SETS, RELATIONS AND FUNCTIONS

Set:

A set is a collection of well-defined objects i.e. the objects follow a given rule or rules.

Elements of a set:

The members of a set are called its elements. If an element x is in set A, we say that x belongs to A and write $x \in A$. If the element x is not in A then we write $x \notin A$.

Examples of sets:

1. The set of vowels in the alphabet of English language.
2. The set of all points on a particular line.

Some special sets:

(i)Finite and infinite sets:

A set A is finite if it contains only a finite number of elements; we can find the exact number of elements in the set. Otherwise, the set is said to be an infinite set.

Example:

Q = set of all rational numbers = $\left\{\frac{p}{q} : p, q \in Z, q \neq 0\right\}$

R = set of all real numbers = {x: x is a rational and an irrational number}

C = set of all complex numbers = $\{x + iy; \quad x, y \in R\}$

(ii) Null set:

A set which does not contain any element is called a null set and is denoted by φ. A null set is also called an empty set.

(iii) Singleton set:

A set which contains only one element is called a singleton set.

(viii) Power set:

The power set of a set A is the set of all of its subsets, and is denoted by $P(A)$ e.g. if $A = \{4, 5, 6\}$ then

$P(A) = \{\varphi, \{4\}, \{5\}, \{6\}, \{4, 5\}, \{5, 6\}, \{4, 5, 6\}\}$.

Note: The null set φ and set A are always elements of $P(A)$.

Theorem: If a finite set has n elements, then the power set of A has 2^n elements.

Operations on sets:

The operations on sets, by which sets can be combined to produce new sets.

(i) Union of sets:

The union of two set A and B is defined as the set of all elements which are either in A or in B or in both. The union of two sets is written as $A \cup B$;

(ii) Intersection of sets:

(i) The intersection of two sets A and B is defined as the set of those elements which are in both A and B and is written as

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

(ii) The intersection of n sets $A_1, A_2 .... A_n$ is written as

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 .... \cap A_n = \{x : x \in A_i \text{ for all } i,\ 1 \le i \le n\}.$$

Disjoint sets:

Two set A and B are said to be disjoint, if there is no element which is in both A and B, i.e. $A \cap B = \varphi$;

- ✓ The properties of the complement of sets are known as **DeMorgan laws**, which are
  - (i) $A^c - B^c = B - A$
  - (ii) $(A \cup B)^c = A^c \cap B^c$
  - (iii) $(A \cap B)^c = A^c \cup B^c$
- ✓ If A and B are not disjoint, then
  - (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
  - (ii) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
  - (iii) $n(A) = n(A - B) + n(A \cap B)$
  - (iv) $n(B) = n(B - A) + n(A \cap B)$

(vi) Cartesian product of sets:

Let a be an arbitrary element of a given set A i.e. $a \in A$ and b be an arbitrary element of B i.e. $b \in B$. Then the pair (a, b) is an ordered pair. Obviously $(a, b) \neq (b, a)$. The cartesian product of two sets A and B is defined as the set of ordered pairs (a, b). The cartesian product is denoted by $A \times B$

$\Rightarrow A \times B = \{(a, b);\ a \in A,\ b \in B\}$.

Relation:

Let A and B be two sets. A relation R from the set A to set B is a subset of the Cartesian product $A \times B$. Further, if $(x, y) \in R$, then we say that x is R-related to y and write this relation as x R y. Hence $R\{(x, y); x \in A,\ y \in B,\ x\ R\ y\}$.

Domain and Range of a relation: Let R be a relation defined from a A set to a set B, i.e.$R \subseteq A \times B$. Then the set of all first elements of the ordered pairs in R is called the domain of R. The set of all second elements of the ordered pairs in R is called the range of R. That is,

D = domain of $R = \{x : (x, y) \in R\}$ or $\{x : x \in A \text{ and } (x, y) \in R\}$,

R^* = range of $R = \{y : (x, y) \in R\}$ or $\{y : y \in B \text{ and} (x, y) \in R\}$.

Clearly $D \subseteq A$ and $R^* \subseteq B$.

FUNCTIONS:

A mapping f: X $\to$ Y is said to be a function if each element in the set X has its image in set Y. Every element in set X should have one and only one image.

Let f: R $\to$ R where y = x^3. Here for each x $\in$ R we would have a unique value of y in the set R

Set 'X' is called domain of the function 'f'.

Set 'Y' is called the co-domain of the function 'f'.

Algebra of Functions:

Let us consider two functions,

$f: D_1 \to R$ and $g: D_2 \to R$. We describe functions f + g, f - g, f.g and f/g as follows:

- $f + g : D \to R$ is a function defined by
- $(f + g)x = f(x)+g(x)$ where $D = D_1 \cap D_2$
- $f - g : D \to R$ is a function defined by
- $(f - g)x = f(x) - g(x)$ where $D = D_1 \cap D_2$
- $f.g: D \to R$ is a function defined by
- by $(f.g)x = f(x). g(x)$ where $D = D_1 \cap D_2$
- $f/g: D \to R$ is a function defined by
- $(f/g)x = \frac{f(x)}{g(x)}$ where $D = \{x : x \in D_1 \cap D_2, g(x) \neq 0\}$

<u>TYPE OF FUNCTION</u>

One-One and Many-One Functions:

When every element of domain of a function has a distinct image in the co-domain, the function is said to be One-One. If there are at least two elements of the domain whose images are the same, the function is known as Many-One.

Onto and Into Functions:

For every point y in b, there is some point x in A such the

f(x) = y. It is called onto function. When the codomain y which is not an image of any element in the domain x, then function is **onto.**

Even and Odd Functions:

- f(x) - f(-x) = 0 for even function and f(x) + f(-x) = 0 for odd functions .

Periodic Function

If f(x) is periodic with period t, then a f(x) +b where a, b $\in$ R (a $\neq$ 0) is also periodic with period t

Some Important function and their domain and range

FUNCTION F(X)	**DOMAIN**	**RANGE**
sinx	$(-\infty,\infty)$	[-1, 1]
cosx	$(-\infty,\infty)$	[-1, 1]
a^x a>1	**R**	$(0, \infty)$
$\log_a x$, a>0 and $\neq 1$ a>0 and $\neq 1$	$(0, \infty)$	$(-\infty, \infty)$
[x]	R	I
$\lvert x \rvert$	R	$[0, \infty)$
{x}	R	[0,1)