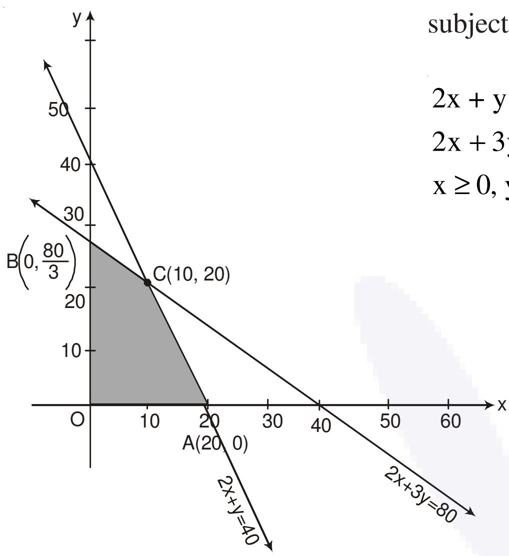
$$= \frac{5}{34}$$ 1

29. Let number of items produced of model A be x and that of model B be y.

LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to



$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 & \end{array}\right\}$$ 2

Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10,\ y = 20.$ $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.

QUESTION PAPER CODE 65/1/2
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. order = 2, degree = 1 $\frac{1}{2}+\frac{1}{2}$

2. (fog) (x) = f(x – 7) = x $\frac{1}{2}$

$\Rightarrow \frac{d}{dx}[(fog)(x)] = 1$ $\frac{1}{2}$

3. $\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ $\frac{1}{2}$

$\Rightarrow x = 3, y = 3$ $\therefore x - y = 0$ $\frac{1}{2}$

4. d.c.'s = <cos 90°, cos 135°, cos 45°> $\frac{1}{2}$

$= < 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k}) + \lambda(2\hat{i}+2\hat{j}-3\hat{k})$ 1

SECTION B

5. As a, b ∈ R ⇒ ab ∈ R ⇒ ab + 1 ∈ R ⇒ a*b ∈ R ⇒ * is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general (a*b)*c ≠ a*(b*c) ⇒ * is not associative. 1

6. $A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ 1

$$A^2-5A=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}-\begin{bmatrix}10&0&5\\10&5&15\\5&-5&0\end{bmatrix}=\begin{bmatrix}-5&-1&-3\\-1&-7&-10\\-5&4&-2\end{bmatrix}$$ 1

7. Let $I=\int\sqrt{1-\sin 2x}\,dx$

$=\int(\sin x-\cos x)dx$ as $\sin x>\cos x$ when $x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ 1

$=-\cos x-\sin x+C$ 1

OR

$I=\int\sin^{-1}(2x).1\,dx$

$=x.\sin^{-1}(2x)-\int\frac{2x}{\sqrt{1-4x^2}}dx$ 1

$=x.\sin^{-1}(2x)+\frac{1}{4}\int\frac{-8x}{\sqrt{1-4x^2}}dx=x\sin^{-1}(2x)+\frac{1}{2}.\sqrt{1-4x^2}+C$ 1

8. $y'=be^{2x}+2y\Rightarrow b=\frac{y'-2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$\frac{e^{2x}.(y''-2y')-(y'-2y).2x^{2x}}{(e^{2x})^2}=0$ 1

$\Rightarrow\ y''-4y'+4y=0$ or $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$ $\frac{1}{2}$

9. Let X: getting an odd number

$p=\frac{1}{2},\ q=\frac{1}{2},\ n=6$ $\frac{1}{2}$

(i) $P(X=5)={}^6C_5\left(\frac{1}{2}\right)^6=\frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X\le 5)=1-P(X=6)=1-\frac{1}{64}=\frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow k = \frac{1}{6}$ 1

10. $A = \{2, 4, 6\}, \quad B = \{1, 2, 3\}, \quad A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2}, \; P(B) = \frac{1}{2}, \; P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

11. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow 1 + |\vec{a} - \vec{b}|^2 = 2(1+1)$

$\Rightarrow |\hat{a} - \hat{b}|^2 = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}$ 1

OR

$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$ 1

$= -30$ 1

12. $I = \int \frac{\tan^2 x . \sec^2 x}{1 - (\tan^3 x)^2} dx$

Put $\tan^3 x = t \Rightarrow I = \frac{1}{3} \int \frac{dt}{1 - t^2}$ 1

$= \frac{1}{6} \log \left| \frac{1+t}{1-t} \right| + C = \frac{1}{6} \log \left| \frac{1 + \tan^3 x}{1 - \tan^3 x} \right| + C$ $\frac{1}{2} + \frac{1}{2}$

