JEE(Main)-2024 | 06 April 2024 (Shift-1 Morning) | Question Paper with Solutions | Memory Based

MATHEMATICS

1. $I = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x \sin^2 x}{\left(\cos^3 x + \sin^3 x\right)^2} dx$

Ans. $\left(\frac{1}{6}\right)$

Sol. $\int_0^{\frac{\pi}{4}} \frac{\cos^2 x \sin^2 x}{\cos^6 x\left(1+\tan^3 x\right)^2} dx$

$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x \tan^2 x}{\left(1+\tan^3 x\right)^2} dx$

$1 + \tan^3 x = t$

$3\tan^2 x \sec^2 x\, dx = dt$

$\Rightarrow \frac{1}{3}\int_1^2 \frac{1}{t^2} dt$

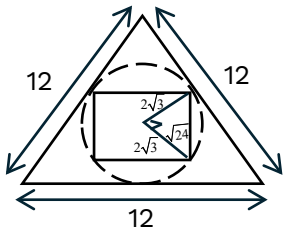
$-\frac{1}{3t}\Big|_1^2 = -\frac{1}{3}\left(\frac{1}{2} - 1\right)$

$= \frac{1}{6}$

2. An equilateral triangle of side 12. A circle is embedded inside the triangle. And a square is embedded inside the circle. If area and perimeter of the square is 'm' and 'n' respectively then find $m + n^2$.

Ans. (408)

Sol.



$r = \frac{\frac{\sqrt{3} \times (12)^2}{4}}{18}$

$r = \frac{\sqrt{3} \times 36}{18}$

$r = 2\sqrt{3}$

Area = 24

Perimeter = $4\sqrt{24}$

$m + n^2 = 24 + 16 \times 24 = 24 \times 17 = 408$

3. Solve: $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$, $y(1)=0$ then y(0) =

Ans. $\frac{1}{2}-\frac{e^{\frac{\pi}{2}}}{2}$

Sol. $\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}$

I.F. = $e^{\tan^{-1}x}$

$$y\cdot e^{\tan^{-1}x}=\int\frac{\left(e^{\tan^{-1}x}\right)^2}{1+x^2}dx$$

$\tan^{-1}x=t$

$=\int e^{2t}dt$

$$\Rightarrow \quad ye^{\tan^{-1}x}=\frac{e^{2\tan^{-1}x}}{2}+c$$

$$y=\frac{e^{\tan^{-1}x}}{2}+c\cdot e^{-\tan^{-1}x}$$

$$x=1,\quad y=0\Rightarrow 0=\frac{e^{\pi/4}}{2}+ce^{-\pi/4}$$

$$c=-\frac{e^{\pi/2}}{2}$$

$$Y(0)=\frac{1}{2}-\frac{e^{\pi/2}}{2}$$

4. Find the range of 'x' for which $f(x)=x^x$ $(x>0)$ is strictly increasing.

Ans. $\left[\frac{1}{e},\infty\right)$

Sol. $y=x^x$

$\ln y=x\ln x$

$$\frac{1}{y}\frac{dy}{dx}=x\cdot\frac{1}{x}+\ln x\cdot 1$$

$$y'=x^x(1+\ln x)$$

$$y'\geq 0\Rightarrow \ln x\geq -1\Rightarrow x\geq e^{-1}$$

$$\therefore x\in\left[\frac{1}{e},\infty\right)$$

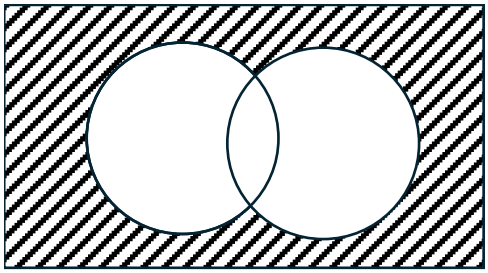
5. Let A = {100, 101, 102, ..., 700}. Find number of numbers in set A which are neither divisible by 3 nor by 4.

Ans. (300)

Sol. No. of numbers divisible by 3 = 200

No. of numbers divisible by 4 = 151

No. of numbers divisible by 12 = 50



∴ No. of numbers neither divisible by 3 nor by 4 = 601 – (200 + 151 – 50) = 300

6. Given that $2x\ell nx\,\frac{dy}{dx}+y=3\ell nx$, and y(1) = 0, find y.

Ans. $y=\ell nx$

Sol. $2x\ln x\frac{dy}{dx}+y=3\ell nx,\quad y(1)=0$

$\Rightarrow \frac{dy}{dx}+\frac{1}{2x\ln x}\cdot y=\frac{3}{2x}$

$I\cdot F.=e^{\frac{1}{2}\int\frac{1}{x\cdot\ln x}dx}=e^{1/2\ln(\ln x)}$ $\quad \left(\begin{array}{c}\ln x=t\\ \frac{1}{x}\cdot dx=dt\end{array}\right)$

$=e^{\ln(\ln x)^{1/2}}=\sqrt{\ln x}$

Solution of Differential Equation -

$y\cdot\sqrt{\ln x}=\int\frac{3}{2}\cdot\frac{\sqrt{\ln x}}{x}dx$ $\quad \left(\begin{array}{c}\ln x=t\\ \frac{1}{x}\cdot dx=dt\end{array}\right)$

$\Rightarrow y\cdot\sqrt{\ln x}=\frac{3}{2}\int\sqrt{t}\cdot dt$

$y\sqrt{\ln x}=\frac{3}{2}\times\frac{2}{3}t^{\frac{3}{2}}+c$

$y\sqrt{\ln x}=(\ln x)^{3/2}+c$

Given y(1) = 0

$\Rightarrow C=0$

$\Rightarrow y\sqrt{\ln x}=\left(\ln x\right)^{3/2}$

Hence, Solution of D.E. $\Rightarrow y=\ln x$

7. Let $A_r=\begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & n(n-1)\end{vmatrix}$ find $2A_{10}-A_8$.

Ans. $(4\alpha+2\beta)$

Sol. $A_r = 2\begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ r & 1 & \frac{n^2}{2}-\frac{\beta}{2} \\ 3r-2 & 3 & n(n-1) \end{vmatrix}$

