OPTICS

Laws of Refraction (Snell's Law)

(i) The incident ray, the refracted ray and the normal to the refracting surface at the point of incidence all lie in the same plane.

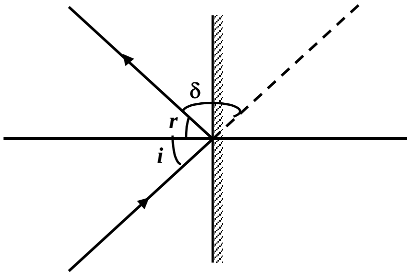
(ii) The ratio of the sines of the angle of incidence (*i*) and that of the angle of refraction (*r*) is a constant quantity for two given media.

Reflection from a Plane-surface

1. When a real object is placed in front of a plane mirror, the image is always erect, *virtual and of same size as the object. It is at same distance behind the mirror as the object is infront of it.*
2. If an object moves *towards* (or *away* from) a plane mirror at speed *v*, the image will also *approach* (or *recede*) at same speed *v*

3. The image formed by a plane mirror suffers **lateral-inversion**

4. **Deviation** (δ) is defined as the angle between directions of incident ray and emergent ray. So if light is incident at an angle of incidence *i*.



Deviation produced by a plane mirror is
$\delta = 180 - 2\theta$

If a ray is incident at an angle *i*, then the deviation is given by

$$\delta = 180 - (i + r) = (180 - 2i)$$

The deviation is maximum for normal incidence.

$\delta_{max} = 180^o$ when $i = 0$

Reflection from Curved Surfaces

Mirror formula:

1. $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ **(4)**

2. Lateral Magnification, $m = \frac{-v}{u}$ **(5)**

6. In case of spherical mirror if $R \to \infty$ (i.e., it becomes plane), $f = R/2 = \infty$, the mirror formula

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ reduces to $\frac{1}{v}+\frac{1}{u}=0$ i.e., $v = -u$

i.e., image is at same distance behind the mirror as the object is infront of it.

7. In case of spherical mirrors if object distance (x_1) and image distance (x_2) are measured from focus instead of pole, $u = (f + x_1)$ and $v = (f + x_2)$ so the mirror formula

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ reduces to $\frac{1}{(f+x_2)}+\frac{1}{(f+x_1)}=\frac{1}{f}$

which on simplification gives

$$x_1x_2 = f^2 \qquad \textbf{(6)}$$

This result is called **'Newton's formula'.**

9. Concave mirror behaves as convex lens (both convergent) while convex mirror behaves as concave lens (both divergent).

10. As **convex mirror** gives erect, virtual and diminished image, field of view is increased. This is why it is used as rear-view mirror in vehicles. Concave mirrors give

enlarged erect and virtual image (if object is between F and P) so are used by dentists for examining teeth.

Apparent shift

$$h' = \frac{h}{\mu}$$

The ***apparent shift*** in the position of the source is

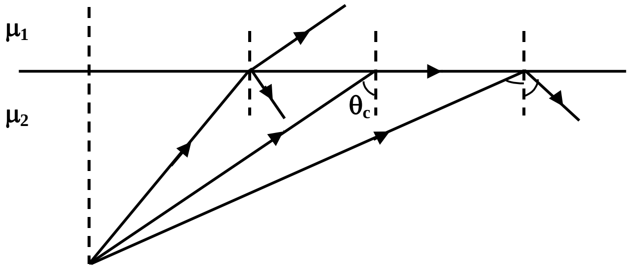
$$s = h - h' = h\left(1 - \frac{1}{\mu}\right) \qquad \textbf{(8)}$$

Total Internal Reflection

Applying Snell's law at the critical angle

$$\mu_2 \sin\theta_c = \mu_1$$

or $$\theta_c = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right) \qquad \textbf{(10)}$$



Critical angle (θ_c) and total internal reflection.

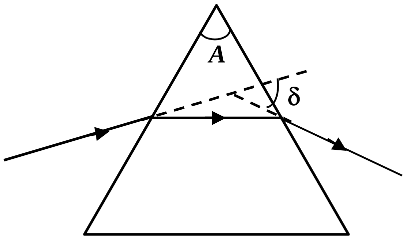
If the rarer medium is air,

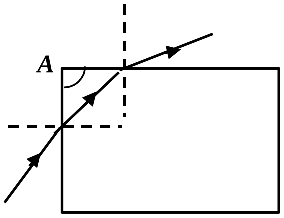
$\mu_1 = 1$.