SECTION C

13. $\tan^{-1}\left(\dfrac{2x+3x}{1-(2x)(3x)}\right)=\dfrac{\pi}{4}$ 1

$\Rightarrow \dfrac{5x}{1-6x^2}=\tan\dfrac{\pi}{4}=1 \Rightarrow 6x^2+5x-1=0$ $1\frac{1}{2}$

$\Rightarrow x=-1 \text{ or } x=\dfrac{1}{6}$ 1

as $x = -1$ does not satisfy the given equation,

$\therefore x = \dfrac{1}{6}$ $\frac{1}{2}$

14. $\log(x^2+y^2)=2\tan^{-1}\left(\dfrac{y}{x}\right)$

differentiating both sides w.r.t. x,

$\dfrac{1}{x^2+y^2}\left(2x+2y\dfrac{dy}{dx}\right)=2\cdot\dfrac{1}{1+\left(\dfrac{y}{x}\right)^2}\cdot\left(\dfrac{x\cdot\dfrac{dy}{dx}-y\cdot 1}{x^2}\right)$ 2

$\Rightarrow \dfrac{2}{x^2+y^2}\left(x+y\dfrac{dy}{dx}\right)=\dfrac{2x^2}{x^2+y^2}\cdot\dfrac{1}{x^2}\cdot\left(x\dfrac{dy}{dx}-y\right)$ 1

$\Rightarrow (x+y)=(x-y)\dfrac{dy}{dx}\Rightarrow\dfrac{dy}{dx}=\dfrac{x+y}{x-y}$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$\Rightarrow \dfrac{du}{dx}-\dfrac{dv}{dx} = 0$ ...(1) 1

Now, $\log u = y.\log x$

$\Rightarrow \dfrac{1}{u}\cdot\dfrac{du}{dx}=y\cdot\dfrac{1}{x}+\log x\cdot\dfrac{dy}{dx}\Rightarrow\dfrac{du}{dx}=x^y\left(\dfrac{y}{x}+\log x\cdot\dfrac{dy}{dx}\right)$ ...(2) 1

Again, $\log v = x \cdot \log y$

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y \cdot 1 \Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$ ...(3) 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x\cdot\log y - x^{y-1}\cdot y}{x^y\cdot\log x - y^{x-1}\cdot x}$$ $\frac{1}{2}$

15. $$I = \int\frac{3x+5}{x^2+3x-18}\,dx = \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{x^2+3x-18}\,dx$$ 1

$$= \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\,dx$$ 1

$$= \frac{3}{2}\log|x^2+3x-18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C$$ 1 + 1

16. Let $I = \int_0^a f(a-x)\,dx$

Put $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

$$I = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$$ $\frac{1}{2}$

II part.

$$I = \int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \quad I = \int_0^{\pi}\frac{(\pi - x)\cdot\sin x}{1+\cos^2 x}\,dx$$

$\Rightarrow \quad 2I = \int_0^{\pi} \frac{\pi \cdot \sin x}{1+\cos^2 x} dx$ $1\frac{1}{2}$

Put cos x = t ⇒ –sinx dx = dt

$\Rightarrow I = -\frac{\pi}{2} \cdot \int_1^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2} \times 2 \times \int_0^1 \frac{dt}{1+t^2}$

$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$ $1\frac{1}{2}$

17. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \frac{-2-32-2}{\sqrt{18}\sqrt{72}} = -1$ 1

⇒ θ = 180° or π $\frac{1}{2}$

Since θ = π so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

18. LHS = $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$= \begin{vmatrix} a+b+c & a+b & a+c \\ -c & a+b & -(a+c) \\ -b & -(a+b) & (a+c) \end{vmatrix}$ $1\frac{1}{2}$

$= (a+b)(a+c)\begin{vmatrix} a+b+c & 1 & 1 \\ -c & 1 & -1 \\ -b & -1 & 1 \end{vmatrix}$ $\frac{1}{2}$

$C_3 \to C_3 + C_2$

$$= (a+b)\,(a+c)\begin{vmatrix} a+b+c & 1 & 2 \\ -c & 1 & 0 \\ -b & -1 & 0 \end{vmatrix}$$ 1

