

29/01/2025

Morning

 **Saral**
Memory Based
Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2025 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.
- (4) **Section - A** : Attempt all questions.
- (5) **Section - B** : Attempt all questions.
- (6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
- (7) **Section - B (21 – 25)** contains 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. **Assertion:** At the peak of mountain, time period of pendulum increases.

Reason: Time period of pendulum increases with decrease in g .

- (1) Assertion is correct, reason is incorrect
- (2) Assertion is incorrect, reason is correct
- (3) Assertion is incorrect, reason is incorrect
- (4) Assertion is correct, reason is correct

Answer (4)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

2. The velocity of a particle moving on a straight line varies with time as $v = At^2 + \frac{Bt}{C+t}$ where A, B, C are constants. Find the dimensions of ABC .

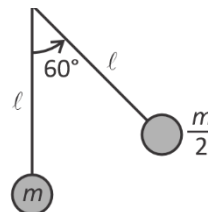
- (1) $L^2 T^{-2}$
- (2) $L^2 T^{-1}$
- (3) $L^2 T^{-3}$
- (4) $L T^{-3}$

Answer (3)

Sol. $[v] = [A][t^2] = \frac{[B][t]}{[C]} = LT^{-1}$

$\Rightarrow [A] = LT^{-3}$
 $[B] = LT^{-1}$
 $[C] = T$
 $[ABC] = L^2 T^{-3}$

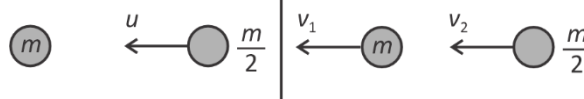
3. A pendulum of mass $\frac{m}{2}$ is released from given situation, find speed of another pendulum after collision ($e = 1$)



- (1) $\sqrt{\frac{3}{2}gl}$
- (2) $\frac{2}{3}\sqrt{gl}$
- (3) $\sqrt{\frac{gl}{3}}$
- (4) $\frac{1}{3}\sqrt{gl}$

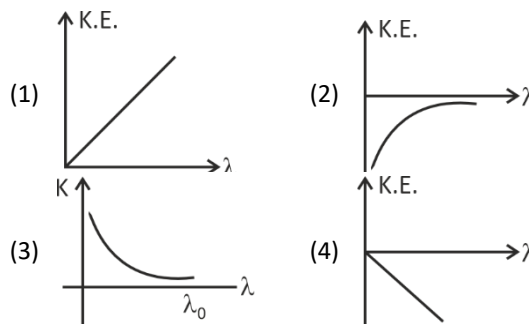
Answer (2)

Sol. Speed before collision = $\sqrt{2 \cdot g \cdot \frac{\ell}{2}} = \sqrt{g\ell}$



$\frac{m}{2}u = mv_1 + \frac{m}{2}v_2$
 $u = 2v_1 + v_2$
 $u = v_1 - v_2$
 $v_1 = \frac{2u}{3} = \frac{2}{3}\sqrt{g\ell}$

4. The graph between wavelengths (λ) of incident light and kinetic energy (K.E.) of photoelectrons in photoelectric effect is

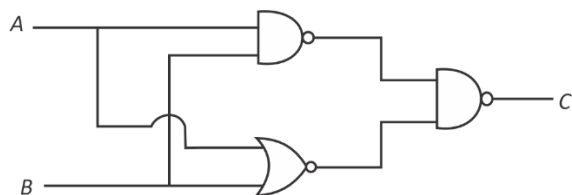


Answer (3)

Sol. $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + KE$ $K = \frac{a}{\lambda} - b$



5. Identify the logic gate represented by the circuit shown below.



- (1) OR Gate (2) NAND Gate
(3) AND Gate (4) NOR Gate

Answer (1)

Sol. $C = \overline{(\overline{AB})(\overline{A+B})}$ De Morgan Rule

$= AB + A + B$ $\overline{\overline{X}Y} = X + Y$

$= A + B$

i.e. OR Gate

6. **Statement-1:** Electromagnetic wave have both energy and momentum.

Statement-2: Rest mass of photon is zero.

- (1) Statement-1 is correct, statement-2 is correct
(2) Statement-1 is correct, statement-2 is incorrect
(3) Statement-1 is incorrect, statement-2 is correct
(4) Statement-1 is incorrect, statement-2 is incorrect

Answer (1)

Sol. Because of radiation pressure, EMW exerts force must carry momentum.

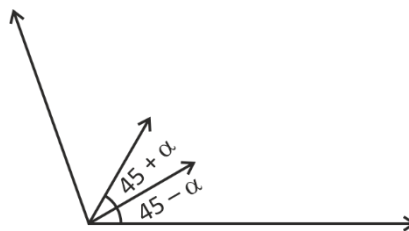
According to special relativity theory, no massive particle can attain speed of light.

7. Two projectile were launched from same position simultaneously only same speed on of the projectile was launched at angle $(45 - \alpha)^\circ$ and the other at an angle of $(45 + \alpha)^\circ$. Find the ratio of maximum height of the projectile.

