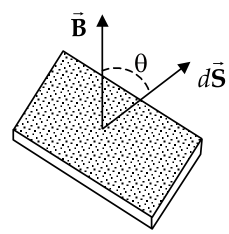
ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

MAGNETIC FLUX

Magnetic flux (Φ_B) through an area $d\vec{S}$ in a magnetic field $\vec{B}$ is defined as

$$\Phi_B = \int \vec{\mathbf{B}}.d\vec{\mathbf{S}}$$

For an elemental area $d\vec{S}$ in a magnetic field $\vec{B}$, the associated magnetic flux is given by



$$d\Phi_B = \vec{\mathbf{B}}.d\vec{\mathbf{S}} = B\, dS \cos\theta$$

As magnetic lines of field are closed curves (i.e., monopoles do not exist), ***total magnetic flux linked with a closed surface is always zero***, i.e.,

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{S}} = 0$$

This law is called **Gauss' law** for magnetism.

INDUCED EMF

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Law I:

Whenever there is a change of flux linked with a circuit, or whenever a moving conductor cuts the flux, an emf is induced in it.

This phenomenon is called ***electromagnetic induction*** and the emf, ***induced emf***. If the circuit is closed the current which flows in it due to induced emf is called ***induced current***.

Law II:

The magnitude of induced emf is equal to the rate of change of flux, i.e.,

$$E = \left|\frac{d\Phi}{dt}\right|$$

The **direction or the sense** of the induced emf (or induced current) is given by Lenz’s law.

LENZ'S LAW

The effect of the induced emf is such as to oppose the change in flux that produces it.

$$\mathrm{E} = -\frac{d\Phi}{dt}$$

Negative sign indicates towards the Lenz's law.

Induced Charge Flow

When a current is induced in the circuit due to the flux change, charge flows through the circuit. The net amount of charge which flows along the circuit is given as:

$$i = \frac{\mathrm{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}$$

Thus, the induced charge is independent of the manner and time in which the flux changes. However, the induced emf and current depend on time.

MOTIONAL EMF

The induced emf,

$$\mathrm{E} = \frac{\Delta\Phi}{\Delta t} = Blv\sin\theta$$

In vector notation $\mathrm{E} = \int\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) d\vec{\mathbf{L}}$

Rotating Coil in Magnetic Field

Induced emf can be produced by changing the orientation of a coil respect to the magnetic field.

$$\Phi = NBS \cos \omega t$$

Where N is the number of turns S is the area of the coil, ω is the angular velocity of rotation.

Self-inductance

It is convenient to express the induced emf in a coil in terms of the current flowing through it rather than the magnetic flux through it,

$$\Phi \propto I \text{ or } \quad \Phi = LI$$

Where, L is a constant of proportionally, called the ***self-inductance*** or simply *inductance* of the coil.

SELF-INDUCTANCE OF A COIL

If a coil of radius R has N turns, its self-inductance of a coil

$$\therefore \quad L = \frac{\Phi}{I} = \frac{\mu_0}{4\pi}(2\pi^2 N^2 R) = \frac{1}{2}\mu_0 \pi N^2 R$$

ENERGY STORED IN AN INDUCTOR

The total energy stored in inductor is given

$$\Rightarrow \boxed{U = \frac{1}{2}LI^2}$$

❖ The ***energy density*** of a magnetic field in free space

$$= \frac{B^2}{2\mu_0}$$

L-R CIRCUIT

(A) Growth of Current

$$\boxed{I = I_0\,(1 - e^{-t/\tau}) \text{ with } I_0 = \frac{E}{R} \quad \text{and} \quad \tau = \frac{L}{R}}$$

Here, I_0 ($= E/R$) is the final value of current I at $t = \infty$, and the quantity τ ($= L/R$) is a constant having dimensions of time. τ is called ***time constant*** of the circuit.

At $t = \tau$, the current is

$$I = I_0 (1 - e^{-1}) = 0.63\ I_0$$

Thus, *in one time constant, the current reaches 63% of the maximum value*

(B) Decay of Current

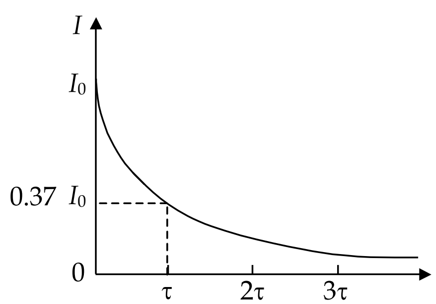
$$\boxed{I = I_0 e^{\frac{-Rt}{L}} = I_0 e^{-t/\tau}}$$

Where $\tau = L/R$ is the *time constant* of the circuit.

We see that the current gradually decreases as time passes. At $t = \tau$,

$$I = I_0/e = 0.37\ I_0$$

The current reduces to 37% of the initial value in one time constant, i.e., 63% of the decay is comlete.



MUTUAL INDUCTION

The appearance of an induced emf in one circuit due to changes in the magnetic field produced by a nearby circuit is called ***mutual induction***.

