PROPERTIES OF SOLIDS AND LIQUIDS

SOLID & ELASTICITY

Stress, Strain and Elastic Moduli

Stress : Restoring force developed/Area.

Strain : Change in dimension/original dimension.

$$\text{Modulus of Elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

- Greater is modulus of elasticity greater is the stress developed *i.e.*, greater is the restoring force.

Young's Modulus

Young's modulus is a measure of the *resistance* of a solid to a *change* in its *length* when a *force* is applied *perpendicular* to a face.

Young's modulus Y for the material of the rod is defined as the ratio of tensile stress to tensile strain.

so

$$\text{Young's Modulus} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$Y = \frac{\sigma}{\varepsilon} = \frac{F_n / A}{\Delta L / L_o} = \frac{F_n L_o}{A \Delta L}$$

Shear Modulus

The *shear stress* is defined as

$$\text{Shear Stress} = \frac{\text{Tangetial force}}{\text{Area}}$$

$$\tau = \frac{F_t}{A}$$

Where A is the *area* of the surface.

Bulk Modulus

Bulk modulus of elasticity $K = \frac{\text{Normal or compressive stress}}{\text{Volumetric strain}} = -V\frac{\Delta P}{\Delta V}$ or $K = -V\frac{dP}{dV}$

$$\text{Compressibility} = \frac{1}{K}$$

Density and Pressure

The ***density*** □□of a substance is defined as the *mass per unit volume* of a sample of the substance.

$$\square = \frac{M}{V}$$

The *SI* units of density are kg m^{-3}.

Specific Gravity

The **specific gravity** of a substance is the ratio of its density to that of water at 4^{o}C, which is 1000 kg/m^3. Specific gravity is a dimensionless quantity numerically equal to the density quoted in g/cm^3. *For example*, the specific gravity of mercury is 13.6, and the specific gravity of water at 100^{o}C is 0.998.

Pressure

The ***pressure*** exerted by a fluid is defined as the *force per unit area* at a point within the fluid.

$$p = \frac{F}{A}$$

The *SI* unit of pressure Nm^{-2} and is also called ***pascal*** (Pa).

Variation of Pressure with Depth

The pressure p decreases with height y from the bottom of the fluid.

In other words, *the pressure p increases with depth h*, i.e.

$$\frac{dp}{dh} = \rho g$$

Pascal Law

$$p = p_o + \rho gh$$

Pressure at any depth h in a fluid may be increased by increasing the pressure p_o at the surface. It is Pascal's ***Law***.

A pressure applied to a confined fluid at rest is transmitted equally undiminished to every part of the fluid and the walls of the container.

Buoyancy: Archimedes' Principle

If a body is *partially* or *wholly* immersed in a fluid, it experiences an upward force due to the fluid surrounding it.

The phenomenon of force exerted by fluid on the body called ***buoyancy*** and the force is called ***buoyant force***.

A body experiences *buoyant force* whether it *floats* or *sinks*, under its own weight or due to other forces applied on it.

Archimedes Principle

A body immersed in a fluid experiences an upward buoyant force equivalent to the weight of the fluid displaced by it.

The Equation of Continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This is called the ***equation of continuity***. It is a statement of the conservation of mass.

If the fluid is *incompressible*, its density remains unchanged. This is a good approximation for liquid, but not for gases. If $\rho_1 = \rho_2$,

$$A_1 v_1 = A_1 v_2$$

The product Av is the *volume rate of flow* (m^3/s). Figure shows a pipe whose cross section narrows.

Bernoulli's Equation

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

SURFACE TENSION

The surface tension is given by

$$T = F/l$$

. Its unit is Newton/meter and the dimensions are $[MT^{-2}]$.

The value of the surface tension of a liquid depends on the temperature of the liquid, as well as on the medium on the other side of the surface. It decreases with rise in temperature and becomes zero at the critical temperature.

Inter atomic Cohesive and Adhesive Forces

The force of attraction between the molecules of the same substance is called cohesive force, and that between the molecules of different substances is called adhesive force.

Surface Energy

Relation between Surface Tension and Work done in increasing the Surface Area:

$$W = T \times \Delta A$$

Or $T = \frac{W}{\Delta A}$

If $\Delta A = 1$ then $T = W$. Then the work done in increasing the surface area by unity will be equal to the surface tension T

Angle of Contact

When the free surface of a liquid comes in contact of a solid, it becomes curved near the place of contact. The angle inside the liquid between the tangent to the solid surface and the tangent to the liquid surface at the point of contact is called the angle of contact for that pair of solid and liquid.

The angle of contact for water and silver is 90^{o}.

Capillarity

Rise or fall of liquid in a tube of fine diameter.

Ascent formula

$$h = \frac{2T}{R\rho g}$$

Where R radius of the capillary tube and □ is the density of water and *g* is the acceleration due to gravity. This shows that as *r* decreases, *h* increases, that is, narrower the tube, greater is the height to which the liquid rises in the tube.

VISCOSITY

When a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'.

Velocity Gradient and Coefficient of Viscosity

According to Newton, the viscous force F acting between two layers of a liquid flowing in stream-lined motion depend upon two factors:

(i) It is directly proportional to the contact-area A of the layers ($F \propto A$).