- (1) $\frac{1 - \sin\alpha}{1 + \sin\alpha}$ (2) $\frac{1 - \sin 2\alpha}{1 + \sin 2\alpha}$
(3) $\frac{1 - \tan\alpha}{1 + \tan\alpha}$ (4) $\frac{1 - \cos\alpha}{1 + \cos\alpha}$

Answer (2)

Sol.



$2gh_1 = 4^2 \sin^2(45 - \alpha)$
 $2gh_2 = 4^2 \sin^2(45 + \alpha)$

$\Rightarrow \frac{h_1}{h_2} = \frac{\left(\frac{\cos\alpha}{\sqrt{2}} - \frac{\sin\alpha}{\sqrt{2}}\right)^2}{\left(\frac{\cos\alpha}{\sqrt{2}} + \frac{\sin\alpha}{\sqrt{2}}\right)^2}$

$\Rightarrow \frac{h_1}{h_2} = \frac{\cos^2\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha}{\cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha}$

$\Rightarrow \frac{h_1}{h_2} = \frac{1 - \sin 2\alpha}{1 + \sin 2\alpha}$

8. A river is flowing with speed 9 km/h. Boat is going downstream. Speed of boat in still water is 27 km/h. A person in boat throws a ball upwards with speed 10 m/s. Find range of the ball as seen by an observer at bank of river

- (1) 10 m (2) 20 m
(3) 25 m (4) $20\sqrt{3}$ m

Answer (2)

Sol. $T = \frac{2u}{g} = \frac{2 \times 10}{10} = 2$ s

$R = (9 + 27) \frac{5}{18} \times 2$

$R = 20$ m

9. Which of two physical quantities have same dimensions?

- (1) Angular momentum and Planck's constant
(2) Torque and moment of inertia
(3) Impulse and surface tension
(4) Momentum and work done

Answer (1)



Sol. (1) $\frac{L}{h} = \frac{mvr}{Et} = \frac{mv^2}{E} \equiv M^0 L^0 T^0$

(2) $\frac{\bar{L}}{I} = \frac{rF \sin \theta}{mr^2} \equiv M^0 L^0 T^{-2}$

(3) $\frac{l}{s} = \frac{Ft}{F/\ell} \equiv LT$

(4) $\frac{p}{\omega} = \frac{mv}{mv^2} = L^{-1} T$

10. If radius of first Bohr's orbit of H-atom is a_0 . Then find the radius of 2nd Bohr's orbit of H-atom.

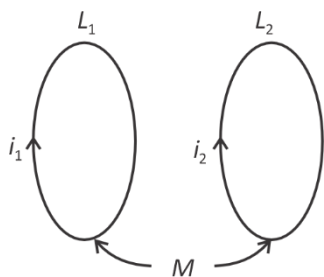
- (1) $8a_0$ (2) $4a_0$
 (3) $2a_0$ (4) $6\pi a_0$

Answer (2)

Sol. $a = \frac{a_0 n^2}{2}$

So, $a(n=2) = 4a_0$

11. Two coils having self-inductance L_1 and L_2 are placed closely such that they have a mutual inductance M . If they carry currents i_1 and i_2 as shown in the figure, then the induced emf in coil 1 is



(1) $-L_1 \left(\frac{di_1}{dt} \right) + M \left(\frac{di_2}{dt} \right)$ (2) $-L_1 \left(\frac{di_1}{dt} \right) - M \left(\frac{di_2}{dt} \right)$

(3) $-L_1 \left(\frac{di_2}{dt} \right) + M \left(\frac{di_1}{dt} \right)$ (4) $-L_1 \left(\frac{di_2}{dt} \right) - M \left(\frac{di_1}{dt} \right)$

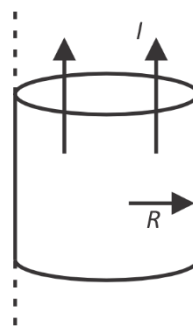
Answer (2)

Sol. $\phi_1 = L_1 i_1 + M i_2$

$$\frac{-d\phi_1}{dt} = -L_1 \left(\frac{di_1}{dt} \right) - M \left(\frac{di_2}{dt} \right)$$

$$\varepsilon_1 = -L_1 \left(\frac{di_1}{dt} \right) - M \left(\frac{di_2}{dt} \right)$$

12. An infinite solid cylindrical wire of radius R carries a current I uniformly distributed along its area. The distance from the centre where the magnetic field is equal to $\frac{\mu_0 I}{4\pi R}$ is



- (1) $\frac{R}{2}$ (2) R
 (3) $4R$ (4) 0

Answer (1)

Sol. $B_{\text{inside}} = \frac{\mu_0 I r}{2\pi R^2}$

$$\Rightarrow r = \frac{R}{2}$$

$$B_{\text{outside}} = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow r = 2R$$

13. When ball is kept under sea at depth 2.5 km. Find percentage change in its volume. If bulk modulus of water is 2×10^9 Pa.