Let the current through the primary winding at any instant be I_1. Then the magnetic flux linking with the secondary winding,

$$\Phi_2 \propto I_1 \quad \text{or} \quad \Phi_2 = MI_1$$

Thus, the induced emf in the secondary is given as

$$E_2 = \frac{d\Phi_2}{dt} \quad \text{or} \quad E_2 = M\frac{dI_1}{dt}$$

Where, M is the constant of proportionally, called the ***coefficient of mutual induction,***

The SI unit of mutual inductance is **henry**. We may define mutual inductance M as follows:

The mutual inductance M of two colds having self-inductance L_1 and L_2 is given by

$$\boxed{M = k\sqrt{L_1 L_2}}$$

Where k is a constant called ***coefficient of coupling***. For tight or close coupling, $k = 1$, and so $M = \sqrt{L_2 L_2}$. For loose coupling $0 < k < 1$, and hence $M < \sqrt{L_1 L_2}$.

Grouping of Coils

(A)Coils in Series

If two coils of inductances L_1 and L_2 are connected in series with coefficient of coupling
$k = 0$, then potential divides, but the current remains same,

$$e = e_1 + e_2 \qquad \text{or} \qquad L_S \frac{dI}{dt} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$

$$\therefore \qquad \boxed{L_S = L_1 + L_2}$$

(B) Coils in Parallel

In parallel combination, the current divides, but the voltage remains the same,

$$I = I_1 + I_2 \Rightarrow \quad \frac{dI}{dt} = \frac{dI}{dt} + \frac{dI_2}{dt}$$

$$\therefore \qquad \frac{e}{L_P} = \frac{e}{L_1} + \frac{e}{L_2} \qquad \left[\text{as } e = L\frac{dI}{dt}\right]$$

$$\text{or} \qquad \boxed{\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}}$$

ALTERNATING CURRENT

$I = I_{max} \sin\omega t$

Where, I is the value of current at time t and I_{max} is the maximum value of the current and ω is the angular frequency (= $2\pi f$, f being frequency).

Different Forms of a.c. emf

The variation of A.C. quantity is sinusoidal hence can be expressed as:

$e = E_{max}\sin \omega t$ as $\omega = 2\pi f$, we have

$e = E_{max}\sin(2\pi ft)$ since $f = 1/T$, we have

$e = E_{max}\sin\left(\frac{2\pi}{T}t\right)$

R. M. S. value of a current

. If I_1, I_2, I_3, …….. I_n are instantaneous currents r. m. s. value will be

$$I_{\max} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots\ldots\ldots + i_n^2}{n}}$$

We can also expresses this value as

$I_{r.m.s.} = I_{max}/\sqrt{2} = 0.707\ V_{max}$

Similarly $V_{r.m.s.} = V_{max}/\sqrt{2} = 0.707\ V_{max}$

Mean or Average Value of an Alternating current

. The alternating currents average value is given by

$I_{av} = \frac{2I_{max}}{\pi}$ or $I_{av} = 0.637\ I_{max}$

A. C. Circuit containing Resistance Only (Resistive Circuit)

$v = V_{max} \sin \omega t$

And the current $I = \frac{v_{max}}{R}$ sin ωt, $I_{max} = \frac{V_{max}}{R}$

Or $i = I_{max} \sin \omega t$

This shows that the current and voltage are in phase.

The power in a circuit is given by $P\ (= V \times I)$

Hence, $P = I_{max} \sin \omega t \times V_{max} \sin \omega t$

Or $P = \frac{1}{2} I_{max} V_{max} (1 - \cos 2\omega t)$

A. C. Circuit Containing Inductor Only (Inductive Circuit)

$$i = I_{\max}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad \text{where} \qquad I_{\max} = \frac{V_{\max}}{\omega L}$$

The quantity ωL is called inductive reactance or reactance and denoted by

$X_L (= \omega L = 2\pi f L)$.

The power in an inductive circuit is given by

$$P = -\frac{1}{2}V_{\max}I_{\max}\sin 2\omega t$$

As there is no constant term in power expression, hence the average power in a pure inductive circuit is zero. This is called wattles power.

A. C. Circuit Containing Capacitance Only (Capacitive Circuit)

$$i = I_{\max}\cos\omega t = I_{\max}\sin\left(\omega t + \frac{\pi}{2}\right)$$

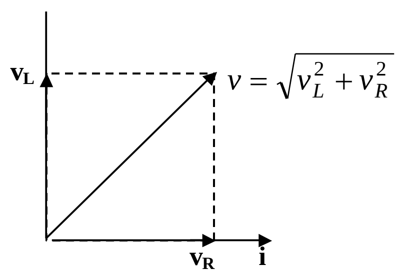
Where $\quad I_{\max} = \frac{V_{\max}}{(1/\omega C)}$

The power in the circuit is given by

$$P = V_{max}\sin\omega t\ I_{max}\cos\omega t \quad = \frac{1}{2}V_{\max}I_{\max}\sin 2\omega t$$

Thus the average power consumed in a pure capacitive circuit is zero.

