



TRIGONOMETRY

Trigonometric Ratios of Compound Angles:

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A.$
- $\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A.$
- $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$
- $\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

Trigonometric Ratios of Multiples of an Angle:

- $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$
- $\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} = 1 - 2 \sin^2 \frac{A}{2} = 2 \cos^2 \frac{A}{2} - 1 = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$
- $\cos^2 A = \frac{1 + \cos 2A}{2}$; $\sin^2 A = \frac{1 - \cos 2A}{2}$

$$\begin{aligned} \blacksquare \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} & \tan A &= \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}} \\ \blacksquare \cot 2A &= \frac{\cot^2 A - 1}{2 \cot A} & \cot A &= \frac{\cot^2 \frac{A}{2} - 1}{2 \cot \frac{A}{2}} \\ \blacksquare \sin 3A &= 3 \sin A - 4 \sin^3 A \\ \blacksquare \cos 3A &= 4 \cos^3 A - 3 \cos A \\ \blacksquare \tan 3A &= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \\ \blacksquare \cot 3A &= \frac{\cot^3 A - 3 \cot A}{3 \cot^2 A - 1} \end{aligned}$$

Sum of sines/cosines in terms of Products:

$$\begin{aligned} \blacksquare 2 \sin A \cos B &= \sin(A + B) + \sin(A - B) \\ \blacksquare 2 \cos A \sin B &= \sin(A + B) - \sin(A - B) \\ \blacksquare 2 \cos A \cos B &= \cos(A + B) + \cos(A - B) \\ \blacksquare 2 \sin A \sin B &= \cos(A - B) - \cos(A + B) \end{aligned}$$

Conversely:

$$\begin{aligned} \blacksquare \sin C + \sin D &= 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \\ \blacksquare \sin C - \sin D &= 2 \sin \frac{C-D}{2} \cos \frac{C+D}{2} \\ \blacksquare \cos C + \cos D &= 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2} \\ \blacksquare \cos C - \cos D &= -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \\ \blacksquare \tan A + \tan B &= \frac{\sin(A+B)}{\cos A \cdot \cos B} \end{aligned}$$

Inverse Trigonometrical functions

Some Important Results:



$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \quad |x| \leq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in \mathbb{R}$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, \quad |x| \geq 1$$

$$\sin^{-1}(-x) = -\sin^{-1}(x)$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$$

$$\tan^{-1}(-x) = -\tan^{-1}(x)$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$$

$$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

$$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1} \frac{1}{x}, \quad x > 0$$

$$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2} = \cot^{-1} \frac{\sqrt{1-x^2}}{x} = \sec^{-1} \frac{1}{\sqrt{1-x^2}}, \quad x > 0$$

$$\cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{\sqrt{1-x^2}}{x} = \operatorname{cosec}^{-1} \frac{1}{\sqrt{1-x^2}}, \quad x > 0$$

$$\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}} = \operatorname{cosec}^{-1} \frac{\sqrt{1+x^2}}{x}, \quad x \in \mathbb{R} \sim \{0\}$$

$$\cot^{-1} x = \cos^{-1} \frac{x}{\sqrt{1+x^2}} = \sec^{-1} \frac{\sqrt{1+x^2}}{x}, \quad x \in \mathbb{R} \sim \{0\}$$

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-x^2}\sqrt{1-y^2}], \quad x \geq 0, \quad y \geq 0$$

$$\cos^{-1} x - \cos^{-1} y = \begin{cases} \cos^{-1} [xy + \sqrt{1-x^2}\sqrt{1-y^2}], & x \geq 0, \quad y \geq 0, \quad x \leq y \\ -\cos^{-1} [xy + \sqrt{1-x^2}\sqrt{1-y^2}], & x \geq 0, \quad y \geq 0, \quad x > y \end{cases}$$

$$\sin^{-1} x + \sin^{-1} y = \begin{cases} \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}], & x \geq 0, \quad y \geq 0, \quad x^2 + y^2 \leq 1 \\ \pi - \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}], & x \geq 0, \quad y \geq 0, \quad x^2 + y^2 > 1 \end{cases}$$



Trigonometric Equations

- $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$,
- $\sin\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{2}$,
- $\sin\theta = -1 \Rightarrow \theta = (4n-1)\frac{\pi}{2}$,
- $\cos\theta = 1 \Rightarrow \theta = 2n\pi$,
- $\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi$,

SOLUTION OF TRIANGLE

In any triangle three sides and three angles are called the elements of the triangle. the three sides $BC = a$, $CA = b$, $AB = c$ and the three angles A , B , C .

(i) $A + B + C = \pi = 180^\circ$

(ii) Area of the ΔABC , $\Delta = \frac{1}{2}$ (base) (height)

SINE FORMULA

In a ΔABC , $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

i.e., sides are proportional to sines of opposite angles.

COSINE FORMULA

In a ΔABC , $a^2 = b^2 + c^2 - 2bc \cos A$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

PROJECTION FORMULA

$$\text{In any } \triangle ABC, \begin{cases} a = b \cos C + c \cos B \\ b = c \cos A + a \cos C \\ c = a \cos B + b \cos A \end{cases}$$

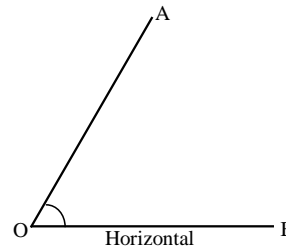
HALF-ANGLE FORMULA

$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} \text{ and } \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

Height and Distance

1. Angle of Elevation:

Consider a point A being observed from a point O (usually called observer) at a lower horizontal level. Draw a horizontal line OB through O in the direction of A. Then, OA is called the line of sight or observation and $\angle AOB$ is called the angle of elevation of point A as seen from O.



2. Angle of Depression:

Consider a point A being observed from a point O at a higher horizontal level $\angle AOB$ here is called the angle of depression of the point A as seen from O. This is also measured w.r.t the horizontal at O.

