ATOMS AND NUCLEI

Rutherford gold–foil experiment

This experiment is also known as Rutherford gold–foil experiment. From his experiment, he found that,

Number of particles (N) scattered through an angle θ are given by

$$N \propto \sin^{-4}\left(\frac{\theta}{2}\right)$$

Distance of closest approach (distance between nucleus and the point at which an α–particle comes to rest) is given by d, where,

$$d = \frac{Ze^2}{\pi\varepsilon_0 m v_i^2}.$$

Distance of closest approach is of the order of 10^{-14}m. Thus nuclear size should be less than this distance. Size of the nucleus is measured in terms of Fermi (1 Fermi = 10^{-15}m).

Impact parameter b, defined as the ‘perpendicular distance of the velocity vector of the α–particle from the centre of the nucleus when particle is far away from the nucleus, is given by

$$b = \frac{Ze^2 \cot\frac{\theta}{2}}{4\pi\varepsilon_0\left(\frac{1}{2}m\nu_i{}^2\right)}$$

Where, θ is angle of scattering.

For head on collision, Impact parameter is zero.

Rutherford Model of the Atom

On the basis of α–particle scattering experiment, Rutherford concluded that:

i) Whole of the positive charge and almost entire mass of the atom is concentrated in a very small core called nucleus.

ii) Electrons are distributed around the nucleus and are continuously revolving around the nucleus in circular orbits.

iii) Atom as a whole is neutral.

Bohr’s Atomic Model

Various postulates of Bohr’s atomic model are :

i) Every electron move around the nucleus in a circular orbit. Necessary centripetal force is provided by the Coulomb’s force of attraction between the negative charge of the electron and positive charge of the nucleus.

i.e., $$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}=\frac{m\nu^2}{r}$$

Where, Z is the atomic number, e is charge of the electron and r is radius of the orbit in which electron is revolving around the nucleus. Mass of the electron is m and ν is the velocity of the electron.

ii) Electrons can revolve around the nucleus only in some fixed orbits. Electrons don't radiate or lose energy while revolving in these stationary orbits.

iii) Only those orbits are permitted for which angular momentum is equal to integral multiple of $\frac{h}{2\pi}$, i.e.,

$m\nu r = n.\frac{h}{2\pi}$.

Where, n is an integer and is known as Principal Quantum Number and h is Planck's constant.

iv) Energy is radiated or absorbed only when an electron jumps form one stationary orbit to another stationary orbit.

If an electron jumps from an orbit of energy E_2 to an orbit of energy E_1, then

$$E_2 - E_1 = h\nu$$

where, ν is the frequency of radiations emitted or absorbed. This is known as Bohr's frequency condition.

Bohr's Theory of Hydrogen Atom

For hydrogen atom, radius of an orbit is given by

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \text{ or } r_n \propto n^2$$

Where, n is the principal quantum number.

For other atoms,

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2 Z}$$ (For hydrogen atom Z = 1)

Or $r_n \propto \frac{n^2}{Z}$

Radii of various orbits are in the ratio of

1 : 4 : 9 : 16 …. etc.

For hydrogen atom radius of first orbit is

$$r_1 = 5.3 \times 10^{-11}\text{m}$$

Velocity of revolving electrons

Velocity of an electron in a particular orbit of hydrogen atom is given by

$$\nu = \frac{e^2}{2\varepsilon_0 nh} \quad \text{or} \quad \nu \propto \frac{1}{n}$$

and for any atom with atomic number Z, it is given by

$$\nu = \frac{Ze^2}{2\varepsilon_0 nh} \quad \text{or} \quad \nu \propto \frac{Z}{n}$$

i.e., electrons in the inner orbits have more velocity as compared to the velocity of an electron in an outer orbit. In terms of fine structure constant, velocity of the electron in hydrogen atom is given by

$$\nu = \alpha \frac{c}{n}.$$

where α is fine structure constant. It is dimensionless quantity and is given by,

$$\alpha = \frac{2\pi e^2}{4\pi\varepsilon_0 ch}.$$

Its value is $\frac{1}{137}$.

