Kinetic Theory of Gases

Kinetic Pressure

$$P = \frac{1}{3}\rho v_{rms}^2$$

where v_{rms} = The ***root mean square (rms)*** speed of the molecules

Kinetic Interpretation of Temperature

From the point of view of kinetic theory, the average kinetic energy of a molecule as

$$K_{av} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

For an ideal gas, the absolute temperature is a measure of the average translational kinetic energy of the molecules.

Different Speeds of Gas Molecules

The motion of molecules in a gas is characterized by any of the following three speeds.

(a) ***Root mean square speed***

$$v_{rms} = \frac{\sqrt{v_1^2 + v_2^2 + \ldots}}{N}$$

$$v_{rms} = \sqrt{\frac{3PV}{\text{mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3P}{\rho}}$$

where M is molecular mass and m is the mass of a single molecule and ρ is the density of gas.

k = Boltzmann constant = 1.38×10^{-23} J/mol.K

(b) ***Most probable speed***

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2P}{\rho}}$$

(c) ***Average speed***

$$v_{av} = \frac{v_1 + v_2 + \ldots\ldots.}{N}$$

$$v_{av} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{8}{\pi}\frac{kT}{m}} = \sqrt{\frac{8}{\pi}\frac{P}{\rho}}$$

INTERNAL ENERGY OF AN IDEAL GAS

For an ideal gas, the *internal energy* U of an ideal gas depends upon temperature T only and is directly proportional to it,

$$U \propto T$$

Specific Heat and Heat Capacity

Heat capacity $C = \frac{Q}{\Delta T}$

The SI unit of heat capacity is JK^{-1}.

Specific heat $Q = mc$ □□□□□where c is called the specific heat of the substance.

C_P, C_V and γ of an ideal gas

(i) Molar specific heat at constant volume $C_V = \frac{dU}{dT} = \frac{R}{\gamma - 1}$

(ii) Molar specific heat at constant pressure $C_P = C_V + R$

(iii) Adiabatic exponent $\gamma = \frac{C_P}{C_V}$

Degree of Freedom *f*

Degree of freedom (f) is defined as the *number of possible independent ways in which a system can have energy*. The independent motions can be *translational*, *rotational* or *vibrational* or any *combination* of them.

(i) A ***monatomic*** gas has 3 degrees of freedom (all translational). Although a monatomic molecules can also rotate but due to its *small* moment of inertia rotational kinetic energy is *insignificant*.

(ii) A ***diatomic*** gas such as H_2, O_2 etc. are made up of two atoms joined rigidly to one another through a bond. A diatomic molecule has *five* degrees of freedom, *three* translational and *two* rotational.

Law of Equipartition of Energy

According to this law the energy of an ideal gas is equally distributed in each degree of freedom. The average kinetic energy per degree of freedom per molecule is $\frac{1}{2}kT$. The kinetic energy per degree of freedom per mole is $\frac{1}{2}RT$. In general, the kinetic energy per molecule is $\frac{1}{2}f\,kT$ and that per mole is $\frac{1}{2}f\,RT$.

Mixture of Gases

Following results are helpful for a mixture of gases.

$$U_{mix} = U_1 + U_2 + \ldots$$

$$M_{mix} = \frac{n_1 M_1 + n_2 M_2 + \ldots}{n_1 + n_2 + \ldots}$$

$$P_{mix} = P_1 + P_2 + \ldots$$

$$(C_V)_{mix} = \frac{n_1 C_{V1} + n_2 C_{V2} + \ldots}{n_1 + n_2 + \ldots}$$

$$(C_P)_{mix} = (C_V)_{mix} + R$$

$$\gamma_{mix} = \frac{(C_P)_{mix}}{(C_V)_{mix}}$$

$$\frac{n_1 + n_2 + n_3 + \ldots}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} + \ldots$$