ELECTROSTATICS

CHARGE

It is the inherent property of certain fundamental particles. It accompanies them wherever they exist. Commonly known charged particles are proton and electron. The charge of a proton is taken as **positive** and that of electron is taken as **negative**. It is represented by symbol *e*.

$e = 1.6 \times 10^{-19}$ coulomb

Positive and negative sign were arbitrarily assigned by **Benzamin Franklin**. This does not mean that charge of proton is greater than charge of electron.

Quantization of Charge

Electric charges appear only in discrete amounts, it is said to be ***quantized***.

Conservation of Charge

For an ***isolated system,*** the total charge remains constant, charge is *neither* created *nor* destroyed, and it is transferred from one body to the other.

COULOMB'S LAW

The force of interaction of two stationary point charges in vacuum is directly proportional to the product of these charges and inversely proportional to the square of their separation

$$F = \frac{kq_1q_2}{r^2}$$

Where, k is a *constant* which depends on the system of units. Its value in SI unit is

$$k = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

The constant is often written in the form

$$k = \frac{1}{4\pi\varepsilon_0}$$

Where, ε_0 is called the ***permittivity constant*** which is numerically equal to

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

ELECTRIC FIELD

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The ***electric field strength*** **(**$\vec{\mathbf{E}}$**)** at a point is defined as the *force per unit charge experienced by a test charge* q_t*, placed at that point.*

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_t}$$

LINES OF FORCE

The ***electric fieldlines*** or ***lines of force*** are helpful in visualizing field patterns. They provide the following basic information:

(a) The direction of the field is along the tangent to a line of force.

(b) The strength or magnitude of the field is proportional to the number of lines that cross a unit area perpendicular to the line.

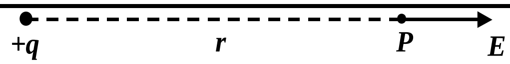
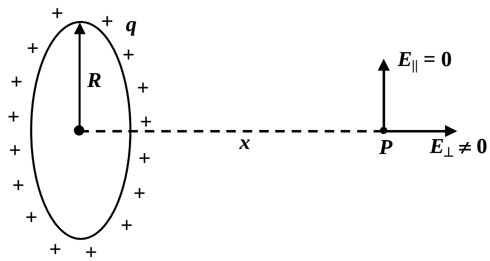
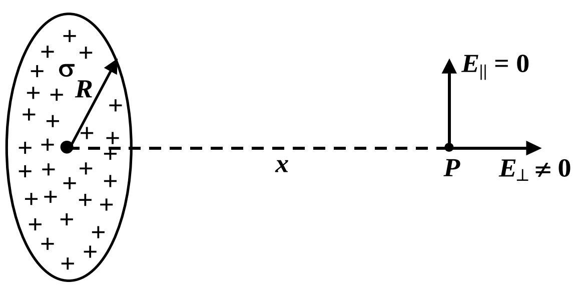
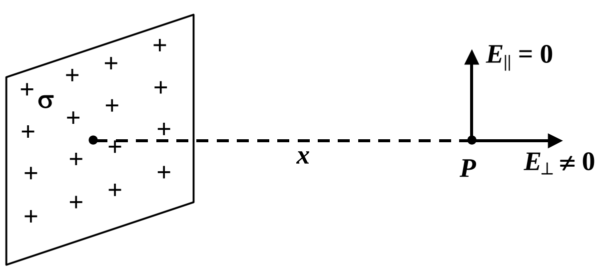
GAUSS' LAW

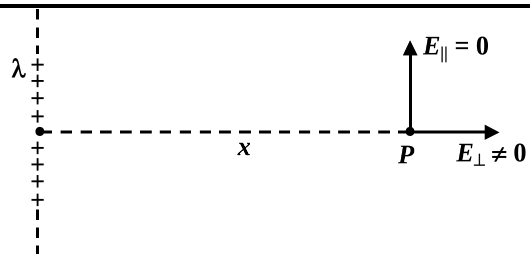
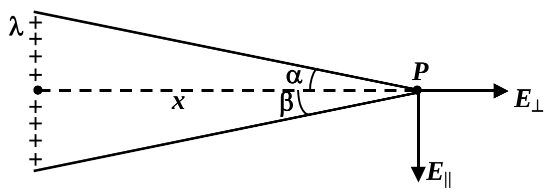
The net flux of $\vec{E}$ through a closed surface equals $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$$

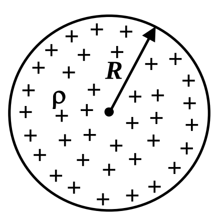
Table 1 Electric field E due to Various Charge Distributions

1. ***Isolated point charge***	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

