KINEMATICS

Displacement

The **displacement** of a particle is defined as the difference between its final position and its initial position. We represent the displacement as Δx.

$$\Delta x = x_f - x_i$$

The subscripts *i* and *f* refer to be *initial* and *final* positions.

Average Velocity and Average Speed

The *average velocity* of an object travelling along the *x*–axis is defined as the ratio of its displacement to the time taken for that displacement.

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Average Acceleration is defined as the ratio of the change in velocity to the time taken.

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Instantaneous Velocity **:** The magnitude of instantaneous velocity at a given instant is called instantaneous velocity at that instant.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

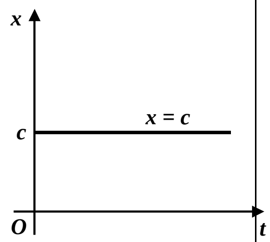
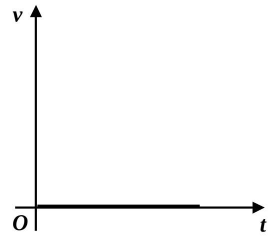
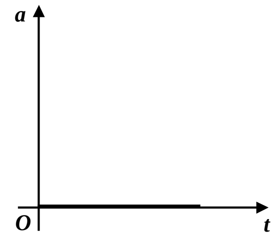
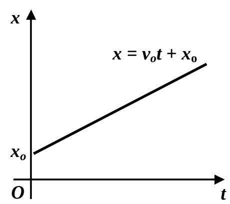
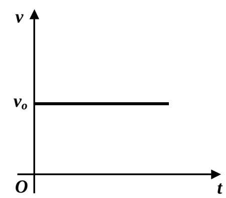
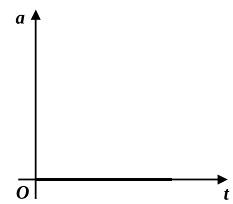
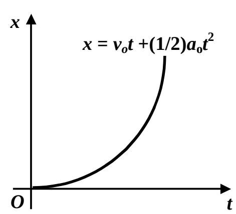
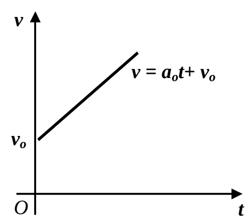
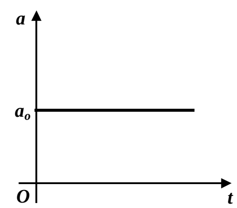
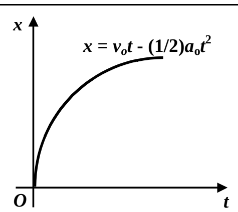
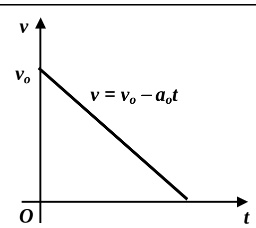
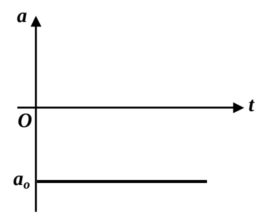
The *instantaneous velocity function* is the derivative with respect to the time of the displacement function.

$$v(t) = \frac{dx(t)}{dt}$$

Instantaneous Acceleration is defined analogous to the method for defining instantaneous velocity. That is, instantaneous acceleration is the value approached by the average acceleration as the time interval for the measurement becomes closer and closer to zero.

The *Instantaneous acceleration* function is the derivative with respect to time of the velocity function

$$a(t) = \frac{dv(t)}{dt}$$

	Displacement	Velocity	Acceleration
1. At rest			
2. Motion with constant velocity			
3. Motion with constant acceleration			
4. Motion with constant deceleration			

MOTION WITH CONSTANT ACCELERATION

The *equations of kinematics* are summarized as

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x = x_0 + \frac{1}{2}(v_0 + v)t$$

Where, x_0 = Initial position coordinate

x = Final position coordinate

v = Final velocity

a = Acceleration (constant)

t = Elapsed time

FREE FALL

Motion that occurs solely under the influence of gravity is called *free fall*.

The *equations of kinematics* may be modified as

$$v = v_0 - gt$$

$$y = y_0 + \frac{1}{2}(v_0 + v)\,t$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

The signs of v and v_0 are determined by their directions relative to the chosen $+y$ axis.

For *two–dimensional motion* in the plane, the x and y components of these equations are:

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$x = x_0 + v_{0x} + \frac{1}{2} a_x t^2 \qquad y = y_0 + y_{0y} t + \frac{1}{2} a_y t^2$$

$$x = x_0 + \frac{1}{2}(v_{0x} + v_x)t \qquad y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

$$v_x^2 = v_{ox}^2 + 2a_x(x - x_0) \qquad v_y^2 = v_{oy}^2 + 2a_y(y - y_o)$$

PROJECTILE MOTION

A projectile motion near the surface of the earth consists of two independent motions, a *horizontal motion at constant speed and a vertical one subject to the acceleration due to gravity*.

The equations of kinematics for projectile motion are

$$x = v_{ox} t \qquad (15)$$

$$v_y = v_{oy} - gt \qquad (16)$$

$$y = y_o + v_{oy}t - \frac{1}{2}gt^2 \qquad (17)$$

$$v_y^2 = v_{oy}^2 - 2g\,(y - y_o) \qquad (18)$$

IMPORTANT

1. The *time of flight* is given by

$$T = \frac{2v_o \sin\theta}{g}$$

2. The *horizontal range* is given by

$$R = \frac{v_o^2 \sin 2\theta}{g}$$

3. The *maximum height* of the projectile is

$$H = \frac{v_o^2 \sin^2\theta}{2g}$$

4. The *trajectory* of a projectile is a parabola

$$y = x\tan\theta - \frac{1}{2}\frac{g}{(v_0\cos\theta)^2}x^2$$

CIRCULAR MOTION

Let us consider a particle which moves in a circular path of radius *r* and constant speed *v*, angular **speed is** given by

$$\omega = v/r$$

Thus, acceleration, $a = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$

Tangential and Normal Components of Acceleration in Two Dimensions

1. The *velocity vector* **is** always *tangential* **to** the path.
2. The *acceleration vector* **may** have two components: one tangential to the path and one perpendicular to the path.
   (a) The component of the acceleration parallel to the path is due to a change in speed. When the speed is increasing, the tangential component a_t points in the *same direction* as the velocity; when the speed is decreasing, the tangential component points *opposite* to the velocity.
   (b) When the path of an object curves, there is a component of the acceleration perpendicular to the velocity. This component of the acceleration a_c points toward the inside of the curve.
   (c) The *total acceleration* is the vector sum of the *tangential* and *centripetal* **components.**