$= 2(a + b)\,(b + c)\,(c + a) = \text{RHS}.$ 1

19. $$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times \left(\sec^2\frac{t}{2} \times \frac{1}{2}\right) = \frac{\cos^2 t}{\sin t}$$ 1

$$\frac{dy}{dt} = \cos t$$ $\frac{1}{2}$

$$\frac{d^2y}{dt^2} = -\sin t \Rightarrow \left.\frac{d^2y}{dt^2}\right]_{t=\frac{\pi}{4}} = -\frac{1}{\sqrt{2}}$$ 1

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$$ $\frac{1}{2}$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \sec^4 t \cdot \sin t$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = 2\sqrt{2}$$ 1

20. Clearly $a \le a\ \forall\ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive. $1\frac{1}{2}$

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2, 1) \notin R$

$\Rightarrow R$ is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ $(\because x_1, x_2 \in N)$ $1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \frac{\sqrt{4x-3}-1}{2}$ 1

21. Let the point of contact be $P(x_1, y_1)$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}}$ (slope of tangent)

$\Rightarrow m_1 = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1-2}}$ 1

also, slope of given line = 2 = m_2

$m_1 = m_2 \Rightarrow x_1 = \frac{41}{48}$ 1

when $x_1 = \frac{41}{48}, y_1 = \sqrt{\frac{41}{16} - 2} = \frac{3}{4}$ $\therefore P\left(\frac{41}{48}, \frac{3}{4}\right)$ $\frac{1}{2}$

Equation of tangent is: $y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

22. Writing $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$ $\frac{1}{2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

Differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$

$\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$ 1

$\Rightarrow \log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log c$ 1

$\Rightarrow v + \sqrt{1 + v^2} = cx \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$

when $x = 1,\ y = 0 \Rightarrow c = 1$ $\frac{1}{2}$

$\therefore\ y + \sqrt{x^2 + y^2} = x^2$ $\frac{1}{2}$

OR

Given equation is $\frac{dy}{dx} + \frac{2x}{1 + x^2} \cdot y = \frac{4x^2}{1 + x^2}$ $\frac{1}{2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = 1 + x^2$ 1

Solution is given by,

$y \cdot (1 + x^2) = \int \frac{4x^2}{1 + x^2} \cdot (1 + x^2)\, dx = \int 4x^2\, dx$ 1

$$\Rightarrow \; y \cdot (1+x^2) = \frac{4x^3}{3} + c$$ $\frac{1}{2}$

when $x = 0,\; y = 0 \Rightarrow c = 0$ $\frac{1}{2}$

$$y \cdot (1+x^2) = \frac{4x^3}{3} \quad \text{or} \quad y = \frac{4x^3}{3(1+x^2)}$$ $\frac{1}{2}$

23. Given lines are: $\frac{x-1}{-3} = \frac{y-2}{\left(\frac{\lambda}{7}\right)} = \frac{z-3}{2}$ and $\frac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5}$ 1

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$$ 1

So, lines are

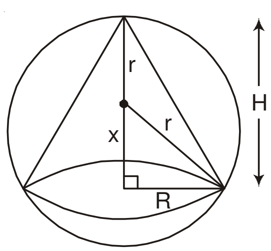
$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{x-3}{2} \quad \text{and} \quad \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$ $\frac{1}{2}$

$$\text{Consider } \Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24.



Correct Figure 1

$r^2 = x^2 + R^2$

Now, $V = \frac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi(r^2 - x^2)(r + x)$$

$$= \frac{1}{3}\pi(r + x)^2 \, (r - x)$$ 1

$$\frac{dV}{dx}=\frac{1}{3}\pi\left[(r+x)^2\,(-1)+(r-x).2(r+x)\right]$$

$$=\frac{1}{3}\pi(r+x)\,(r-3x)$$ 1

$$\frac{dV}{dx}=0 \Rightarrow x=-r \quad \text{or } x=\frac{r}{3}$$ $\frac{1}{2}$

(Rejected)

$$\frac{d^2V}{dx^2}=\frac{1}{3}\pi\left[(r+x)(-3)+(r-3x)\right]=-\pi H<0$$ 1

$\Rightarrow$ V is maximum when $x=\frac{r}{3}$.