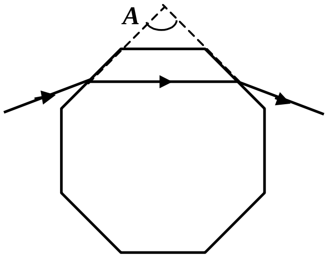
Then, the values of critical angle for glass and water are given below.

PRISM

It is an optical medium bounded by two plane refracting surfaces inclined to each other at a suitable angle.







Deviation Produced by a Prism

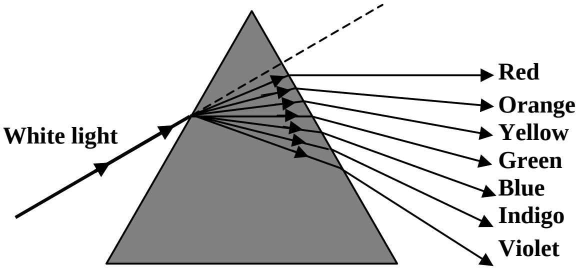
The angle of deviation δ is given by

$$\delta = A(\mu - 1)$$

The refractive index of the prism is given by

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left[\frac{\delta_{\min} + A}{2}\right]}{\sin\left[\frac{A}{2}\right]} \qquad \textbf{(11)}$$

Dispersion of Light by a Prism



The dispersion of light takes place because the *refractive index* μ*of the medium depends on the wavelength of light*. An approximate empirical relation is given by **Cauchy's formula**,

$$\mu = A + \frac{B}{\lambda^2}$$

Where, A and B are constants.

Total ***angular dispersion*** is

$\theta = \delta_v - \delta_r$.

Where, δ_v and δ_r are the deviations for violet and red light respectively.

Dispersive Power of a Prism

The ratio of dispersion to mean deviation is called dispersive power,

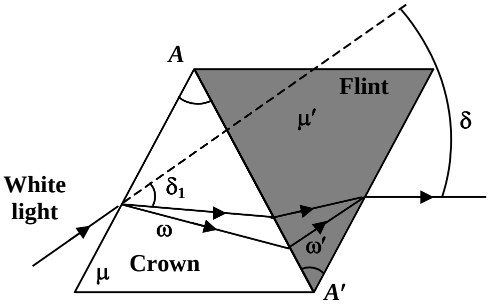
$$\omega = \frac{\theta}{\delta_y} = \frac{\delta_v - \delta_r}{\delta_y}$$

Where δ_y is the deviation of *yellow light* (whose wavelength is considered as mean of all the wavelengths present)

For a prism, $\delta = (\mu - 1)A$

$$\therefore \qquad \omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \qquad \textbf{(12)}$$

Deviation without Dispersion (Achromatic Prism)



It is possible to combine two prisms of different materials in such a way that each cancels the dispersion due to the other. Thus, the net dispersion is zero but a deviation is produced. The required condition is

$(\mu_v - \mu_r)\, A = (\mu'_v - \mu'_r)\, A'$

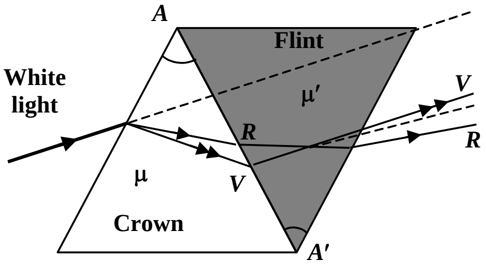
which is equivalent to

$\omega\delta = \omega'\delta'$ **(13)**

where δ is the deviation for the mean ray.

Dispersion without Deviation

Two prisms can be combined in such a way that the deviation of the mean ray produced by one is equal and opposite to that produced by the other. Such a combination is called a ***direct vision prism***. The required condition is



$(\mu - 1)\, A = (\mu' - 1)\, A'$ **(14)**

REFRACTION ON CURVED SURFACES

1. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

2. Lateral Magnification, $m = \frac{\mu_1 v}{\mu_2 u}$

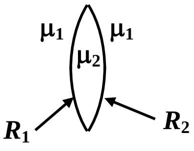
LENSES

Lens formula

1. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

2. Lateral Magnification, $m = \frac{v}{u}$

3. *Lensmaker's* formula



$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Power of a Lens

Power of a lens is defined as $P = \frac{1}{f}$ where □ is the refractive index of the medium and f is the focal length of the lens in that medium. The unit of power is dioptre.

