GRAVITATION

NEWTON'S LAW OF GRAVITATION

The force of interaction between any two particles having masses m_1 and m_2 separated by a distance r is attractive and acts along the line joining the particles. The magnitude of the force is given by

$$F = \frac{Gm_1m_2}{r^2}$$

where G is a universal constant. Its numerical value is 6.67 × 10^{-11} Nm^2/kg^2.

Variation in 'g' (Acceleration due to gravity)

(a) **With altitude**

As for an external point a spherical distribution of mass behaves as if the whole of its mass were concentrated at the centre, i.e., $g = I = (GM/r^2)$

so at the surface of earth

$$g = GM/R^2$$

and for a height h above the surface of earth

$$g' = [GM/(R + h)^2] \quad [\text{as } r = R + h]$$

$$\therefore \quad \frac{g'}{g} = \frac{R^2}{(R+h)^2} \quad \text{i.e.} \quad g' = \frac{g}{[1+(h/R)]^2}$$

So with increase in height 'g' decreases. However, if h << R

$$g' = g\left[1+\frac{h}{R}\right]^{-2}, \quad \text{i.e.,} \quad g' = g\left[1-2\frac{h}{R}\right]$$

(b) **With depth**

As in case of spherical distribution of mass for an internal point

$$g = I = (GM/R^3)r$$

So, at the surface of earth $g = (GM/R^2)$

and for a point at a depth d below the surface,

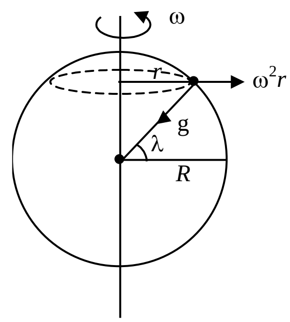
$$g' = \frac{GM}{R^3}(R-d) \quad [\text{as } r = R - d]$$

$$\frac{g'}{g} = \left(\frac{R-d}{R}\right), \quad \text{i.e.,} \quad g' = g\left[1-\frac{d}{R}\right]$$

So, with increase in depth below the surface of earth 'g' decreases and at the centre of earth it becomes zero.

(c) Due to rotation of earth

The earth rotates from west to east on its axis. Everybody on the surface of earth experiences a centrifugal force in the reference frame of the earth. The effective value of acceleration due to gravity at a place of latitude λ is given by



$$g' = \sqrt{g^2 + (\omega^2 r)^2 + 2g\omega^2 r\cos(\pi - \lambda)}$$

Now $\omega^2 r << g$

$\therefore \quad g' \cong \sqrt{g^2 - 2g\omega^2 R\cos^2\lambda} \qquad (\because r = R\cos\lambda)$

or $\quad g' \approx g - \omega^2 R\cos^2\lambda$

At equator $\lambda = 0^o$ i.e. $g' \approx g - \omega^2 R$

At the poles $\lambda = 90^o$ i.e. $g' = g$

Note that the vector g′ is not exactly towards the center of earth.

GRAVITATIONAL POTENTIAL ENERGY

Assuming potential energy at infinity to be zero

$$U = -\frac{GMm}{r}$$

The above equation gives the potential energy of a particle of mass *m* separated from the center of earth by a distance *r*.

MECHANICAL ENERGY OF AN ORBITING BODY

Consider a satellite of mass *m* orbiting around earth in a circular orbit of radius *r*. The total mechanical energy of the system (*earth* + *satellite*) is the sum of its *potential energy* and the *kinetic energy* of the satellite.

$$E = \frac{1}{2}mv_0^2 - G\frac{Mm}{r}$$

where *M* is the mass of earth, and v_0 is the orbital velocity of the satellite.

The *orbital velocity* of a satellite is given by

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2} \qquad \text{or} \quad v_0 = \sqrt{\frac{Gm}{r}}$$

The *kinetic energy* is, therefore, given by

$$K = \frac{1}{2}mv_0^2 = +G\frac{Mm}{2r}$$

ESCAPE SPEED

It is the *minimum* velocity required to escape from the gravitational field of the planet.

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

KEPLER'S LAWS

First Law

The planets move around the sun in elliptical orbits with the sun at one focus.

Second Law

The line joining the sun to a planet sweeps out equal areas in equal time.

Third Law

The square of the period of planet is proportional to the cube of its mean distance from the sun.

The mean distance turns out to be the semi-major axis, *a*.

Mathematically, $T^2 \propto a^3$

or $T^2 = \kappa a^3$

where κ is a constant that applies to all planets.