$$H=r+x=r+\frac{r}{3}=\frac{4r}{3}$$ $\frac{1}{2}$

Maximum volume $V=\frac{1}{3}\pi\left(r+\frac{r}{3}\right)^2\left(r-\frac{r}{3}\right)=\frac{32}{81}\pi r^3$ 1

25. $|A|=-1\neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A=\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}$$ 2

$$A^{-1}=\frac{1}{|A|}\cdot \text{adj } A=\begin{bmatrix}0 & 1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}$

Now, $X=A^{-1}B$ 1

$$=\begin{bmatrix}0 & 1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix}\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$ 1

$\Rightarrow x = 1, y = 2, z = 3$ $\frac{1}{2}$

OR

$A = I.A$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to R_2 + R_1, R_3 \to R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{3}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5/3 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1/3 & 1/3 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5/3 & -4/3 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{1}{3}R_3, R_2 \to R_2 - \frac{5}{3}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$ 1

26. Let E_1 : item is produced by A
E_2 : item is produced by B
E_3 : item is produced by C
A : defective item is found. 1

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$$ 1

$$P(A/E_1) = \frac{1}{100}, P(A/E_2) = \frac{5}{100}, P(A|E_3) = \frac{7}{100}$$ 1

$$P(E_1|A) = \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}}$$ 2

$$= \frac{5}{34}$$ 1

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k}) = 17$ 1

Equation of required parallel plane is

$$\vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k}) = (4\hat{i}+3\hat{j}+\hat{k})\cdot(5\hat{i}+2\hat{j}-3\hat{k})$$ 1

$$\Rightarrow \vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k}) = 23$$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

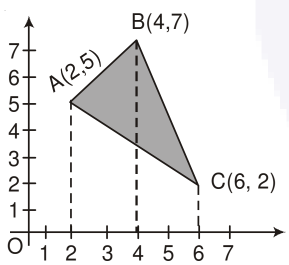
$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i}+\hat{j}+\hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) = $\frac{|-2+1+4|}{\sqrt{(-1)^2+1^2+1^2}} = \sqrt{3}$ 1

28.



Correct Figure 1

Equation of AB : $y = x + 3$

Equation of BC : $y = \frac{-5x}{2} + 17$

Equation of AC : $y = \frac{-3x}{4} + \frac{13}{2}$

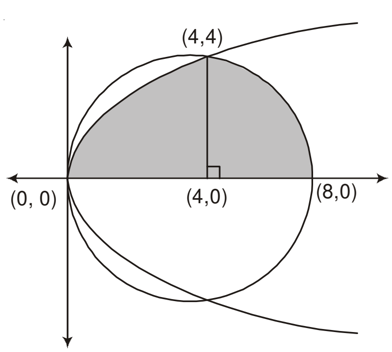
$1\frac{1}{2}$

Required Area $= \int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2}+17\right)dx - \int_2^6 \left(\frac{-3x}{4}+\frac{13}{2}\right)dx$ $1\frac{1}{2}$

$$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4}+17x\right]_4^6 - \left[\frac{-3x^2}{8}+\frac{13x}{2}\right]_2^6$$ $1\frac{1}{2}$

$= 7$ $\frac{1}{2}$

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

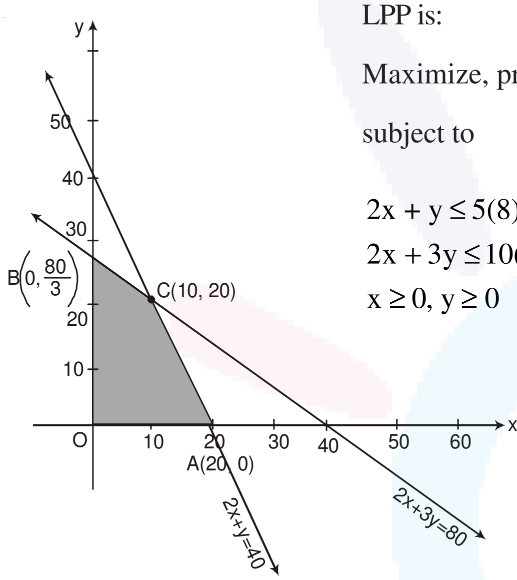
$$\text{Required Area} = \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2-(x-4)^2}\,dx \qquad 1\tfrac{1}{2}$$

$$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2}+\frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8 \qquad 1\tfrac{1}{2}$$

$$= \left(4\pi+\frac{32}{3}\right) \qquad 1$$

Note: A student may also arrive at the answer $\left(8\pi+\frac{64}{3}\right)$ which is double $\left(4\pi+\frac{32}{3}\right)$ because of 'about x-axis'. He/she may be given full marks.