Resistance and inductance in series A. C. Circuit



The resultant voltage is given by

$$v = \sqrt{v_L^2 + v_R^2}$$

and $i = \dfrac{v}{\sqrt{R^2 + X_L^2}}$

The term $\sqrt{R^2 + X_L^2}$ is called impedance of the circuit and denoted by Z (its units are ohm) i.e.

$$Z = \sqrt{R^2 + X_L^2}$$

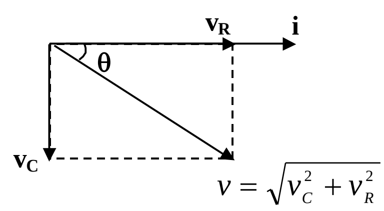
Average power in R and L in series circuit when A. C. current passes is given by

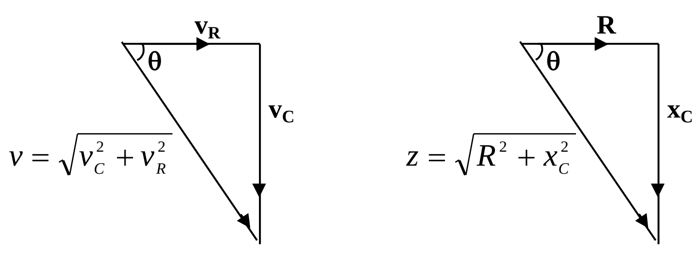
$$P = V_{r.m.s.} I_{r.m.s.} \cos \phi$$

where ϕ is the angle by which vector V leads the vector I.

Resistance and Capacitance in Series A. C. Circuit

The voltage and current vectors are as shown in figure. The voltage and impedance triangles are shown in figure are given as





(a) (b)

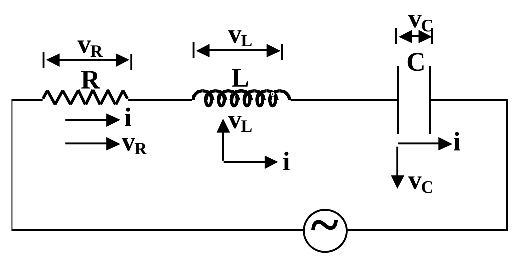
$v = \sqrt{v_R^2 + v_c^2}$ and $Z = \sqrt{R^2 + X_C^2}$

where $X_C = 1/\omega$ C is capacitive reactance.

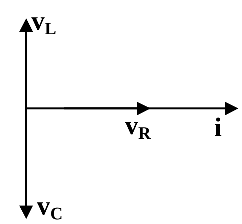
The power consumed $p = v.\ I = V_{r.m.s.} I_{r.m.s.} \cos$□

RLC Series A. C. Circuit

A circuit having resistance R, inductance L and capacitance C in series with an A. C. source shown in figure.



$$v = \sqrt{v_R^2 + (v_L - v_C)^2}$$



and the impedance is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

When ωL > 1/ωC

The net reactance ωL - $\frac{1}{\omega C}$ is positive hence ϕ is also positive as a result current will lag behind voltage or voltage leads the current.

When ωL = 1/ωC

The net reactance is zero hence $\phi = 0$ and the voltage and current are in phase and impedance is equal to the resistance.

When ωL < 1/ωC

The net reactance $\omega L - \frac{1}{\omega C}$ is negative hence ϕ is also negative and the current leads the applied emf.

Series resonance circuit

The effective reactance is inductive or capcitive depending upon $X_L > X_C$ or $X_L < X_C$. The inductive reactance X_L is directly proportional to the frequency while the capacitive reactance is inversely proportional to the frequency. At certain frequency both reactances become equal and this frequency is called resonance frequency. At resonance frequency

$X_L = X_C$ or $\omega L = 1/\omega C$ i.e.

$$\omega_r = \sqrt{\frac{1}{LC}} \quad \text{or} \quad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \text{ Hz}$$

The current and voltages are in phase as $Z = R$. Such circuits are called acceptor circuits. The ratio of v_L or v_C with applied voltage at resonant frequency is called *Q*-factor or voltage magnification and is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

TRANSFORMERS

The primary winding has N_1 turns, and the secondary winding has N_2 turns. When the magnetic flux changes because of changing currents in the two coils, the resulting induced emf's are

$$\varepsilon_1 = -N_1 \frac{d\Phi_B}{dt} \quad \text{and} \quad \varepsilon_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2}$$

$\frac{V_2}{V_1} = \frac{N_2}{N_1}$ (Terminal voltages of transformer primary and secondary)

where V_1 and V_2 are either the amplitudes or the rms values of the terminal voltages. By choosing the appropriate turns ratio N_2/N_1, we may obtain any desired secondary voltage from a given primary voltage.

Efficiency of a Transformer

In an ordinary transformer, there is some loss of energy due to primary resistance, hysteresis in the core, eddy currents in the core etc. The efficiency of a transformer is defined as

$$\eta = \frac{\text{output power}}{\text{input power}}$$

Efficiencies of the order of 99% can be easily achieved.