Equivalent Focal Length of Lens Combination

If n number of lenses of focal lengths f_1, f_2, f_3,…f_n are joined together then the equivalent focal length of the combination is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\ldots+\frac{1}{f_n}$$

In terms of power $P = p_1 + p_2 + \ldots + p_n$

Sign Convention

Focal length of converging lens is taken as positive and that of the diverging lens is taken as negative.

$$\frac{1}{F}=\frac{1}{f_l}+\frac{1}{f_m}+\frac{1}{f_l}=\frac{2}{f_l}+\frac{1}{f_m}$$

where f_l= focal length of lens and

f_2 = focal length of mirror

Optical Instruments

Magnifying power of a simple microscope

The magnifying power of simple microscope is

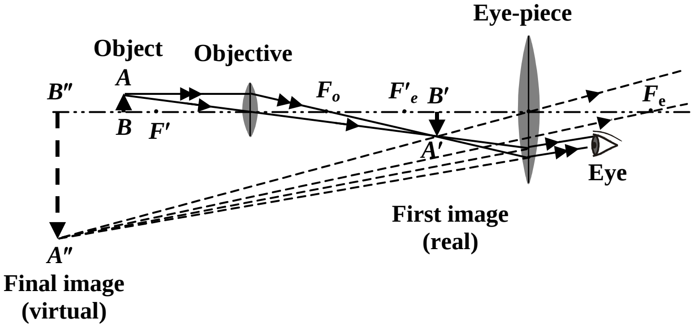
$$M=1+\frac{D}{f}$$

The smaller the focal length *f*, the larger is the magnifying power *M*.

Compound Microscope

(i) *Objective* — a convex lens of very short focal length.

(ii) *Eye-piece*— a convex lens of slightly longer focal length.



The final image *A″B″* is inverted, virtual, enlarged and at a distance *D* to ∞ from eye. The magnifying power is minimum when final image is at ∞,

$$(M)_{\min} = -\frac{v}{u}\frac{D}{f_e}$$

The *MP* is maximum, when final image is at distance *D*,

$$(M)_{\max} = -\frac{v}{u}\left[1+\frac{D}{f_e}\right]$$

Astronomical Telescope

A telescope is used to see the details of distant objects. The magnifying power M for normal adjustment is

$$M = \frac{f_o}{f_e}$$

WAVE OPTICS

Huygen’s Principle

1. Every point on a wavefront vibrates in same phase with same frequency.
2. Every point on a wavefront acts like a new independent source and sends at a spherical wave, called a secondary wave.
3. Wave fronts move in space with the velocity of wave in that medium.

Coherent Sources

Two sources are ***coherent*** if they have the *same frequency* and a *constant phase difference*. In this case, the total intensity I is not just the sum of individual intensities $I_1$and

I_2 due to the two sources but includes an interference term whose magnitude depends on the phase difference.

$$I = I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2}\cos\phi}_{\text{Interference term}}$$

where ϕ is the phase difference.

INTERFERENCE: YOUNG'S DOUBLE SLIT EXPERIMENT

Fringe Width

It is defined as the distance between two successive *maxima* or *minima*.

$$\therefore \quad \omega = y_{n+1} - y_n = (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$$

or $\boxed{\omega = \frac{\lambda D}{d}}$

Optical Path

Optical path length–

$$L = c_o \times \left[\frac{d}{c}\right] = \mu d \qquad \left(\text{because } \mu = \frac{c_o}{c}\right)$$

Phase Difference = $\frac{2\pi}{\lambda}$ (optical path difference)

Intensity Distribution

When two coherent light waves of intensity I_1 and I_2 with a constant phase difference ϕ superimpose, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