29. Let number of items produced of model A be x and that of model B be y.



LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to

$2x + y \le 5(8)$ i.e., $2x + y \le 40$

$2x + 3y \le 10(8)$ i.e., $2x + 3y \le 80$

$x \ge 0,\ y \ge 0$ 2

Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	$350 \leftarrow$ maximum	

Maximum profit = ₹ 350

when $x = 10,\ y = 20.$ $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.

QUESTION PAPER CODE 65/1/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $3A = \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ $\frac{1}{2}+\frac{1}{2}$

2. Order = 2, degree = 2 $\frac{1}{2}+\frac{1}{2}$

3. (fof) (x) = f(x + 1) = x + 2 $\frac{1}{2}$

$\frac{d}{dx}(fof)(x) = 1$ $\frac{1}{2}$

4. d.c.'s = <cos 90°, cos 135°, cos 45°> $\frac{1}{2}$

$= < 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$ 1

SECTION B

5. $I = \int \sin x \cdot \log(\cos x)\, dx$

$\cos x = t \Rightarrow I = -\int \log t \cdot 1dt$ 1

$= -\left[t.\log t - \int \frac{1}{t}.t dt\right]$ $\frac{1}{2}$

$= t(1 - \log t) + C = \cos x(1 - \log(\cos x)) + C$ $\frac{1}{2}$

6. Let $f(x) = (1 - x^2) . \sin x \cos^2 x$

as $f(-x) = -f(x) \Rightarrow$ f is odd function. 1

$\therefore I = 0$ 1

OR

$I = \int_{-1}^{2} \frac{|x|}{x} dx = \int_{-1}^{0} -1dx + \int_{0}^{2} 1dx$ 1

$= -1 + 2 = 1$ 1

7. As a, b $\in$ R $\Rightarrow$ ab $\in$ R $\Rightarrow$ ab + 1 $\in$ R $\Rightarrow$ a*b $\in$ R $\Rightarrow$ * is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general (a*b)*c $\neq$ a*(b*c) $\Rightarrow$ * is not associative. 1

8. $2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 1

$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ 1

9. A = {2, 4, 6}, B = {1, 2, 3}, A $\cap$ B = {2}

Now, $P(A) = \frac{1}{2}, \; P(B) = \frac{1}{2}, \; P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

10. $y' = be^{2x} + 2y \Rightarrow b = \frac{y' - 2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$\frac{e^{2x}.(y'' - 2y') - (y' - 2y).2x^{2x}}{(e^{2x})^2} = 0$ 1

$\Rightarrow y'' - 4y' + 4y = 0$ or $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ $\frac{1}{2}$

11. Let X: getting an odd number

$p=\frac{1}{2},\ q=\frac{1}{2},\ n=6$ $\frac{1}{2}$

(i) $P(X=5)={}^{6}C_{5}\left(\frac{1}{2}\right)^{6}=\frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X\le 5)=1-P(X=6)=1-\frac{1}{64}=\frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow\ k=\frac{1}{6}$ 1

12. Given $|\hat{a}+\hat{b}|=1$

As $|\hat{a}+\hat{b}|^2+|\hat{a}-\hat{b}|^2=2(|\hat{a}|^2+|\hat{b}|^2)$ 1

$\Rightarrow\ 1+|\vec{a}-\vec{b}|^2=2(1+1)$

$\Rightarrow\ |\hat{a}-\hat{b}|^2=3 \Rightarrow |\hat{a}-\hat{b}|=\sqrt{3}$ 1

OR

$[\vec{a}\ \ \vec{b}\ \ \vec{c}]=\begin{vmatrix}2 & 3 & 1\\ 1 & -2 & 1\\ -3 & 1 & 2\end{vmatrix}$ 1

$= -30$ 1

SECTION C

13. LHS = $\begin{vmatrix}a & b & c\\ a-b & b-c & c-a\\ b+c & c+a & a+b\end{vmatrix}$

$R_1 \to R_1+R_3$ and taking $a+b+c$ common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$ 1

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a-2b+c & b-2c+a & c-a \\ b-a & c-b & a+b \end{vmatrix}$$ 1

