VECTORS ALGEBRA

Scalar and Vector Algebra.

Scalars: Scalars are mathematical entities which have only a magnitude (and no direction)

Vectors: Vectors are mathematical entities which have both a magnitude and a direction.

Vectors Additions

Note that addition is **commutative** i.e., $\bar{a}+\bar{b}=\bar{b}+\bar{a}$.
Also, $\bar{a}+(\bar{b}+\bar{c})=(\bar{a}+\bar{b})+\bar{c}$ i.e. the addition of vectors obeys the **associative** law. If $\bar{a}$ and $\bar{b}$ are collinear, their sum is still obtained in the same manner although we do not have a triangle or a parallelogram in this case.
For adding more than two vectors, we have a **polygon law of addition** which is just an extension of the triangle law.

- Two vectors (non-zero and non-collinear) constitute a plane.Their sum or difference also lies in the same plane. Three vectors are said to be coplanar if their line segments lie in the same plane or are parallel to the same plane.
- $\left||\bar{a}|-|\bar{b}|\right| \leq |\bar{a}+\bar{b}| \leq |\bar{a}|+|\bar{b}|$.

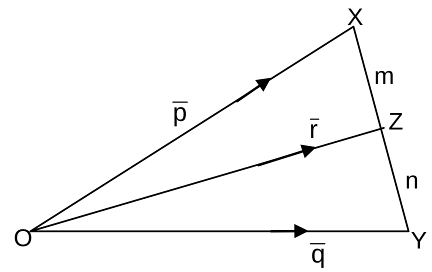
Section Formula:

The position vecture of a point X = $\vec{p}$

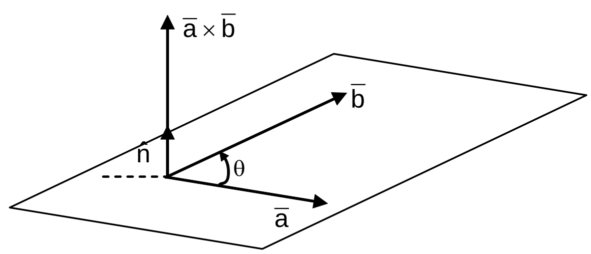
and that of another point Y = $\bar{q}$

If the line joining P and Q is divided by a point Z in the ratio of m:n (internally or externally), then

$\bar{r} = \dfrac{m\bar{q} + n\bar{p}}{m+n}$



VECTOR (OR CROSS) PRODUCT OF TWO VECTORS

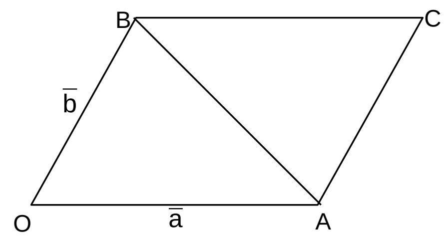


- $\bar{a} \times \bar{a} = \bar{o} \Longrightarrow \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \bar{o}$
- $\bar{a} \times \bar{b} = -(\bar{b} \times \bar{a})$ (non-commutative)
- $\bar{a} \times (\bar{b} + \bar{c}) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$ (Distributive)
- $(l\bar{a}) \times (m\bar{b}) = lm(\bar{a} \times \bar{b})$
- $\bar{a} \times \bar{b} = \bar{o} \Longleftrightarrow \bar{a}$ and $\bar{b}$ are collinear (if none of $\bar{a}$, or $\bar{b}$ is a zero vector)
- $\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$
- $\hat{j} \times \hat{i} = -\hat{k}, \quad \hat{k} \times \hat{j} = -i, \quad \hat{i} \times \hat{k} = -\hat{j}$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$= (a_2b_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$

- Any vector perpendicular to the plane of $\bar{a}$ and $\bar{b}$ is $\lambda\ (\bar{a}\times\bar{b})$ where λ is a real number. Unit vector perpendicular to $\bar{a}$ and $\bar{b}$ is $\pm\ \frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$

- $|\bar{a}\times\bar{b}|$ denotes the area of the parallelogram OACB, whereas area of $\Delta OAB = \frac{1}{2}|\bar{a}\times\bar{b}|$



Scalar (or Dot) Product of Two Vectors

The scalar product of $\bar{a}$ and $\bar{b}$ written as $\bar{a}\cdot\bar{b}$ is defined to be the number $|\bar{a}||\bar{b}|\cos\theta$ where θ is the angle between the vectors $\bar{a}$ and $\bar{b}$ i.e. $\bar{a}\cdot\bar{b} = ab\cos\theta$.

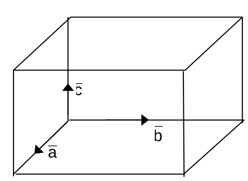
Properties

$\bar{a}\cdot\bar{b} =$ $(a\cos\theta)b$ = (projection of $\bar{a}$ on $\bar{b}$) b = (projection of $\bar{b}$ on $\bar{a}$) a

- $\bar{a}\cdot\bar{b} = 0 \Leftrightarrow \bar{a},\ \bar{b}$ are perpendicular to each other $\Rightarrow \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Scalar Triple Product

It is defined for three vectors $\bar{a},\ \bar{b},\ \bar{c}$, in that order as the scalar $(\bar{a}\times\bar{b})\cdot\bar{c} = [\bar{a}\ \ \bar{b}\ \ \bar{c}]$, which can also be written



simply as $\bar{a}\times\bar{b}\cdot\bar{c}$. It denotes the volume of the parallelopiped formed by taking a, b, c as the co-terminus edges. i.e. V = magnitude of $\bar{a}\times\bar{b}\cdot\bar{c} = |\bar{a}\times\bar{b}\cdot\bar{c}|$.

Properties

- $\bar{a}\times\bar{b}\cdot\bar{c} = \bar{a}\cdot\bar{b}\times\bar{c}$, i.e. position of dot and cross can be interchanged without altering the product. Hence it is also represented by $[\bar{a}\quad\bar{b}\quad\bar{c}]$.
- $[\bar{a}\quad\bar{b}\quad\bar{c}] = [\bar{b}\quad\bar{c}\quad\bar{a}] = [\bar{c}\quad\bar{a}\quad\bar{b}]$
- $[k\bar{a}\quad\bar{b}\quad\bar{c}] = k[\bar{a}\quad\bar{b}\quad\bar{c}]$

Vector Triple Product

It is defined for three vectors $\bar{a},\bar{b},\bar{c}$ as the vector $\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}$
In general, $\bar{a}\times(\bar{b}\times\bar{c}) \neq (\bar{a}\times\bar{b})\times\bar{c}$.

$\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\times\bar{b})\times\bar{c}$ if some or all of $\bar{a}$, $\bar{b}$, $\bar{c}$ are zero vectors or $\bar{a}$ and $\bar{c}$ are collinear.