Apply $R_1 \to R_1 - R_2$

$A_r = 2\begin{vmatrix} 0 & 0 & \alpha+\beta/2 \\ r & 1 & \frac{n^2}{2}-\frac{\beta}{2} \\ 3r-2 & 3 & n(n-1) \end{vmatrix}$

$\therefore A_r = 2\left(\left(\alpha+\frac{\beta}{2}\right)(3r-3r+2)\right)$

$A_r = 4\left(\alpha+\frac{\beta}{2}\right)$

$A_8 = 4\left(\alpha+\frac{\beta}{2}\right)$

$A_{10} = 4\left(\alpha+\frac{\beta}{2}\right)$

$2A_{10} - A_8 = 4\left(\alpha+\frac{\beta}{2}\right) = 4\alpha+2\beta$

8. In an octagon how many triangles are possible so that no side of triangle is side of octagon?

Ans. (16)

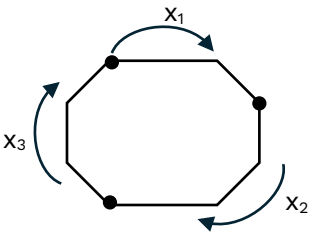
Sol. Method – 1

No. of possible triangles = ${}^6c_3 - {}^4c_1$

= 20 – 4

= 16

Method – 2



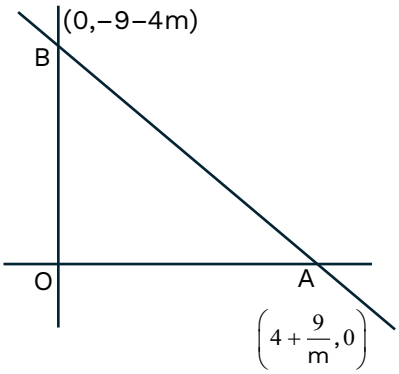
$x_1 + x_2 + x_3 = 5 \qquad x_1 \geq 1,\ x_2 \geq 1,\ x_3 \geq 1$

No. of possible triangles = $\frac{4c_2 \times 8c_1}{3} = \frac{8\times 6}{3} = 16$

9. A variable line is passing through (4, – 9), slope of line is positive and it make intercepts on x and y-axis on point A and B. Find the minimum area of triangle OAB.

Ans. (72)

Sol. $y + 9 = m(x - 4)$



$$\text{Area of } \Delta OAB = \left|\frac{1}{2}\left(4+\frac{9}{m}\right)(-9-4m)\right|$$

$$= \frac{1}{2}\left|36+16m+\frac{81}{m}+36\right|$$

$$= \frac{1}{2}\left|72+16m+\frac{81}{m}\right|$$

Area of ΔOAB will be minimum when, $m = \frac{9}{4}$

$$\text{Area of } \Delta OAB_{(min)} = \frac{1}{2}|72+36+36|$$

$$= \frac{1}{2}(144) = 72$$

10. If mean of 20 observation is 10, SD = 2. One of the observation which is 12 is replaced by 8. Find the value of new SD?

Ans. $\sqrt{3.96}$

Sol. $\frac{\sum x_i}{20} = 10 \quad \sum x_i = 200$

$\sum x_{i\,new} = 196$

new mean = $\frac{196}{20}$

$\frac{\sum x_i^2}{20} - 100 = 4$

$\sum x_i^2 = 2080$

$\sum x_{i\,new}^2 = 2080 - 80 = 2000$

$\sigma^2 = \frac{2000}{20} - \left(\frac{196}{20}\right)^2$

$100 - 96.04$

$\sqrt{3.96}$

11. Let $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{x^2 - 2x - 15}{x^2 - 4x + 9}$ then f is

(1) one-one onto (2) many-one onto

(3) many-one into (4) one-one into

Ans. (3)

Sol. $y = \frac{x^2 - 2x - 15}{x^2 - 4x + 9}$

for, $x^2 - 4x + 9$

$D < 0$, cannot be factorised

$\therefore$ f(x) is many one.

$yx^2 - 4xy + 9y = x^2 - 2x - 15$

$x^2(y-1) + x(-4y+2) + 9y + 15 = 0$

$D < 0$ as $x \in R$

$(-4y+2)^2 - 4(y-1)(9y+15) < 0$

$16y^2 + 4 - 16y - 36y^2 + 36y - 60y + 60 < 0$

$-20y^2 - 40y + 64 < 0$

$-5y^2 - 10y + 16 < 0$

$5y^2 + 10y - 16 > 0$

$D < 0$

$\therefore$ Range $\neq$ R

$\therefore$ not onto

$\therefore$ f(x) is many-one into.

12. A company have two branches A and B. 'A' produce 60% of total production and remain by 'B'. Branch 'A' produce 80% good quality product and branch 'B' produce 90% good quality product. Randomly a product is selected and that was of good quality. P is the probability that selected product is from branch 'B' find value of 126P.

Ans. (54)

Sol. Production of A = 60%

Production of B = 40%

Probability that product is of Branch $A = \frac{60}{100}$

Probability that product is of Branch $B = \frac{40}{100}$

Let E be the probability that the product is good.

Total probability that the product is good

$= \frac{60}{100} \times \frac{80}{100} + \frac{40}{100} \times \frac{90}{100}$

Now

$$P\left(\frac{B}{E}\right)=\frac{P(B)\cdot P\left(\frac{E}{B}\right)}{P(A).\,P\left(\frac{E}{A}\right)+P(B)\cdot P\left(\frac{E}{B}\right)}=\frac{\frac{40}{100}\times\frac{90}{100}}{\frac{60}{100}\times\frac{80}{100}+\frac{40}{100}\times\frac{90}{100}}$$

$$=\frac{0.4\times0.9}{0.6\times0.8+0.4\times0.9}$$

$$=\frac{3}{7}$$

$$\therefore 126\,P\left(\frac{B}{E}\right)=54$$

$126P = 54$

13. Find the shortest distance between two lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$.

Ans. $(4\sqrt{3})$

Sol.

$$\frac{\begin{vmatrix}-4 & 16 & 0\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}}=\frac{192}{\sqrt{256+256+256}}$$

$$=4\sqrt{3}$$

14. If in the expansion of $(x + y)^n$

$T_2 = 15$

$T_3 = 10$

$T_4 = \frac{10}{3}$

for n = 5. Find the value of $\left(n^3+x^5+243y^5\right)$

Ans. (143)