$= (a + b + c) [(a - 2b + c) (c - b) - (b - 2c + a) (b - a)]$ 1

$= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc =$ RHS. 1

14. $\tan^{-1}\left(\frac{4x+6x}{1-(4x)(6x)}\right) = \frac{\pi}{4}$ 1

$\Rightarrow \frac{10x}{1-24x^2} = 1 \Rightarrow 24x^2 + 10x - 1 = 0$ $1\frac{1}{2}$

$\Rightarrow x = \frac{1}{12}$ or $-\frac{1}{2}$ 1

as $x = -\frac{1}{2}$ does not satisfy the given equation, so $x = \frac{1}{12}$ $\frac{1}{2}$

15. Clearly $a \le a \ \forall \ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow$ R is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive. $1\frac{1}{2}$

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2, 1) \notin R$

$\Rightarrow$ R is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ $\quad (\because x_1, x_2 \in N)$ $1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \frac{\sqrt{4x-3}-1}{2}$ 1

16. Let the point of contact be $P(x_1, y_1)$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}}$ (slope of tangent)

$\Rightarrow m_1 = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1 - 2}}$ 1

also, slope of given line = 2 = m_2

$m_1 = m_2 \Rightarrow x_1 = \frac{41}{48}$ 1

when $x_1 = \frac{41}{48}, y_1 = \sqrt{\frac{41}{16} - 2} = \frac{3}{4}$ $\quad \therefore P\left(\frac{41}{48}, \frac{3}{4}\right)$ $\frac{1}{2}$

Equation of tangent is: $y-\frac{3}{4}=2\left(x-\frac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y-\frac{3}{4}=\frac{-1}{2}\left(x-\frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

17. $\log(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)=2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right)$$ 2

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right)=\frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right)$$ 1

$$\Rightarrow (x+y)=(x-y)\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{x+y}{x-y}$$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx}-\frac{dv}{dx}=0 \quad ...(1)$$ 1

Now, $\log u = y.\log x$

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx}=y\cdot\frac{1}{x}+\log x\cdot\frac{dy}{dx}\Rightarrow\frac{du}{dx}=x^y\left(\frac{y}{x}+\log x\cdot\frac{dy}{dx}\right) \quad ...(2)$$ 1

Again, $\log v = x \cdot \log y$

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx}=x\cdot\frac{1}{y}\cdot\frac{dy}{dx}+\log y\cdot 1\Rightarrow\frac{dv}{dx}=y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right) \quad ...(3)$$ 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)-y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)=0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx}=\frac{y^x\cdot\log y-x^{y-1}\cdot y}{x^y\cdot\log x-y^{x-1}\cdot x}$$ $\frac{1}{2}$

18. $y=(\sin^{-1}x)^2$

$$\Rightarrow y'=2\cdot\sin^{-1}x\cdot\frac{1}{\sqrt{1-x^2}}$$ 1

$$\Rightarrow \sqrt{1-x^2}\cdot y'=2\sin^{-1}x$$

$$\Rightarrow \sqrt{1-x^2}\cdot y''+y'\cdot\frac{1}{2\sqrt{1-x^2}}(-2x)=\frac{2}{\sqrt{1-x^2}}$$ 2

$$\Rightarrow (1-x^2)\cdot y''-xy'=2 \text{ or } (1-x^2)\cdot\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0.$$ 1

19. Let $I=\int_0^a f(a-x)\,dx$

Put a – x = t ⇒ –dx = dt $\frac{1}{2}$

$$I=-\int_a^0 f(t)\,dt=\int_0^a f(t)\,dt=\int_0^a f(x)\,dx$$ $\frac{1}{2}$

II part.

$$I=\int_0^\pi\frac{x\sin x}{1+\cos^2x}dx$$

$$\Rightarrow \quad I=\int_0^\pi\frac{(\pi-x)\cdot\sin x}{1+\cos^2x}dx$$

$$\Rightarrow \quad 2I=\int_0^\pi\frac{\pi\cdot\sin x}{1+\cos^2x}dx$$ $1\frac{1}{2}$