For first orbit, velocity of electron is $\frac{1}{137}$ times the velocity of light and it is the maximum velocity which an electron can attain in a hydrogen atom and this comes out to be 2.19 $\times 10^6 \text{ms}^{-1}$

Orbital frequency of electron:

It is denoted by ν and is reciprocal of time period and for hydrogen atom it is given by

$$\nu = \frac{me^4}{4\varepsilon_0{}^2 n^3 h^3}.$$

For innermost orbit of hydrogen atom

$$\nu = 65.8 \times 10^{14}\text{Hz}.$$

Kinetic energy of the electron in nth orbit

$$\text{K.E.} = \frac{me^4}{8\varepsilon_0{}^2 n^2 h^2}$$

K.E. is also given by

$$\text{K.E.} = \frac{e^2}{8\pi\varepsilon_0 r}, \text{ i.e., K.E.} \propto \frac{1}{r}.$$

Potential energy of the electron in nth orbit

$$\text{P.E.} = \frac{-Ze^2}{4\pi\varepsilon_0 r}$$

P.E. is also given by

$$= \frac{-me^4}{8\varepsilon_0{}^2 n^2 h^2}$$ and for hydrogen atom

$\text{P.E.} \propto -\frac{1}{r}.$

Potential energy is numerically twice the K.E.

Total energy of the electron in an orbit

Total energy of the electron in the nth orbit is given by

$$\text{T.E.} = \text{P.E.} + \text{K.E.}$$

$$= \frac{me^4}{8\varepsilon_0{}^2n^2h^2} - \frac{me^4}{8\varepsilon_0{}^2n^2h^2} = -\frac{me^4}{8\varepsilon_0{}^2h^2}\left(\frac{1}{n^2}\right)$$

K.E. is numerically equal to total energy .

Time Period

Time period of the electron in an orbit is given by Kepler's law, i.e.,

$$T^2 \propto r^3$$

Frequency of the energy emitted

If an electron jumps from a higher orbit n_2 to a lower orbit n_1 then frequency of the radiations emitted is given by

$\nu = \frac{me^4}{8\varepsilon_0 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and wave number (no. of waves per unit length) denoted by $\bar{\nu}$ is given by,

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} = \frac{me^4}{8\varepsilon_0^2 c h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ or}$$

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where, R = $\frac{me^4}{8\varepsilon_0^2 c h^3} = 10973700\,\text{m}^{-1}$ and is known as Rydberg's constant.

∴ Frequency of the emitted radiations, can also be given as

$$v = Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Energy of electron in nth orbit

In hydrogen like atoms, energy in nth orbit is given by

$$E_n = -R\,ch\frac{Z^2}{n^2}$$

i.e. $E_n \propto -\frac{Z^2}{n^2}$

and for hydrogen atom

$$E_n = \frac{-Rch}{n^2} = \frac{21.76\times10^{-19}}{n^2} J$$

$$= \quad \frac{-13.6}{n^2} \text{eV (electron volt)}$$

13.6 eV is also known as one Rydberg.

$n = \alpha$ then $E_\alpha = \frac{-13.6}{\alpha} = 0$

HYDROGEN SPECTRUM

In emission line spectrum of hydrogen atom, various lines are obtained at different regions of the spectrum. Each group of line is called series of spectral lines and these series are named after the names of their discoverer.