Sol. $T_2 = {}^nC_1(x)^{n-1}y = 15$

$T_3 = {}^nC_2x^{n-2}y^2 = 10$

$T_4 = {}^nC_3x^{n-3}y^3 = \frac{10}{3}$

${}^5C_1x^4y = 15$ $\qquad$ $5c_2x^3y^2 = 10$

$5x^4y = 15$ $\qquad$ $10\,x^3y^2 = 10$

$x^4y = 3$ $\qquad$ $x^3y^2 = 1$

$x = 9^{1/5}$ $\qquad$ $y = \frac{1}{x^{3/2}}$

$$\therefore y=\frac{1}{3^{3/5}}$$

$\therefore \quad n^3 + x^5 + 243y^5$

$= 125 + 9 + \frac{243}{27}$

= 125 + 9 + 9

= 143

15. Let S = {1, 2, 3, ..., 20} be a given set. Relation R_1 is define as R_1 = {(x, y) : 2x – 3y = 2} and R_2 as R_2 = {(x, y) : 4x = 5y} (x, y ∈ S). If m denotes number of elements required to make R_1 symmetric and n denote the number of elements to make R_2 symmetric. Then find m + n.

Ans. (10)

Sol. $R_1 = \{(x, y) : 2x - 3y = 2\}$

$y = \frac{2(x-1)}{3}$

$\therefore R_1$ = {(4,2), (7,4), (10,6), (13,8), (16,10), (19,12)}

To make it Symmetric

(2,4), (4,7), (6,10), (8,13), (10,16), (12,19) are required

$\therefore$ m = 6

R_2 = {(x,y) : 4x = 5y}

$y = \frac{4}{5}x$

R_2 = {(5,4), (10,8), (15,12), (20,16)}

To make it symmetric

(4,5), (8,10), (12,15), (16,20) are required

$\therefore$ n = 4

m + n = 10

16. Given a function $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right); & x \neq 0 \\ 0; & x = 0 \end{cases}$, then find $f''\left(\frac{2}{\pi}\right)$

Ans. $\left(\frac{24 - \pi^2}{2\pi}\right)$

Sol. $f'(x) = x^3 \cdot \cos\left(\frac{1}{x}\right)\left(-1/x^2\right) + \sin(1/x) \cdot 3x^2$

$f'(x) = -x \cdot \cos\left(\frac{1}{x}\right) + 3x^2 \sin(1/x)$

$f''(x) = x \cdot \sin\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) - \cos\left(\frac{1}{x}\right) + 3x^2 \cdot \cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) + 3\sin\left(\frac{1}{x}\right).2x$

$f''(x) = -\frac{1}{x}\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) - 3\cos\left(\frac{1}{x}\right) + 6x\sin\left(\frac{1}{x}\right)$

$f''(x) = -\frac{1}{x}\sin\left(\frac{1}{x}\right) - 4\cos\left(\frac{1}{x}\right) + 6x \cdot \sin\left(\frac{1}{x}\right)$

$$f''\left(\frac{2}{\pi}\right) = -\frac{\pi}{2}\times 1 - 0 + \frac{12}{\pi}$$

$$f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$$

17. Let α, β be the distinct roots of the quadratic equation $x^2 - (t^2 - 5t + 6)x + 1 = 0$ and $a_n = \alpha^n + \beta^n$, then the minimum value of $\frac{a_{2023}+a_{2025}}{a_{2024}}$ is

Ans. $\left(-\frac{1}{4}\right)$

Sol. $x^2 - \left(t^2 - 5t + 6\right)x + 1 = 0$

$$\therefore a_{2025} - \left(t^2 - 5t + 6\right)a_{2024} + a_{2023} = 0$$

$$\Rightarrow \frac{a_{2023}+a_{2025}}{a_{2024}} = \left(t^2 - 5t + 6\right)$$

$$\Rightarrow \frac{a_{2023}+a_{2025}}{a_{2024}} = t^2 - 5t + \frac{25}{4} - \frac{25}{4} + 6$$

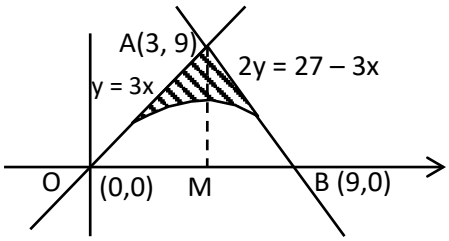
$$\Rightarrow \frac{a_{2023}+a_{2025}}{a_{2024}} = \left(t-\frac{5}{2}\right)^2 + \left(-\frac{1}{4}\right)$$

$\therefore$ Minimum value is $-\frac{1}{4}$

18. Let the area of the region enclosed by curves $y = 3x$, $2y = 27 - 3x$ and $y = 3x - x\sqrt{x}$ be A. Then 10A is equals to

Ans. (162)

Sol.



Area of triangle AOB = $\frac{1}{2}\times 9\times 9$

$$= \frac{81}{2}$$

Area of curve = $\int_0^9 \left(3x - x^{3/2}\right)dx$

$$= \left[3\frac{x^2}{2} - \frac{2}{5}x^{5/2}\right]_0^9$$

$$= 3\times\frac{81}{2} - \frac{2}{5}\times 243$$

$$= 243\left[\frac{1}{2} - \frac{2}{5}\right] = 243\left[\frac{5-4}{10}\right]$$

$$= \frac{243}{10}$$

Area of required region = $\frac{81}{2} - \frac{243}{10}$

$$= \frac{405-243}{10} = \frac{162}{10}$$

$$= 16.2$$

19. If $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$. Then n =

Ans. (47)

Sol. $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$

$$\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{12}}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{7}{11}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{23}{24}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{\frac{23}{24}+\frac{1}{n}}{1-\frac{23}{24n}}\right) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{23n+24}{24n-23}\right) = \frac{\pi}{4}$$

$$\frac{23n+24}{24n-23} = 1$$

$23n + 24 = 24n - 23$

$n = 47$

PHYSICS

1. A particle mass m is situated on earth surface, find minimum kinetic energy so that it can escape from earth's surface.