	
2. ***A Ring of Charge*** 	$E_{\parallel} = 0$ $E_{\perp} = \frac{1}{4\pi\varepsilon_0} \frac{qx}{(R^2 + x^2)^{3/2}}$
3. ***A Disc of Charge*** 	$E_{\parallel} = 0$ $E_{\perp} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$ Where σ is the surface charge density
4. ***Infinite Sheet of Charge*** 	$E_{\perp} = \frac{\sigma}{2\varepsilon_o}$ where σ is the surface charge density
5. ***Infinitely Long Line of Charge***	$E_{\parallel} = 0$ $E_{\perp} = \frac{\lambda}{2\pi\varepsilon_o r}$

<table>
<tr><td>
</td><td>where λ is the linear charge density</td></tr>
<tr><td>6. Finite Line of Charge

</td><td>$E_{\perp}=\frac{\lambda}{4\pi\varepsilon_o x}[\sin\alpha+\sin\beta]$

$E_{\parallel}=\frac{\lambda}{4\pi\varepsilon_o x}[\cos\beta-\cos\alpha]$

where λ is the linear charge density</td></tr>
<tr><td>7. Uniformly charged sphere

</td><td>Inside $0 \le r \le R$

$\vec{\mathbf{E}}=\frac{\rho\vec{\mathbf{r}}}{3\varepsilon_o}$

Outside $r \ge R$

$\vec{\mathbf{E}}=\frac{\rho r}{3\varepsilon_o}\left(\frac{R}{r}\right)^3\hat{r}$

where ρ is the volume charge density.</td></tr>
</table>

POTENTIAL

Electric potential, ΔV is defined as ***the change in electrostatic potential energy per unit charge.***

$$\Delta V = \frac{\Delta U}{q}$$

The SI unit of electric potential is the volt (V).

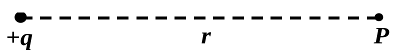
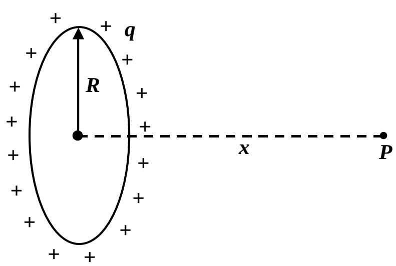
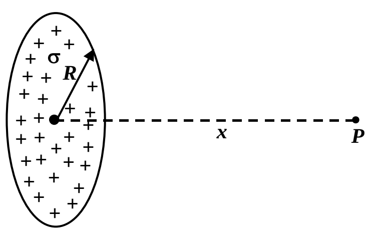
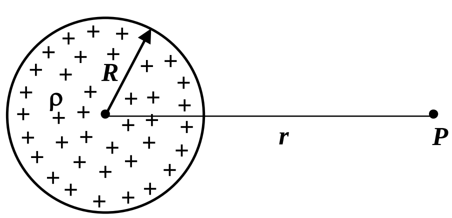
Relationship between E and V

We know that

$$\Delta V = \frac{W_{ext}}{q}$$

Now $\quad W_{ext} = \int \vec{\mathbf{F}}_{ext} \cdot \mathbf{d}\vec{\mathbf{s}}$

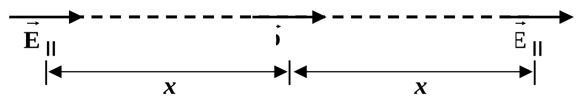
Table 2 *Electric Potential V due to Various Charge Distribution*

1. ***Isolated Charge***  	$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$
2. ***A Ring of Charge***  	$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{\sqrt{R^2 + x^2}}$
3. ***A Disc of Charge***  	$V = \frac{\sigma}{2\varepsilon_o}\left[\sqrt{R^2 + x^2} - x\right]$
4. ***A sphere of Charge***  	**Inside** $0 \le r \le R$ $V = \frac{\rho R^2}{6\varepsilon_o}\left[3 - \frac{r^2}{R^2}\right]$ **Outside** $r \ge R$

	$V = \frac{\rho R^3}{3\varepsilon_o}\left(\frac{1}{r}\right)$ where ρ is the volume charge density.

1. Electric field intensity due to Dipole

(i) *Along the axis*



The $\vec{E}_{\parallel}$ is parallel to $\vec{p}$.

$$\vec{E}_{\parallel} = \frac{2k\vec{p}}{x^3}$$

The direction of electric field along the axis is in the same direction as that of the dipole moment.

(ii) *Along the bisector*

$$\vec{E}_{\perp} = -\frac{k\vec{p}}{y^3}$$

The direction of electric field along the bisector is opposite to that of the dipole moment.