Maximum number of spectral lines that can be emitted when an electrons is in the nth orbit are :

$$\frac{n(n-1)}{2}$$

S.No.	Name of the series	Region	Origin When an electron jumps from	Wavelength of first line of the series Max. λ	Min. λ
1.	Lyman series	UV	n_2 = 2,3,4..etc. to n_1 = 1	$\frac{4}{3R}$	$\frac{1}{R}$
2.	Balmer series	Visible	n_2 = 3, 4, 5,..etc. to n_1= 2	$\frac{36}{5R}$	$\frac{4}{R}$
3.	Paschen series	Near Infrared	n_2 = 4, 5, 6,..etc. to n_1 = 3	$\frac{144}{7R}$	$\frac{9}{R}$
4.	Brackett series	Middle Infrared	n_2 = 5, 6, 7,..etc. to n_1 = 4	$\frac{400}{9R}$	$\frac{16}{R}$
5.	Pfund series	Far Infrared	n_2 = 6,7, 8, ..etc. to n_1 = 5	$\frac{900}{11R}$	$\frac{25}{R}$

SIZE OF THE NUCLEUS

Radius of Nucleus is defined as the distance from the centre of the nucleus at which density of nucleus matter decreases to one half of its maximum value at the centre.

$$R = R_0(A)^{1/3}$$

Where, A is the mass number and $R_0 = 1.1 \times 10^{-15}$m. Nuclear radii vary from 1F to 10F.

Nuclear volume is given by

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

Or $V \propto A$ i.e. Nuclear volume is proportional to its mass number.

Density of the nucleus is 2.29×10^{17} kg/m^3. Such high density are found in white dwarf stars.

Nuclear density does not depend on A and density of all nuclei is almost same. This is not uniformly distributed in the nucleus. It is maximum at the centre and decreases as we move away from the centre of the nucleus.

ATOMIC MASS UNIT (amu)

It is defined as $\frac{1}{12}^{\text{th}}$ of the mass of carbon atom C^{12}

$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg.}$$

Also, $1 \text{ amu} = 931.5 \text{ MeV}$

MASS DEFECT

Mass defect appears in the form of energy and is given by

$$\Delta M = ZM_P + (A-Z)M_n - M'$$

Where, M_P = mass of a proton, M_n = mass of a neutron

And M' = Mass of the nucleus.

BINDING ENERGY

B E is given by

$$B = [ZM_H + (A–Z) M_n – M]c^2$$

Where $M_H = M_P + M_e$ = mass of the hydrogen atom.

$M = M' + ZMe$ = mass of the atom

and c = velocity of light.

Binding Energy per nucleon, $\bar{B}$, or average binding energy is given by $\bar{B} = \frac{B}{A} = \frac{[ZM_H + (A - Z)M_n - M]c^2}{A}$

$\bar{B}$ gives the idea about the nuclear stability.

For nuclei with A = 1 to 39, $\bar{B} < 8.5$ MeV/nucleon.

For nuclei with A = (40–120) $\bar{B} \approx 8.5$ MeV/nucleon.

With the exception of iron (A = 56) for which $\bar{B} \approx 8.8$ MeV/ nucleon and for nuclei with

A > 121, it decreases slowly upto U–238 (7.6MeV) and the decrease is fast beyond U–238 indicating that elements are unstable.

NUCLEAR FORCES

Nuclear forces are due to the exchange of meson (Yukawa 1935).

Properties of these forces are:

(i) They are complex in nature.

(ii) They are strongly attractive.

(iii) They are charge independent.

(iv) They are spin dependent.

(v) They are short range forces. These are maximum at a distance of 1.5Fermi.

(vi) They are 10^{37} times stronger than gravitational forces.

(vii) They are 10^{2} times stronger electrostatic forces.

RADIOACTIVITY

Phenomenon of spontaneous emission of radiations by the nucleus of a substance is called radioactivity.

RADIOACTIVE DECAY LAW

Radioactive decay law states that ‘rate of decay of radioactive nuclei is proportional to the number of nuclei present and is independent of the age of these nuclei' or $\frac{-dN}{dt} = \lambda N_t$, where N_t is the number of radioactive nuclei left after time t and λ is decay constant or disintegration constant of the substance. Negative sign indicates that as t increase N_t decreases. Relation between N_t and N_0 is

$$N_t = N_0 e^{-\lambda t}$$

Where, N_0 is the number of nuclei of radioactive substance at time t = 0. Above equations explains the decay of the nucleus.