Ans. $\frac{GMm}{R}$

Sol. $V_e = \sqrt{\frac{2GM}{R}}$

Kinetic energy = $\frac{1}{2}mV_e^2$

$= \frac{1}{2}m \times \frac{2GM}{R} \Rightarrow \frac{GMm}{R}$

2. Which of the following do not explain wave theory of particle?

(1) Reflection (2) Diffraction

(3) Photoelectric effect (4) Interference

Ans. (3)

Sol. Photoelectric effect can not be explain by wave theory of light.

3. Which of the above phenomena represent particle nature

(1) Interference (2) Diffraction (3) Polarisation (4) Photelectric effect

Ans. (4)

Sol. Photoelectric effect can not be explain by wave theory of light.

4. In a prism the ratio of minimum deviation and prism angle is 1 and refractive index of prism is $\sqrt{3}$. Find the prism angle.

Ans. A = 60°

Sol. Given $\mu = \sqrt{3}$

$= \frac{\delta_{min}}{A} = 1$

$= \delta_{min} = A$

$= \mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$

$= \sqrt{3} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\frac{A}{2}}$

$= \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$= \sqrt{3} = 2\cos\frac{A}{2}$

$= \frac{\sqrt{3}}{2} = \cos\frac{A}{2}$

$= \frac{A}{2} = 30°$

A = 60°

5. Speed of wave in a medium is 1.5×10^8 m/s. Relative permittivity of medium (μ_r) is 2. Find the value of relative permeability.

Ans. 2

Sol. $C = \frac{1}{\sqrt{\mu\varepsilon_0}}$ and $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$\mu = \mu_0\mu_r$, $\varepsilon = \varepsilon_0\varepsilon_r$

$$\Rightarrow 1.5\times10^8 = \frac{1}{\sqrt{\mu_0\mu_r \times \varepsilon_0 \times \varepsilon_r}}$$

$$\Rightarrow \mu_0\varepsilon_0 \times \mu_r\varepsilon_r = \frac{1}{(1.5\times10^8)^2}$$

$$\Rightarrow \frac{1}{C^2}\times 2 \times \varepsilon_r = \frac{1}{(1.5\times10^8)^2}$$

$$\Rightarrow \varepsilon_r = \frac{1}{2}\times\left[\frac{C}{1.5\times10^8}\right]^2$$

$$\Rightarrow \varepsilon_r = \frac{1}{2}\times\left(\frac{3\times10^8}{1.5\times10^8}\right)^2 = \frac{1}{2}\times(2)^2 = 2$$

6. There is a pulley block system where $m_2 > m_1$ and acceleration of block m_1 is $\frac{g}{\sqrt{2}}$ upward. Find the ratio of m_1 to m_2.

Ans. $3-2\sqrt{2}$

Sol. $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$

$$\Rightarrow \frac{g}{\sqrt{2}} = \left(\frac{1-\frac{m_1}{m_2}}{1+\frac{m_1}{m_2}}\right)g$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1-\left(\frac{m_1}{m_2}\right)}{1+\left(\frac{m_1}{m_2}\right)}$$

$$\Rightarrow 1+\frac{m_1}{m_2} = \sqrt{2} - \sqrt{2}\left(\frac{m_1}{m_2}\right)$$

$$\Rightarrow \left(\sqrt{2}+1\right)\frac{m_1}{m_2} = \sqrt{2}-1$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$= 3-2\sqrt{2}$

7. A particle is performing SHM with A = 0.6 m and time period (T) = π. Find the maximum velocity.

Ans. 1.2 m/s

Sol. T = π

$$\frac{2\pi}{\omega} = \pi$$

ω = 2 rad/s

Maximum velocity = ωA = 2 × 0.6 = 1.2 m/s.

8. Find the ratio of shortest wavelength of Lyman to Balmer series?

Ans. $\frac{1}{4}$

Sol. $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda_L} = RZ^2\left(\frac{1}{1^2} - \frac{1}{\infty}\right) \quad \text{.....(i)}$$

And

$$\frac{1}{\lambda_B} = RZ^2\left(\frac{1}{2^2} - \frac{1}{\infty}\right) \quad \text{.....(ii)}$$

From (i) and (ii)

$$\frac{\lambda_L}{\lambda_B} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$$

9. The initial velocity of a particle is 100 m/s. After some time it changes to 40 m/s. What is the percentage change in it kinetic energy? Mass of particle is 40 gm.

Ans. −84%

Sol. $\% \Delta K = \frac{K_f - K_i}{K_i} \times 100$

$$= \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\frac{1}{2}mv_i^2} \times 100$$

$$= \left[\left(\frac{V_f}{V_i}\right)^2 - 1\right] \times 100$$

$$= \left[\left(\frac{V_f}{V_i}\right)^2 - 1\right] \times 100$$

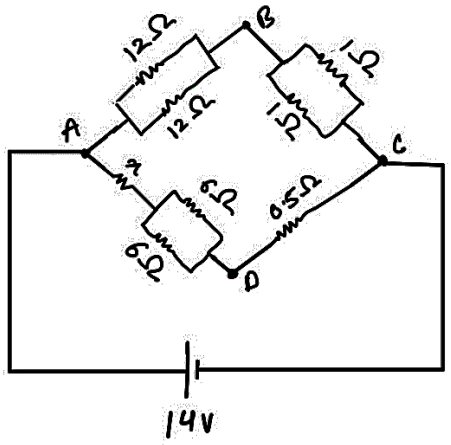
$$= \left[\left(\frac{2}{5}\right)^2 - 1\right] \times 100$$

$$= \left[\frac{4}{25} - 1\right] \times 100$$

$$= -\frac{21}{25} \times 100$$

= −84% (−ve sign indicate it will decrease)

10. In the circuit below potential at B and potential at D are same, then find the value of resistance x.



Ans. x = 3Ω

Sol. If $V_B = V_D$

$R_{AB} \times R_{CD} = R_{BC} \times R_{AD}$ (wheatstone bridge type)

As shown in circuit

$R_{AB} = 6\Omega$

$R_{BC} = \frac{1}{2}\Omega$

$R_{CD} = 0.5\Omega$

$R_{AB} = x + 3\Omega$

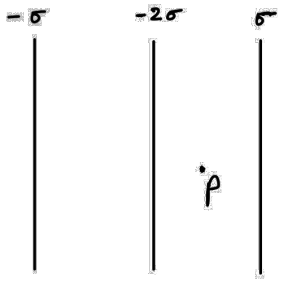
Using formula

$6(0.5) = (0.5)(x + 3)$

$\Rightarrow x + 3 = 6$

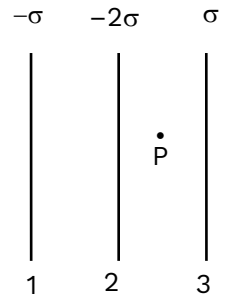
$x = 3\Omega$

11. There are 3 infinite sheets of charge density $-\sigma$, -2σ and σ respectively. Then find the electric field at point P (as shown in figure) [Sheets are non-conducting].



