PERMUTATION AND COMBINATIONS

FUNDAMENTAL PRINCIPLE OF COUNTING

1. MULTIPLICATION PRINCIPLE OF COUNTING

If a job can be done in *m* ways, and when it is done in any one of these ways another job can be done in *n*, then both the jobs together can be done in *mn* ways.

2. ADDITION PRINCIPLE OF COUNTING

If a job can be done in *m* ways and another job can be done in *n* ways then either of these jobs can be done in $m + n$ ways.

PERMUTATIONS

Each of different arrangement which can be made by taking some or all of a number of things is called a permutation.

1. COUNTING FORMULAE FOR PERMUTATION

To find the value of ^nP_r

$$^nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1)$$

$= \frac{n!}{(n-r)!}$ (using factorial notation $n! = n(n-1)\ldots\ldots$ 3.2.1.) where $0 \le r \le n$.

In particular

- The number of permutations of *n* different things taken all at a time $= {}^nP_n = n!$
- $^nP_0 = 1$, $^nP_1 = n$ and $^nP_{n-1} = {}^nP_n = n!$
- $^nP_r = n\,({}^{n-1}P_{r-1})$ where $r = 1, 2, \ldots\ldots n$.

3. PERMUTATION OF *n* DISTINCT OBJECT WHEN REPETITION IS ALLOWED

- The number of permutations of n different things taken r at time when each thing may be repeated any number of times is n^r.

4. ARRANGEMENT OF *n* THINGS WHEN ALL ARE NOT DISTINCT

- The number of permutations of n things taken all at a time, where x are alike of one kind, y are alike of second kind and z are alike of third kind and the rest $n-(x+y+z)$ are all distinct is given by

$$\frac{n!}{x!\,y!\,z!} \quad (x+y+z \leq n)$$

CIRCULAR PERMUTATIONS

In the event of the given n things arranged in a circular or even elliptical permutation –and in this case the first and the last thing in the arrangement are indistinguishable – the number of permutations is $(n-1)!$.

NUMBER OF CIRCULAR PERMUTATIONS OF *n* DIFFERENT THINGS TAKEN *r* AT A TIME

CASE I: If clockwise and anticlockwise orders are taken as different, then the required number of circular permutations $= \frac{{}^{n}P_{r}}{r}$.

CASE II: If clockwise and anticlockwise orders are taken as not different, then the required number of circular permutations $= \frac{{}^{n}P_{r}}{2r}$

COMBINATIONS

Each of different grouping or selections that can be made by some or all of a number of given things without considering the order in which things are placed in each group, is called combinations.

1. COUNTING FORMULAE FOR COMBINATIONS

The number of combinations of n different things taken r at a time is given by ${}^{n}C_{r}$ or $C(n, r)$

$${}^{n}C_{r} = \frac{n!}{(n-r)!\ r!} \qquad (0 \le r \le n)$$

as $\quad {}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$

Key results on ${}^{n}C_{r}$

- ${}^{n}C_{0} = {}^{n}C_{n} = 1$
- ${}^{n}C_{1} = n$ There are n ways to select one thing out of n distinct things.
- ${}^{n}C_{r} = {}^{n}C_{n-r}$
- If n is odd then the greatest value of ${}^{n}C_{r}$ is ${}^{n}C_{\frac{n+1}{2}}$ or ${}^{n}C_{\frac{n-1}{2}}$.

- If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.

6. SELECTION FROM DISTINCT/IDENTICAL OBJECTS

(I) SELECTION FROM DISTINCT OBJECTS

- The number of ways (or combinations) of selection from n distinct objects, taken at least one of them is

$${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n = 2^n - 1$$

(II) SELECTION FROM IDENTICAL OBJECTS

The number of ways of selections of atleast one out of $a_1 + a_2 + a_3 + \ldots\ldots + a_n + k$ objects, where a_1 are alike of one kind, $\ldots\ldots\ldots a_n$ are alike of nth kind and k are distinct is

$$(a_1 + 1)\,(a_2 + 1)\,\ldots\ldots\ldots(a_n + 1)\,2^k - 1.$$

8. DIVISION OF DISTINCT OBJECT IN TO GROUPS

In the case of grouping we have the following. If $m + n + p$ things are divided into 3 groups one containing m, the second n and the third p things; number of groupings is

$${}^{(m+n+p)}C_m \cdot {}^{(n+p)}C_n \cdot {}^pC_p$$

$$= \frac{(m+n+p)!}{m!\,n!\,p!}$$ where m, n, p are distinct natural numbers.

In general, the number of ways in which mn different things can be divided equally into m distinct groups is $\frac{(mn)!}{(n!)^m}$ when order of groups is important.

9. DIVISION OF IDENTICAL OBJECTS INTO GROUPS

The number of ways of division or distribution of n identical things into r different groups is ${}^{n+r-1}C_{r-1}$ or ${}^{n-1}C_{r-1}$ according as empty groups are allowed or not allowed.

10. ARRANGEMENTS IN GROUPS

The number of ways of distribution and arrangement of n distinct things into r different groups is $n!\ {}^{n+r-1}C_{r-1}$ or $n!\ {}^{n-1}C_{r-1}$ according as empty groups are allowed or not allowed.