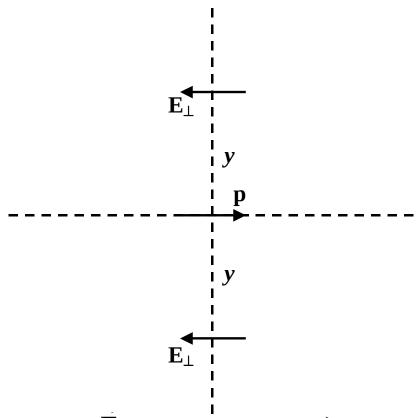
2. Electric Potential Due to a Dipole Moment

(i) Along the axis

$$V_{\parallel} = \frac{2kp}{x^2}$$

(ii) Along the bisector

$$V_{\perp} = 0$$

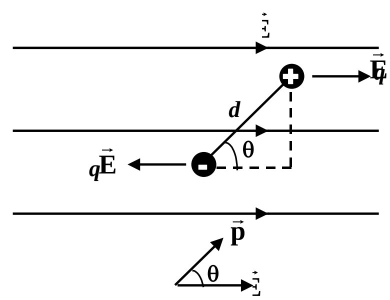


The $\vec{E}_{\perp}$ is anti-parallel to $\vec{p}$

3. Dipole in an External Uniform Field

(i) Torque

If a dipole is oriented at an angle θ to an uniform electric field as shown in the figure, the charges experience equal and opposite forces. So there is no net force on the dipole. However, *there is a net torque on the dipole.*



An electric dipole experiences a torque in an electric field.

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \textbf{(20)}$$

The *magnitude* of the torque is

$$\tau = pE\sin\theta$$

(ii) Potential Energy

The potential energy of a dipole in an external field is given by

$$U = -\vec{\mathbf{p}}.\vec{\mathbf{E}}$$

CAPACITORS

A *capacitor* is a device that stores electrical energy. The capacitance of the capacitor is defined as the *magnitude of the charge on one plate divided by the magnitude of the potential difference V between them*

$$C = \frac{q}{V}$$

Capacitance depends on the size and shape of the plates and the *material between them*. It does not depend on q or V individually. The SI unit of capacitance is the **farad (F)**.

1 farad = 1 coulomb/volt

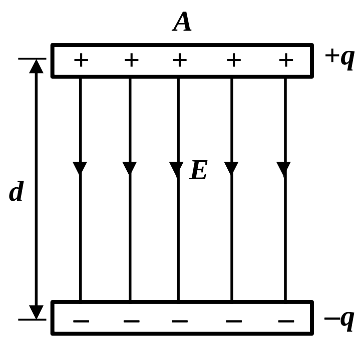
1. Parallel Plate Capacitor

$$\sigma = \frac{q}{A}$$

$$E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A}$$

$$V = Ed = \frac{qd}{\varepsilon_o A}$$

$$C = \frac{q}{V} = \frac{\varepsilon_o A}{d}$$



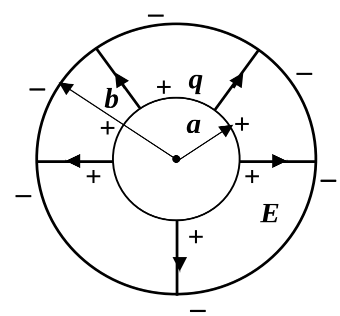
Parallel plate capacitor

2. Spherical Capacitor

$$V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\therefore C = \frac{q}{V} = \frac{4\pi\varepsilon_o}{\frac{1}{a} - \frac{1}{b}} = \frac{4\pi\varepsilon_o}{\frac{1}{a} - \frac{1}{b}}$$

or $$C = \frac{4\pi\varepsilon_o ab}{b - a}$$

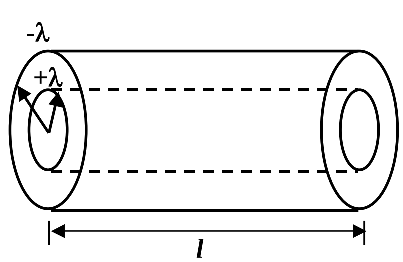


Spherical capacitor

3. Cylindrical Capacitor

$$E = \frac{\lambda}{2\pi\varepsilon_o r} a \le r \le b$$

$$V = V_a - V_b = -\int_b^a E\,dr$$



Cylindrical capacitor

or $V = \frac{\lambda}{2\pi\varepsilon_o} \ln\left|\frac{b}{a}\right|$

$$C = \frac{q}{V} = \frac{\lambda l}{V} = \frac{2\pi\varepsilon_o l}{\ln\left|\frac{b}{a}\right|} \qquad \textbf{(26)}$$

Energy stored in a Capacitor

The energy stored in a capacitor is equal to the work done to charge it.

$$dW = Vdq = \left(\frac{q}{C}\right) dq$$

The charge moves through the wires, not across the gap between the plates.

The total work done to transfer charge Q is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{1}{2} CV^2$$

Since the charge on each plate is unaffected the capacitance in the presence of the dielectric is

$$C = \frac{q_0}{A} = \frac{kq_o}{V_o} = kC_o$$

The capacitance of the capacitor increases by a factor k.