Ans. $E_{net\,P} = \frac{2\sigma}{\varepsilon_0}$

Sol.



$\xleftarrow{E_3=\frac{\sigma}{2\varepsilon_0}}$

$\xleftarrow{E_1=\frac{\sigma}{2\varepsilon_0}}$

$\xleftarrow{E_2=\frac{2\sigma}{2\varepsilon_0}}$

Net electric field at P

$\longleftarrow$P

$$E_{net} = \frac{4\sigma}{2\varepsilon_0}$$

$$\Rightarrow E_{net\,P} = \frac{2\sigma}{\varepsilon_0}$$

12. He gas and O_2 gas are at same temperature find the ratio of their rms speed of molecules.

Ans. $2\sqrt{2}$

Sol. $$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} = \propto \frac{1}{\sqrt{M}}$$

$$\frac{(V_{rms})_{He}}{(V_{rms})_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{He}}}$$

$$\sqrt{\frac{32}{4}}$$

$$= 2\sqrt{2}$$

13. Which of the following materials is not semiconductor.

(1) Germanium (2) Silicon (3) Graphite (4) Copper oxide

Ans. (3)

Sol. Theoretical.

14. If the 4 masses m, m/2, 2m, 4m have same momentum. Which of the following will have maximum kinetic energy?

Ans. $\frac{m}{2}$

Sol. $$KE = \frac{P^2}{2m}$$

$$KE \propto \frac{1}{m}$$

$\frac{m}{2}$ has maximum K.E.

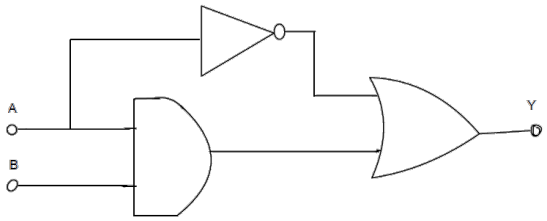
15. **Match the column**

Quantity	**Dimensional Formula**
(i) Torque	(a) $[M^1L^2T^{-2}]$
(ii) Magnetic field	(b) $[M^1A^{-1}T^{-2}]$
(iii) Magnetic moment	(c) $[M^0L^2T^0A^1]$
(iv) Permeabilty	(d) $[M^1L^1T^{-2}A^{-2}]$

Ans. (i)→(a); (ii)→(b); (iii)→(c); (iv)→(d)

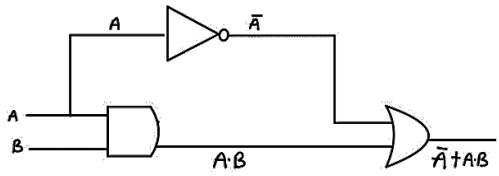
Sol. Theoritical.

16. Find out the truth table?



Ans. Below (Truth table)

Sol.



Truth table

A	B	$\bar{A}$	A.B	Final
0	0	1	0	1
0	1	1	0	1
1	0	0	0	0
1	1	0	1	1

17. Given $T = 2\pi\sqrt{\frac{m}{K}}$, if m decreases by 1% and time period (T) increases by 2%. Find percentage change in K?

Ans. K decreases by 5%

Sol. $T = 2\pi\sqrt{\frac{m}{K}}$

$$\frac{dT}{T} = \frac{1}{2}\frac{dm}{m} - \frac{1}{2}\frac{dK}{K}$$

$$\frac{dT}{T} = \frac{1}{2}(-1\%) - \frac{1}{2}\left(\frac{dK}{K}\right)$$

$$2\% = -\frac{1}{2} - \frac{1}{2}\left(\frac{dK}{K}\right)$$

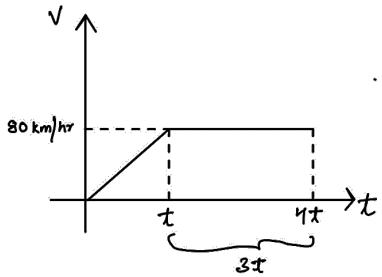
$$\frac{dK}{K} = -5\%$$

K decreases by 5%.

18. A train starting from rest first accelerates up to speed 80 km/h for time t then it moves with a constant speed for time 3t. The average speed of the train for this duration of journey will be:

Ans. 70 km/h

Sol.



$$V_{average} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{\left(\frac{1}{2} \times t \times 80\right) + (80 \times 3t)}{4t}$$

$$= \frac{40t + 240t}{4t}$$

$$= \frac{280}{4} = 70 \text{ km/h}$$

19. A big drop is made out of 1000 small drops, if the ratio of total surface energy of droplets and surface energy of big drop is 10/x. Then find the value of x.

Ans. x = 100

Sol. $V_i = V_f$

$$\Rightarrow 1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\Rightarrow R^3 = 1000r^3$$

$$\Rightarrow R^3 = (10r)^3$$

$$\Rightarrow R = 10r$$

$$\frac{\text{surface energy final}}{\text{surface energy initial}} = \frac{10}{x}$$

$$\Rightarrow \frac{T(4\pi R^2)}{1000 \times T(4\pi r^2)} = \frac{10}{x}$$

$$\Rightarrow \frac{(10r)^2}{1000 \times r^2} = \frac{10}{x}$$

$$\Rightarrow \frac{100}{1000} = \frac{10}{x} \Rightarrow x = 100$$

20. The frequency of electron in the first Bohr orbit in the H-atom is:

Ans. $\nu = 6530 \times 10^{12}$ Hz

Sol. $$\nu = \frac{h}{2\pi mr}$$

$$\nu = \frac{v}{2\pi r} = \frac{L}{4\pi^2 mr^2}$$

Solving by putting the values, $\nu = 6530 \times 10^{12}$ Hz

21. While measuring diameter of a wire using a screw gauge the following readings were noted as main scale reading is 1 mm and circular scale reading is equal to 42 division. Pitch of screw gauge is 1mm and it has 100 divisions on circular scale. The diameter of wire is $\frac{x}{50}$ mm. The value of x is:

Ans. 71

Sol. $$\text{Least count} = \frac{\text{Pitch}}{\text{No of division on circular scale}}$$

$$= \frac{1\text{mm}}{100}$$

= 0.01 mm

Diameter of wire = 1 mm + 42 × 0.01 mm
= 1.42 mm

22. A hydrogen atom having E energy in ground state, when it is revolving in radius of orbit r = 8.48 Å. Its energy become $\frac{E}{x}$. Find the value of x.