DECAY CONSTANT (λ)

It is defined as the reciprocal of the time in which number of nuclei left is $\frac{1}{e}$ times the number of nuclei at t = 0

i.e. if $\lambda = \frac{1}{t}$ then $N_t = \frac{N_0}{e}$

HALF LIFE (T)

Time in which number of nuclei left are half of their original number is called half-life ($T_{1/2}$) and is given by

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$

MEAN LIFE (τ)

It is the average life of all the nuclei of a radioactive substance as the number of nuclei in the substance reduces from N_0 to 0, and is given by $\tau = \frac{1}{\lambda}$

RATE OF DECAY (R)

Rate of decay is defined as the number of nuclei disintegrating per second. Or $R_t = \lambda N_t$

UNITS OF RADIOACTIVITY

Units of radioactivity are:

(i) Becquerel (Bq), (ii) Curie (Ci), (iii) Rutherford.

1Becquerel = one disintegration / sec

1 curie = 3.7×10^{10} disintegration / sec

1 millicurie = 3.7×10^{7} disintegrations / sec

1 microcurie = 3.7×10^{4} disintegrations /sec

1 Rutherford = 10^{6} disintegrations / sec.

NUCLEAR DECAY

(i) **Alpha decay.** It is given by the reaction

$${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}$$

and available energy is given by

$$Q = [m_X - m_Y - m_\alpha]c^2$$

(ii) **Beta decay (Minus)**

${}_{Z}X^{A} \rightarrow {}_{Z} + {}_{1}Y^{A} + {}_{-1}\beta^{0} + \bar{\nu}$ (anti neutrino) and available K. E. is given by

$$Q = [m_X - m_Y]c^2.$$

(iii) **Beta Decay (Plus)**

${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + {}_{1}\beta^{0}$(positive) + ν(neutrino) and available K. E is given by

$$Q = [m_X - m_Y - zm_e]c^2$$

(iv) **Electron capture**

$$_{Z}X^{A} + {}_{-1}e^{0} \rightarrow {}_{Z-1}Y^{A} + v$$

and available K. E. is given by

$$Q = [m_X - m_Y]c^2$$

CHAIN REACTION

Chain reaction can be controlled (nuclear reactor) or uncontrolled (atomic bomb). e.g.

$_{92}U^{235} + {}_{0}n^{1}(\text{slow}) \rightarrow {}_{92}U^{236} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_{0}n^{1} +$ Q(200MeV)

Half life of a neutron is 12 minute and they disintegrate as :

$$_{0}n^{1} \rightarrow {}_{1}p^{1} + {}_{-1}\beta^{0} \rightarrow {}_{1}p^{1} + {}_{-1}e^{0} + \bar{v}.$$

On the basis of their energy, they are classified as :

(a) Slow neutrons with energies < 1eV.

(b) Thermal neutrons with energies in the range1eV to 1.2 MeV.

(c) Fast neutrons with energies >1.2MeV.

NUCLEAR FISSION

Process of disintegration of a heavy nucleus with two or more moderate nuclei of comparable masses is called nuclear fission. Fission can be controlled (reactor) or uncontrolled (atomic bomb) e.g.

$${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236} \rightarrow {}_{56}Ba^{141} + {}_{56}Kr^{92} + 3{}_{0}n^{1} + Q(200\ \text{MeV})$$

Here energy released per nucleon is ≈ 0.85 MeV.

NUCLEAR FUSION

Process of combining two or more lighter nuclei to form a heavy nucleus is called nuclear fusion. A typical nuclear reaction in fusion is:

$$4{}_{1}H^{1} \rightarrow {}_{2}He^{4} + 2{}_{1}e^{0}\ (\text{positron}) + Q(26.7\text{MeV})$$

Here energy, released per nucleon is 6.68 MeV.