In YDSE, usually the intensities I_1 and I_2 are equal i.e. $I_1 = I_2 = I_o$

Then $I = 2I_o(1 + \cos\phi)$

or $I = 4I_o\cos^2(\phi/2)$

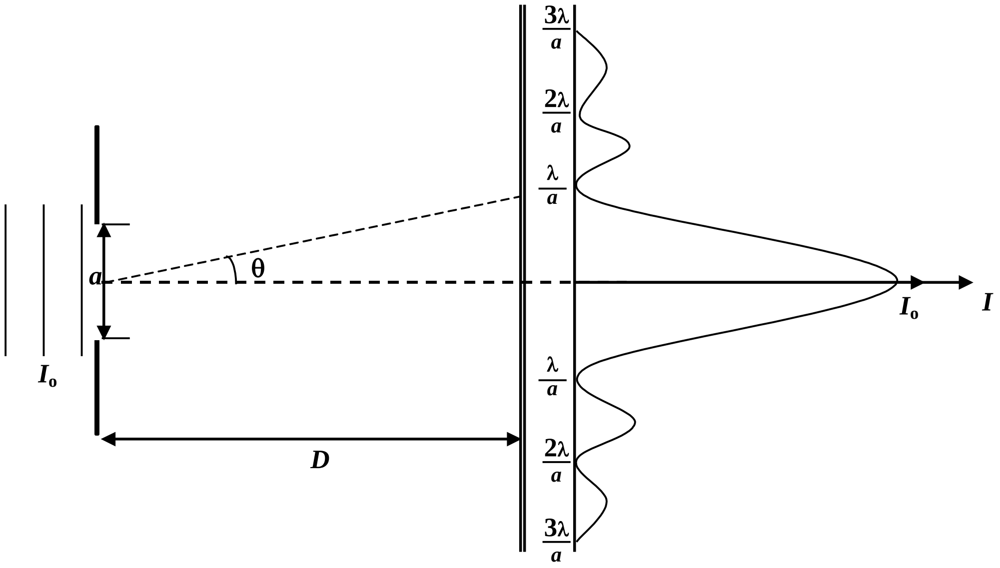
For *maxima*: $\phi = 2n\pi \Rightarrow I_{max} = 4I_o$

For *minima*: $\phi = (2n - 1)\pi \Rightarrow I_{min} = 0$

DIFFRACTION

When light passes through an opening of size comparable to its wavelength it no longer propagates in straight path but spreads out through the opening. This spreading action of waves is called ***diffraction***.

When the double slit in YDSE is replaced by a single narrow slit, a broad pattern with a central bright region is observed. On both sides, there are *alternate dark* and *bright* regions, the intensity becoming weaker away from the centre.



Light intensity distribution on the screen when it passes through an opening of size comparable to its wavelength.

The figure shows the *intensity distribution* on the screen due to diffraction of light waves of wavelength λ through a rectangular opening of width *a*.

The *angular position* (θ) of *n*th diffraction minima is given by

$a \sin \theta = n\lambda$

where, $n = 1, 2, 3, \ldots\ldots\ldots$

The mathematical expression for *intensity distribution* on the screen is given by

$$I = I_o\left(\frac{\sin\phi}{\phi}\right)^2$$

where $\phi = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi}{\lambda}\left(\frac{ay}{D}\right)$

POLARISATION

Polarisation

An unpolarised light is symmetrical about its direction of propagation. The phenomenon of restricting the oscillation to a particular plane is called polarisation. If a wave has oscillations in all possible directions it is called unpolarised light. If oscillations of light wave are restricted only to a single direction in a plane perpendicular to the direction of propagation it is called as plane of polarized light.

Production of Polarised light

Polarised light can be produced by

(i) refraction (ii) reflection (iii) dichorism (Nicol prism) or by scattering.

Polarisation by reflection – Brewsters Law

When unpolarised light is incident at certain angle i_p pm a transparent surface of refractive index μ, then reflected light gets plane polarised. The refractive index is related to the $i_p$called polarizing angle by a relation known as Brewster law $\mu = \tan i_p$and the refracted and reflected rays are perpendicular to each other i.e. $i_p + r = 90$, where r is the refraction angle to the plate.

Nicol's prism

This device used for producing and analyzing plane polarised light is based on the phenomenon of double refraction. When unpolarised light beam passes through a calcite crystal the refracted ray splits into two refracted rays. One is ordinary ray, which has vibrations perpendicular to the principal section and obeys normal refraction laws. The other have vibrations in principal section and does not obey ordinary laws of refraction is called extra-ordinary ray. This phenomenon is known as double refraction and produces both rays as plane polarised.

Malus Law

When polarised light is passed through an analyzer its intensity depends upon the angle (θ) between plane of transmission of polariser and analyzer and is given by Malus Law i.e.

$$I = I_o\cos^2\theta$$

For unpolarised light as average value of $\cos^2$□ is ½, the intensity is given by

$$I = I_o/2$$

Polaroids

These are devices used for producing strong beam of plane polarised light. These are not natural crystals rather are artificially made polarizing material and consist of thin sheets of plastics with a thin layer of tiny crystal of quiminerodosulphate between them.