Ans. x = 16

Sol. $r = 0.529 \frac{n^2}{z^2}$ Å (for hydrogen z = 1)

$r = 0.529\ n^2$

$\Rightarrow 8.48\ Å = 0.529n^2$

$n^2 = 16$

$n = 4$

and we know that,

$T.E. = -13.6 \frac{z^2}{n^2}$ eV

At $n = 1 \Rightarrow$ T.E. = −13.6 = E

At $n = 4 \Rightarrow T.E. = -\frac{13.6}{4^2} = \frac{E}{x}$

$= \frac{E}{x} = \frac{E}{16}$

x = 16

23. Energy incident on metal surface is 2.48 eV and the stopping potential is 0.5 V. Find the work function.

Ans. ϕ = 1.98 eV

Sol. $eV_s = K.E_{max}$

$V_s = \frac{h\nu - \phi}{e}$

0.5 eV = 2.48 eV − ϕ

ϕ = (2.48 − 0.5)eV

ϕ = 1.98 eV

24. **Statement-1:** Inductor has maximum current at resonance frequency.
Statement-2: Current in purely resistive circuit can never be less than current in series in LRC circuit.
(1) Only statement -1 is correct.
(2) Only statement -2 is correct.
(3) Both of the statements are correct.
(4) None of the statements is correct

Ans. (3)

Sol. Theoretical

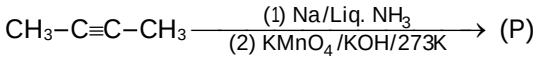
25. A thin spherical shell (conducting) having charge density σ. Find the electric field at the surface of the shell.

Ans. $E = \frac{\sigma}{\varepsilon_0}$

Sol. Theoretical

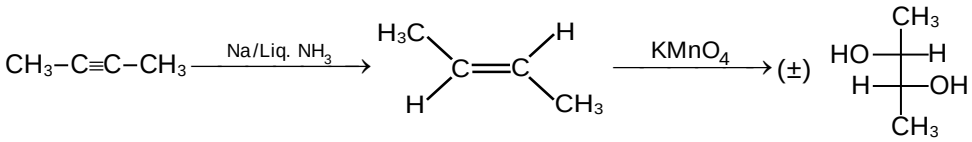
CHEMISTRY

1. Total number of O-atoms in the product (P) formed



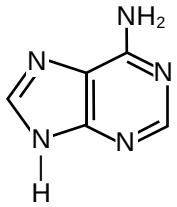
Ans. (02.00)

Sol.

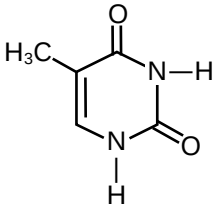


2. Which N-base is not present in D.N.A.

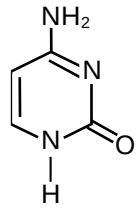
(1)



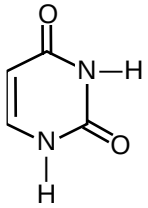
(2)



(3)



(4)



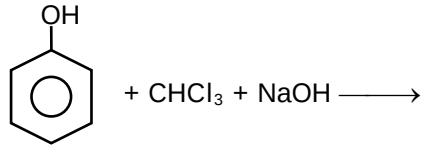
Ans. (4)

Sol. (1) Adenine (2) Thymine

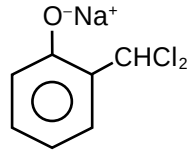
(3) Cytosine (4) Uracil

In D.N.A. N-base is present in Adenine, Thymine, Cytosine, Guanine

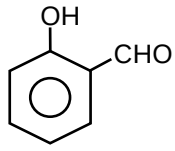
3. In given reaction which one is correct intermediate



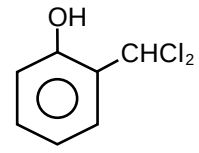
(1)



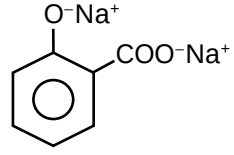
(2)



(3)

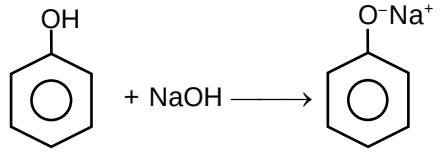


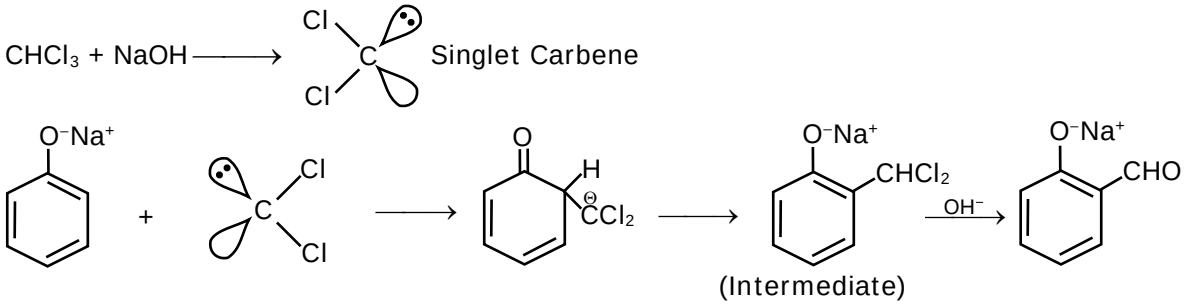
(4)



Ans. (1)

Sol. Given reaction is Reimer Tiemann reaction





4. Match the following

Hybridisation	Structure
(P) sp^3d^2	(A) Octahedral
(Q) sp^3	(B) Trigonal Bipyramidal
(R) dsp^2	(C) Tetrahedral
(S) sp^3d	(D) Square planar

(1) P→A, Q→C, R→D, S→B (2) P→B, Q→A, R→C, S→D

(3) P→B, Q→D, R→A, S→C (4) P→C, Q→A, R→D, S→B

Ans. (1)

Sol. P→A, Q→C, R→D, S→B is the correct match.

