MATHEMATICAL INDUCTIONS

The word induction means the method of reasoning about a general statement from the conclusion of particular cases. Inductions starts with observations. It may be true but then it must be so proved by the process of reasoning. Else it may be false but then it must be shown by finding a counter example where the conjecture fails.

Principle of Mathematical Induction: The proposition $P(n)$ involving a natural number n is assumed to be true for all $n \in N$, follows the following three steps:

Step -I (Verification step)

Actual verification of the proposition $P(n)$ for the starting value of $n = i$.

Step -II (Induction step)

Assuming that if $P(n)$ is true for $n = k$; $k \geq i$, prove that it is also true for $n = k + 1$.

Step -III (Generalization step)

Combining the above two steps leads to the conclusion that $P(n)$ is true for all integers $n \in N$.

PROPOSITION

A statement which is either true or false is called a proposition or statement.

$P(n)$ denotes a proposition whose truth value depends on natural variable '*n*'.

For example $1^2+2^2+3^2+......n^2 = \frac{n(n+1)(2n+1)}{6}$ is a proposition whose truth value depends on natural number *n*.

We write $P(n): 1^2+2^2+3^2+.......n^2 = \frac{n(n+1)(2n+1)}{6}$, where $P(4)$ means $1^2+2^2+3^2+4^2 = \frac{4(4+1)(8+1)}{6}$

To prove the truth of proposition $P(n)$ depending on natural variable *n*, we use mathematical induction.

FIRST PRINCIPLE OF MATHEMATICAL INDUCTION

The statement $P(n)$ is true for all $n \in N$ if

(i) $P(1)$ is true.

(ii) $P(m)$ is true $\Rightarrow$ $P(m+1)$ is true.

The above statement can be generalized as $P(n)$ is true for all $n \in N$ and $n \geq k$ if

(i) $P(k)$ is true.

(ii) $P(m)$ is true $(m > k)$ $\Rightarrow$ $P(m+1)$ is true.

APPLICATION OF FIRST PRINCIPLE OF MATHEMATICAL INDUCTION

To prove any statement $P(n)$ to be true for all $n \geq k$ with the help of first principle of mathematical induction we follow the following procedure:

Step (i)Check if the statement is true or false for $n = k$.

Step (ii) Assume the statement is true for $n = m$.

Step (iii) Prove the statement is true for $n = m + 1$.