Put cos x = t ⇒ –sinx dx = dt

$$\Rightarrow \ I = -\frac{\pi}{2}\cdot\int_{1}^{-1}\frac{dt}{1+t^2} = \frac{\pi}{2}\times 2\times\int_{0}^{1}\frac{dt}{1+t^2}$$

$$= \ \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$ $1\frac{1}{2}$

20. $I = \int \frac{\cos x}{(1+\sin x)\,(2+\sin x)}dx.$ Put sin x = t $\frac{1}{2}$

$$= \int\frac{dt}{(1+t)\,(2+t)} = \int\left(\frac{1}{1+t} - \frac{1}{2+t}\right)dt$$ 2

$$= \ \log\left|\frac{1+t}{2+t}\right| + c = \log\left|\frac{1+\sin x}{2+\sin x}\right| + c$$ $1+\frac{1}{2}$

21. I.F. $= e^{-\int\frac{2x}{1+x^2}dx} = \frac{1}{1+x^2}$ 1

Solution is given by,

$$y\cdot\left(\frac{1}{1+x^2}\right) = \int\frac{x^2+2}{1+x^2}\,dx$$ $1\frac{1}{2}$

$$y\cdot\frac{1}{1+x^2} = \int\left(1+\frac{1}{1+x^2}\right)dx = x + \tan^{-1} x + c$$ $1\frac{1}{2}$

or $y = (1 + x^2)\,(x + \tan^{-1}x + c)$

OR

Given equation can be written as

$$\int\frac{dy}{2e^{-y}-1} = \int\frac{dx}{x+1}$$

$$\Rightarrow \ \int\frac{e^y}{2-e^y}dy = \int\frac{dx}{x+1}$$ 1

$$\Rightarrow \ -\log|2-e^y| + \log c = \log|x+1|$$ $1\frac{1}{2}$

$\Rightarrow (2 - e^y)\,(x + 1) = c$

When $x = 0,\ y = 0 \Rightarrow c = 1$ 1

$\therefore$ Solution is $(2 - e^y)(x + 1) = 1$ $\frac{1}{2}$

22. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\,\sqrt{72}} = -1$ 1

$\Rightarrow \theta = 180^\circ$ or π $\frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

23. Given lines are: $\dfrac{x-1}{-3} = \dfrac{y-2}{\left(\frac{\lambda}{7}\right)} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ 1

As lines are perpendicular,

$(-3)\left(\dfrac{-3\lambda}{7}\right) + \left(\dfrac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$ 1

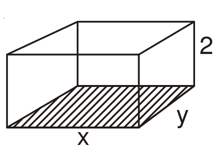
So, lines are

$\dfrac{x-1}{-3} = \dfrac{y-2}{1} = \dfrac{x-3}{2}$ and $\dfrac{x-1}{-3} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ $\frac{1}{2}$

Consider $\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

24. $V = 2xy \Rightarrow 2xy = 8$ (given) 1



$\Rightarrow y = \dfrac{4}{x}$

Now, cost, $C = 70xy + 45 \times 2 \times (2x + 2y)$ 1

$= 280 + 180x + \frac{720}{x}$ 1

$\frac{dC}{dx} = 180 - \frac{720}{x^2}$ 1

$\frac{dC}{dx} = 0 \Rightarrow x = 2m$ $\frac{1}{2}$

$\frac{d^2C}{dx^2} = \frac{1440}{x^3} = 180 > 0$ at $x = 2$ $\frac{1}{2}$

$\Rightarrow$ C is minimum at $x = 2m$. $\frac{1}{2}$

Minimum cost $= 280 + 180(2) + \frac{720}{2} =$ ₹ 1,000 $\frac{1}{2}$

25. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ 2

$$\therefore \; A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore X = A^{-1} \cdot B$ 1

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ 1

$\Rightarrow x = 3, \; y = 1, \; z = 2$ $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_2 \to R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 5R_3$

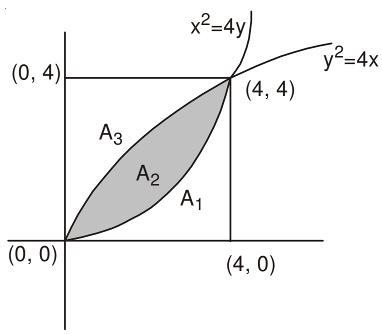
$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{6}{5}R_3, R_2 \to R_2 + \frac{2}{5}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