5. Find the sum of magnetic moment of basic and amphoteric oxides of Cr.

CrO, Cr_2O_3, CrO_3

Ans. (08.77)

Sol. CrO → Basic → Cr^{+2} → d^4, μ = 3.87 B.M.

Cr_2O_3 → Amphoteric → Cr^{3+} → d^3, μ = 4.90 B.M.

Sum of magnetic moment = 3.87 + 4.90 = 8.77 ≃ 9 B.M.

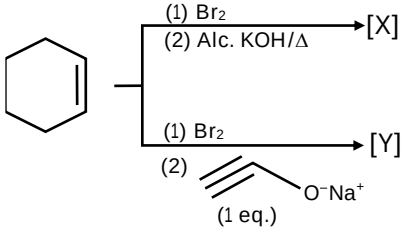
6. For nucleophilic addition reaction most reactive aldehyde is?

(1) HCHO (2) CH_3—CHO (3) C_2H_5—CHO (4) C_3H_7—CHO

Ans. (1)

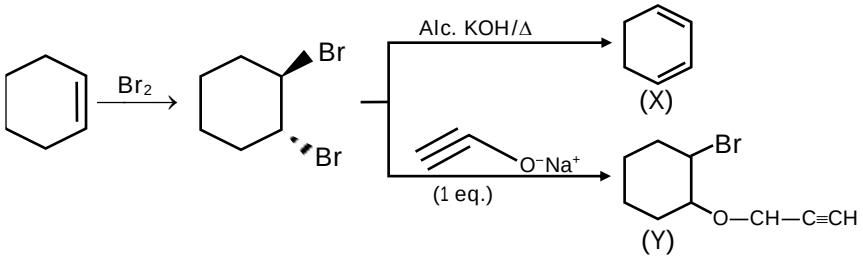
Sol. (i) Minimum hindrance increase rate of nucleophilic addition reaction of aldehyde

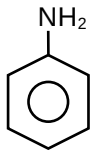
(ii) +I group around carbonyl decrease rate of nucleophilic addition reaction.

7. 

Find sum of total π-electrons in product [X] and [Y]

Ans. (08.00)

Sol. 

8. 

9.3 (gm)

$\xrightarrow[0^\circ C]{NaNO_2+HCl}$ 100%(X) Yield $\xrightarrow{Ph-OH}$ 100%(Y) Yield

Find out mass of (Y)

Ans. (19.80)

Sol. $X = Ph-\overset{+}{N}_2$

Y = Ph—N=N—Ph—OH

Molecular weight of Y = 198 gm/mol

Mole of $Ph-NH_2 = \frac{9.3}{93} = 0.1$

100% Yield mole formed of X and Y is 0.1

Mass of Y = 198 × 0.1 = 19.8

9. Which of the following are element of lanthanide series.

Eu, Cm, Cr, Yb, Lu, Cd

Ans. (3)

Sol. Eu, Yb, Lu

10. $[Co(CN)_6]^{3-}$, $[Co(NH_3)_5Cl]^{2+}$, $[Co(NH_3)_5(H_2O)]^{3+}$, $[Co(H_2O)_4]^{2+}$

I II III IV

Arrange them in increasing order of wave number absorbed

(1) I > III > II > IV (2) I > II > III > IV (3) IV > III > II > I (4) I > II > IV > III

Ans. (1)

Sol. Δ_o order of given compounds is I > III > II > IV

11. Match the Lists

List–I	List–II
(P) CCl_4	(A) Antiseptic
(Q) DDT	(B) Refrigerator
(R) CFC	(C) Insectiside
(S) CHI_3	(D) Fire extinguisher

(1) P→C, Q→D, R→B, S→A (2) P→D, Q→C, R→B, S→A

(3) P→B, Q→C, R→D, S→A (4) P→A, Q→B, R→D, S→C

Ans. (2)

Sol. P→D, Q→C, R→B, S→A is the correct match.

12. Statement–I: 2,4,6-trinitrotoluene is known as picric acid

Statemetn–II: Phenol can be converted into picric acid by addition of concentrated HNO_3 in phenol-2,4-disulphonic acid.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement-I and Statement-II are false

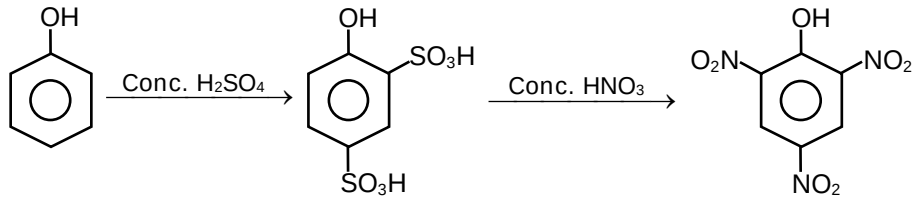
(2) Statement-I is false but Statement-II is true

(3) Both Statement–I and Statement-II are true

(4) Statement-I is true but Statement-II is false

Ans. (2)

Sol.



(Picric acid)

13. Match the following

Compound	Structure
(P) SF_4	(A) T-shape
(Q) NH_4^+	(B) See-saw
(R) BrO_3^-	(C) Tetrahedral
(S) BrF_3	(D) Pyramidal

(1) P→B, Q→C, R→D, S→A (2) P→B, Q→A, R→C, S→D

(3) P→B, Q→D, R→A, S→C (4) P→C, Q→A, R→D, S→B

Ans. (1)

Sol. P→B, Q→C, R→D, S→A is the correct match of structure.

14. $KMnO_4 + C_2O_4^{2-} \xrightarrow{H^+} A + B$

Find change in oxidation state of Mn.