(ii) It is directly proportional to the velocity-gradient $\Delta v_x/\Delta z$ between the layers ($F \propto \Delta v_x/\Delta z$).

Combining both these laws, we have

$$F \propto A\frac{\Delta v_x}{\Delta z}$$

Or $$F = \pm \eta A\frac{\Delta v_x}{\Delta z}$$

Where η is a constant called 'coefficient of viscosity' of the liquid.

Dimensions and Unit of Coefficient force Viscosity:

From the above formula, we have

$$\eta = \frac{F}{A(\Delta v_x / \Delta z)}$$

$\therefore$ Dimensions of η = $\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \frac{[MLT^{-2}]}{[L^2T^{-1}]}$ = $[ML^{-1}T^{-1}]$

Its unit is kg/ (meter-second).

Effect of Temperature on Viscosity: The viscosity of liquids decreases with rise in temperature. On the other hand, the viscosity of gases increases with rise in temperature.

Steady Flow of Liquid through a Capillary Tube: Poiseuille's Formula

Poiseuille gave a formula for the volume of a liquid flowing per second through a capillary tube, is given by

$$q = \frac{\pi p a^4}{8\eta k l}$$

Where η is the coefficient of viscosity of the liquid. This is known as 'Poiseuille's formula'.

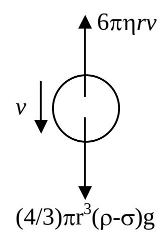
Effective Force on a Body Falling in a Liquid: Stoke's Law

Stokes showed that if a small sphere of radius r is moving with a terminal velocity v through a perfectly homogeneous medium (liquid or gas) of infinite extension, then the viscous force acting on the sphere is

$$F = 6\pi\eta r v$$

Where η is the coefficient of viscosity of that medium.

Calculation of Terminal Velocity:



Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas), whose density is σ and coefficient of viscosity η. When it attains a terminal velocity v, it is subjected to two forces:

(i) Effective force acting downward

$$= V(\rho - \sigma)g = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

(ii) Viscous force acting upward

$$= 6\pi\eta r v$$

Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

$$6\pi\eta r v = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

Or $$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Thus, terminal velocity of the ball is directly proportional to the square of its radius.

THERMAL EXPANSION

When the temperature of a body increases, its size increases.

(1) Coefficient of linear expansion is given by

$$\alpha = \frac{\Delta L}{L\Delta T}$$

$L_{□}$ = L_0 (1 + □□)

(2) Coefficient of superficial expansion is given by

$$\beta = \frac{\Delta A}{A\Delta T}$$

$A_{□}$ = A_0 (1 + □□)

(3) Coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V\Delta T}$$

or $V_{□}$ = V_0 (1 + □□□)

An Isotropic body expands equally in all directions and we can obtain the following relations

□ = 3□, □ = 2□□□

Specific Heat and Heat Capacity

Heat capacity $C = \frac{Q}{\Delta T}$

The SI unit of heat capacity is JK^{-1}.

Specific heat $Q = mc$ □□□□□where c is called the specific heat of the substance.

HEAT CONDUCTION

The transfer of energy arising from the temperature difference between adjacent parts of a body is called ***heat conduction***.

The fundamental law of heat conduction

$$\frac{dQ}{dt} == -kA\frac{dT}{dx}$$

Here $\frac{dQ}{dt}$ is the time rate of heat transfer across the area A, $\frac{dT}{dx}$ is called the ***temperature gradient***, and k is a constant of proportionality called the ***thermal conductivity***.

HEAT RADIATION

The process in which heat is transferred from one place to the other without any intervening medium is called ***radiation***.

Basic Definitions

(i) Perfectly Black Body

A body which absorbs all the radiations incident on it is called a *perfectly* black body.

(ii) Absorptive Power of a surface (*a*)

The ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same time is called the ***absorptive power*** *(a)* of the surface.

(vii) Kirchhoff's Law

The *ratio* of *emissive power* to *absorptive power* is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature. That is,

$$\left(\frac{e}{a}\right)_1 = \left(\frac{e}{a}\right)_2 = \ldots\ldots\ldots\ldots\ldots\ldots = \left(\frac{e}{a}\right)_{perfectly\ black\ body}$$

or $\frac{e}{a}$ = constant

Similarly, $\frac{e_\lambda}{a_\lambda}$ = constant

This *implies* that a good *absorber* is a good *emitter* also. In other words, a good *absorber* of a particular wavelength λ is a good *emitter* of that particular wavelength λ.

(viii) Stefan's law

The radiant energy emitted by a perfectly black body per second per unit area (emissive power) is directly proportional to the fourth power of the absolute temperature of the body. $E = \sigma T^4$

For any other surface, $E = e\sigma T^4$

Where e is the emissivity of the surface, and

σ is the Stefan's constant = $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$.

(IX) Newton's law of cooling

The rate of cooling is proportional to temperature difference between the body and its surroundings provided the temperature difference is not very large from the surrounding. This is called ***Newton's Law of Cooling***.

(X) Wien's Displacement Law

Products of wavelengths λ_m corresponding to *maximum* spectral radiance and temperature T of the body in kelvin is constant.

i.e. $\lambda_m T = b =$ constant

Or $\lambda_m \propto \frac{1}{T}$

Where b is Wien's constant having value of 2.89×10^{-3} m-K