26. Correct Figure 1



Point of intersection are (0, 0) and (4, 4) 1

here, $A_1 = \int_0^4 \frac{x^4}{4}dx = \frac{16}{3}$ ...(1) 1

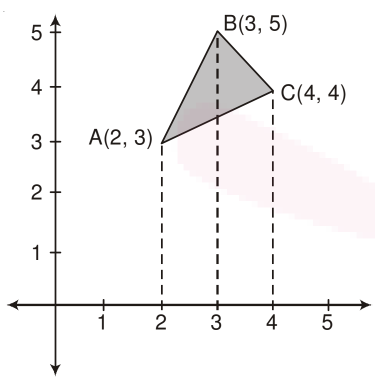
$A_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx = \frac{16}{3}$ ..(2) $1\frac{1}{2}$

$A_3 = \int_0^4 \frac{y^2}{4}dy = \frac{16}{3}$ ...(3) 1

From (1), (2) and (3), $A_1 = A_2 = A_3$. $\frac{1}{2}$

OR

Correct Figure 1



$$\left.\begin{aligned}&\text{Equation of } AB: y = 2x - 1\\&\text{Equation of } BC: y = -x + 8\\&\text{Equation of } AC: y = \frac{1}{2}(x+4)\end{aligned}\right\}$$ $1\frac{1}{2}$

Required Area $= \int_2^3 (2x-1)dx + \int_3^4 (-x+8)dx - \int_2^4 \left(\frac{x+4}{2}\right)dx$ $1\frac{1}{2}$

$= \left[x^2 - x\right]_2^3 + \left[\frac{-x^2}{2} + 8x\right]_3^4 - \frac{1}{2}\left[\frac{x^2}{2} + 4x\right]_2^4$ $1\frac{1}{2}$

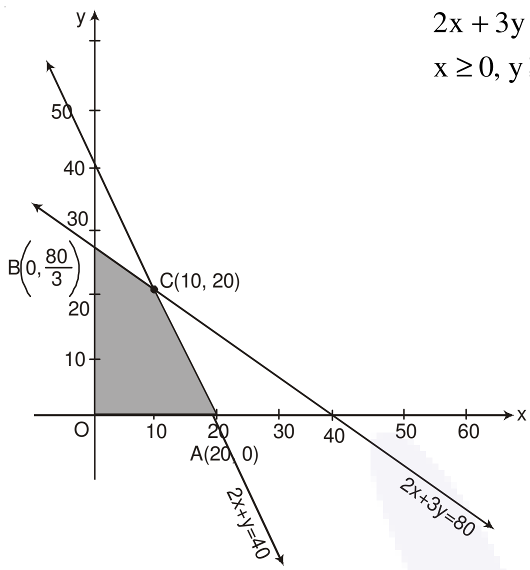
$= 4 + \frac{9}{2} - 7 = \frac{3}{2}$ $\frac{1}{2}$

27. Let number of items produced of model A be x and that of model B be y.

LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to



$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 & \end{array}\right\}$$ 2

Correct Figure 2

Corner point	$z = 15x + 10y$	
$A(20, 0)$	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
$C(10, 20)$	$350 \leftarrow$ maximum	

Maximum profit = ₹ 350

when $x = 10,\ y = 20$. $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.

28. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be a(x + 1) + b(y – 3) + c(z + 4) = 0 ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, 2a – 2b + 4c = 0 or a – b + 2c = 0 ...(2) 1

Also, a + 2b – c = 0 ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

∴ Required plane is –3(x + 1) + 3(y – 3) + 3(z + 4) = 0

∴ –x + y + z = 0

Also vector equation is: $\vec{r}.(-\hat{i}+\hat{j}+\hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) = $\frac{|-2+1+4|}{\sqrt{(-1)^2+1^2+1^2}} = \sqrt{3}$ 1

29. X = no. of kings = 0, 1, 2 $\frac{1}{2}$

$$P(X=0) = P(\text{no king}) = \frac{48}{52}\times\frac{47}{51} = \frac{188}{221}$$ 1

$$P(X = 1) = P(\text{one king and one non-king}) = \frac{4}{52}\times\frac{48}{51}\times 2 = \frac{32}{221}$$ 1

$$P(X = 2) = P(\text{two kings}) = \frac{4}{52}\times\frac{3}{51} = \frac{1}{221}$$ 1

Probability distribution is given by

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$\frac{1}{2}$

Now, Mean = $\Sigma X\cdot P(X) = \frac{34}{221}$ or $\frac{2}{13}$ 1

and $Var(X) = \Sigma X^2\cdot P(X) - [\Sigma X\cdot P(X)]^2$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{6800}{48841} \text{ or } \frac{400}{2873}$$ 1