(1) 5 (2) 4 (3) 3 (4) 6

Ans. (1)

Sol. $KMnO_4 + C_2O_4^{2-} \xrightarrow{H^+} Mn^{2+} + CO_2$

$Mn^{+7} \Rightarrow Mn^{+2}$

Change = 5

15. Match the following

Compound	Structure
(P) SO_2Cl_2	(A) Paramagnetic
(Q) NO	(B) Diamagnetic
(R) NO_3^-	(C) Tetrahedral
(S) I_3^-	(D) Linear

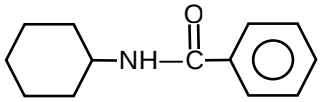
(1) P→B,C; Q→A,D; R→B; S→B,D (2) P→A,B; Q→B,C; R→B; S→B,D

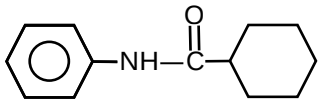
(3) P→B, Q→D, R→A, S→C (4) P→C, Q→A, R→D, S→B

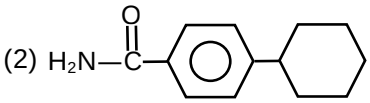
Ans. (1)

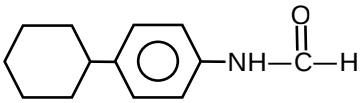
Sol. P→B,C; Q→A,D; R→B; S→B,D is the correct match of properties

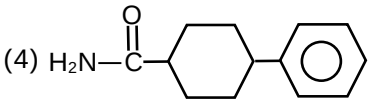
16. Which one is correct metamer of



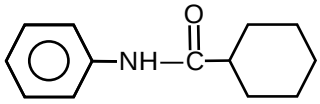
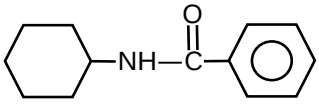
(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol.  is the correct metamer of 

17. Given a NaOH solution of molality = 3m having density = 1.12gm/ml.
Find the molarity of solution

Ans. (3M)

Sol. Molality = 3m

3 moles in 1kg solvent

Weight of 3 moles of NaOH $\Rightarrow$ 3x40 = 120g

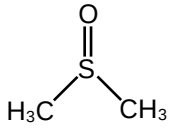
Mass of solution = 1000 + 120 = 1120gm

Density = W/V

V = w/density = 1120/1.12 = 1000ml = 1L

Molarity = moles/volume = 3/1 = 3M

18. Functional group in sulphonic acid

(1) $-SO_3H$ (2) $-SO_4H$ (3) $-SO_2$ (4) 

Ans. (1)

Sol. In sulphonic acid $-SO_3H$ group is present.

19. Find the number of process in which electron gain enthalpy is negative

(A) $Al(g) + e^- \longrightarrow Al^-(g)$

(B) $Be(g) + e^- \longrightarrow Be^-(g)$

(C) $O(g) + 2e^- \longrightarrow O^{2-}(g)$

(D) $Na(g) + e^- \longrightarrow Na^-(g)$

(E) $N(g) + e^- \longrightarrow N^-(g)$

Ans. (02.00)

Sol. Al and Na are only metals (and rest others are non metal) so, they do not accept electron thus:

$Al(g) + e^- \longrightarrow Al^-(g)$

$Na(g) + e^- \longrightarrow Na^-(g)$

Have negative electron gain enthalpy

20. A gas initially at 298K and 5 atm expands adiabatically until it is in equilibrium with constant external pressure of 1 atm. The final temperature (in kelvin) of the gas is: (Given $C_V = \frac{5}{2}R$.)

Ans. (230.0)

Sol. $$\frac{T_2 - T_1}{\gamma - 1} = -P_{ext}\left(\frac{T_2}{P_2} - \frac{T_1}{P_1}\right)$$

$$\frac{T_2 - 298}{1.4 - 1} = -1\left(\frac{T_2}{1} - \frac{298}{5}\right)$$

$$T_2 - 298 = 23.84 - 0.4T_2$$

$$T_2 = 230$$

21. Match the following

Cation	Group Reagent
(P) Al^{3+}	(A) Dilute HCl
(Q) Mn^{2+}	(B) H_2S gas with dil. HCl
(R) Pb^{2+}	(C) NH_4OH with NH_4Cl
(S) Cu^{2+}	(D) H_2S gas with NH_4OH

(1) P→C, Q→D, R→A,B; S→B
(2) P→B, Q→A, R→C, S→D
(3) P→B, Q→D, R→A, S→C
(4) P→A,B; Q→A, R→D, S→B

Ans. (1)

Sol. P→C, Q→D, R→A,B; S→B is the correct match.

22. Assertion (A): Ga is used in thermometer
Reason (R): Melting point of Ga is low where as boiling point is high
(1) Both A and R are correct and R is the correct explanation of A.
(2) Both A and R are correct, but R is not the correct explanation of A.
(3) A is correct but, R is incorrect.
(4) R is correct but, A is incorrect.

Ans. (1)

Sol. Both A and R are correct and R is the correct explanation of A.

23. For the first order reaction, find ratio for completion of 99.9% to completion of 90% of the reaction.

Ans. (3)

Sol. $\ln[A]_t = \ln[A]_0 - kt$

$$Kt = \ln\frac{[A]_0}{[A]_t}$$

$$\frac{kt_{99.9} = \ln\frac{1}{0.001}}{kt_{90} = \ln\frac{1}{0.1}}$$

$$\frac{t_{99.9}}{t_{90}} = \frac{\ln 10^{-3}}{\ln 10^{-1}}$$

$$= 3$$

24. During electrolysis of dilute solution if we add H_2O. What happen to molar conductivity

(1) Increase (2) Remains unchanged

(3) Decrease (4) Depend on electrolyte

Ans. (1)

Sol. On dilution molar conductivity increases

25. A sample contain mixture of helum and oxygen gas the ratio of root mean square speed of helium and oxygen sample is.

(1) $\frac{1}{4}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{2\sqrt{2}}{1}$ (4) $\frac{1}{32}$

Ans. (3)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M_O}}$

$V_{rms} \propto \frac{1}{\sqrt{M_O}}$

$\frac{V_{He}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{He}}} = \sqrt{\frac{32}{4}} = 2\sqrt{2} : 1$

26. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of the lyman series of hydrogen atom is

(1) 4 : 1 (2) 1 : 4 (3) 1 : 2 (4) 2 : 1

Ans. (1)

Sol. $n = \infty \rightarrow n = 1$

$\frac{h_C}{\lambda} = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{\lambda_1}{\lambda_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{1}\right)^2 = 4 : 1$