ROTATIONAL MOTION

ROTATIONAL KINEMATICS

Angular displacement θ, which is given by

$$\theta = \frac{s}{r}$$

The ***average angular velocity*** of the body for a finite time interval is given by

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

The unit of angular velocity is *radian per second* (rad/s).

The ***instantaneous angular velocity*** is defined as

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

In terms period T and frequency *f* the angular velocity is given by

$$\omega = \frac{2\pi}{T} = 2\pi f$$

The *relation* between ***linear speed*** and ***angular speed*** is given by

$$\omega = \frac{v}{r}$$

Or $v = \omega r$

Although all particles have the same angular velocity, their speeds increase linearly with distance from the axis of rotation.

Equations of rotation kinematics

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

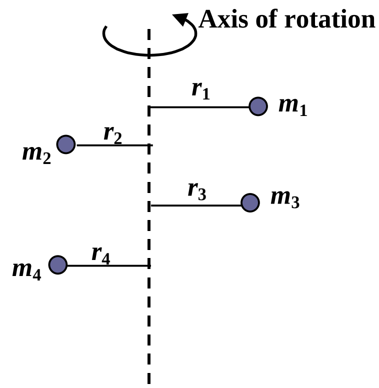
The above equations are called the ***equations of rotation kinematics*** for *constant* angular acceleration.

MOMENT OF INERTIA

For a ***discrete system*** of particles the moment of inertia is defined as

$$I = \Sigma m_i r_i^2$$

Where m_i is the mass of the i^{th} particle and r_i is the perpendicular distance of the i^{th} particle from the axis of rotation.



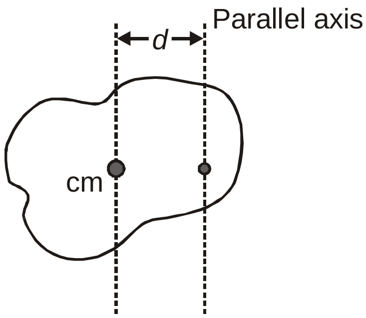
A discrete system of particles

The Parallel Axis Theorem

It states that the moment of inertia of a body about an axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of perpendicular distance between the two axes.

$$I = I_{cm} + md^2$$

I_{cm} = Moment of inertia of the body about its centre of mass



I = Moment of inertia of the body about a parallel axis

m = Total mass of the body

d = Perpendicular distance between two parallel axes.

Perpendicular Axis Theorem

Let the moment of inertia about the x and y axes to be I_x and I_y. The perpendicular axis theorem states that

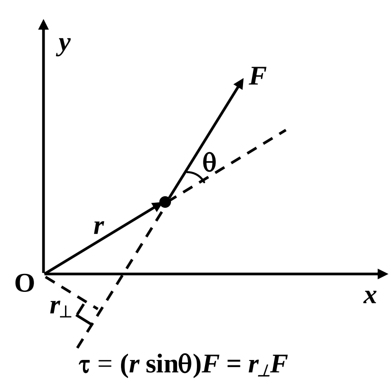
$$I_z = I_x + I_y$$

TORQUE

The ***torque*** of a force F that acts at a distance r from the origin is defined as

$$\tau = r\,F\,\sin\theta$$

Where θ is the angle between th



NEWTON'S SECOND LAW

Since torque is a rotational analog of force, therefore, Newton's second law for rotational motion is given by

$$\tau_{net} = I\alpha$$

ROTATIONAL WORK AND ENERGY

The ***rotational work*** done by a force about the fixed axis of rotation is defined as

$$W_{rot} = \int \tau d\theta$$

Where τ is the torque produced by the force, and $d\theta$ is the infinitesimally small angular displacement about the axis.

The ***rotational kinetic energy*** of a body about a fixed rotational axis is defined as

$$K_{rot} = \frac{1}{2} I\omega^2$$

Where I is the moment of inertia about the axis.

Work – Energy Theorem

In complete analog to the work energy theorem for the translator motion, it can be stated for rotational motion as:

$$W_{rot} = \Delta K_{rot}$$

The net rotational work done by the forces is equal to the change in rotational kinetic energy of the body.

Rotational Power

In complete analog with the linear motion, the instantaneous *rotational power* is defined as

$$P_{rot} = \frac{dW_{rot}}{dt} = \boldsymbol{\tau}.\boldsymbol{\omega}$$

ROLLING MOTION

In a pure rolling motion.

$v_c = \omega R$

Where ω is the angular velocity of the wheel about its center of mass, and v_c is the linear velocity of the *center of mass*.

Since rolling is a combination of translation of the center and rotation about the center, therefore, velocity of any point on the rim is the vector sum

$$\boldsymbol{v} = \boldsymbol{v}_c + \boldsymbol{v}'$$

Where $\boldsymbol{v}_c$ the velocity of the center of mass, and v is′ is the velocity of the particle *with respect* to center of mass.

Kinetic Energy of a Rolling Body

The ***total kinetic energy*** of a rolling body is given by

$$K = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2$$

In pure rolling motion, $v_c = \omega R$

$$\therefore \quad K = \frac{1}{2} m(\omega R)^2 + \frac{1}{2} I_c \omega^2$$

Or $K = \frac{1}{2} (I_c + mR^2)\, \omega^2$

ANGULAR MOMENTUM

Angular momentum about the origin is defined as

$$\vec{L} = \vec{r} \times \vec{p}$$

The *magnitude* of momentum is given by

$$L = rp \sin\theta$$

Conservation of Angular Momentum

If $\tau_{ext} = 0$ $\qquad \dfrac{d\boldsymbol{L}}{dt} = 0$

Thus, $\boldsymbol{L}$ = constant

Angular Impulse

Angular impulse is defined as

$$\tau = \int \tau_{ext} dt$$