- (1) 2% (2) 1.5%
 (3) 1.25% (4) 2.75%

Answer (3)



Sol. $\beta = \frac{\Delta P}{-\frac{\Delta V}{V}} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta P}{\beta}$

$$= \frac{10^3 \times 10 \times 2500}{2 \times 10^9} \times 100$$

$$= \frac{25}{20}$$

$$= 1.25\%$$

14. Heat given to 0.5 moles of a monoatomic gas at constant pressure is 500 J. Initial temperature of gas was 27°C. Find value of ΔU and ΔT .

- (1) 300 J, 48°C (2) 150 J, 24°C
 (3) 180 J, 16°C (4) 210 J, 18°C

Answer (1)

Sol. At constant pressure,

$$\Delta Q = nC_p \Delta T$$

$$500 = \frac{n \cdot 5}{2} R \Delta T$$

$$\Delta U = nC_v \Delta T = \frac{3}{2} nR \Delta T$$

$$= \frac{3}{2} \times 200$$

$$= 300 \text{ J}$$

$$\Delta T = \frac{200 \times 3}{0.5 \times 25}$$

$$\Delta T = 48$$

15. **Assertion:** A negative potential is required to stop the photoelectron.

Reason : Speed of electron decreases when a negative potential is applied in a photo cell.

- (1) Assertion is correct but Reason is false
 (2) Assertion is correct and Reason is also correct
 (3) Assertion is false but Reason is correct
 (4) Assertion is false and Reason is also false

Answer (2)

Sol. Conceptual

16. If electric dipole of dipole moment \vec{P} is placed in electric field \vec{E} with $\vec{P} \parallel \vec{E}$. It is rotated slightly (and slowly) and released. Find the time period of oscillation of dipole (moment of inertia of dipole is I).

(1) $T = 2\pi \sqrt{\frac{I}{PE}}$

(2) $T = \frac{1}{2\pi} \sqrt{\frac{PE}{I}}$

(3) $T = 2\pi \sqrt{\frac{IE}{P}}$

(4) $T = \frac{1}{2\pi} \sqrt{\frac{PI}{E}}$

Answer (1)



$$\tau_{(R)} = -(\vec{P})(\vec{E})\sin\theta \approx -|\vec{P}||\vec{E}|\theta$$

$$\alpha = -\omega^2\theta = -\frac{PE}{I} \cdot \theta$$

$$\Rightarrow T = 2\pi \sqrt{\frac{I}{PE}}$$

17. In adiabatic process of closed system, work done by the gas depends explicitly on

- (1) Change in volume
 (2) Change in pressure
 (3) Change in temperature
 (4) Change in number of moles

Answer (3)

Sol. $\Delta\theta = \Delta V + \Delta W \Rightarrow \Delta W = -\Delta V$

$$W = -\frac{\mu R \Delta T}{\gamma - 1} = -\frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1)$$

Only Change in temperature
Both on change in pressure and volume



18. Match the correct option for List-I and List-II, where symbols have usual meanings.

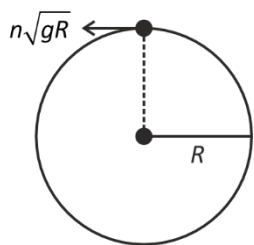
	List-I		List-II
(A)	Electric field inside the spherical shell	(i)	$\frac{\sigma}{2\epsilon_0}$
(B)	Electric field just outside the spherical shell	(ii)	$\frac{\sigma}{\epsilon_0}$
(C)	Electric field inside the charged parallel plate capacitor	(iii)	0
(D)	Electric field of infinite charge sheet	(iv)	$\frac{2\sigma}{\epsilon_0}$

- (1) A-(iii), B-(ii), C-(iv), D-(ii)
- (2) A-(iii), B-(ii), C-(ii), D-(i)
- (3) A-(iii), B-(ii), C-(ii), D-(iv)
- (4) A-(iv), B-(iii), C-(i), D-(ii)

Answer (2)

19. A particle is able to complete the vertical circular motion with speed $n\sqrt{gR}$ at top-most point. Find the ratio of

$$\frac{KE_{\text{Bottom}}}{KE_{\text{Top}}}$$



- (1) $\frac{n^2 + 4}{n}$
- (2) $\frac{n}{n^2 + 4}$
- (3) $\frac{n^2 + 2}{n}$
- (4) $\frac{n^2 + 4}{n^2}$

Answer (4)

Sol. $V_{\tau} = n\sqrt{gR}$

$$V_{\text{Bottom}}^2 = V_{\tau}^2 + 4gR = n^2gR + 4gR$$

$$\frac{KE_{\text{Bottom}}}{KE_{\text{Top}}} = \frac{gR(n^2 + 4)}{gRn^2} = \frac{n^2 + 4}{n^2}$$

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In a hydraulic lift, the two sides have areas $A_1 = 25 \text{ cm}^2$ and $A_2 = 100 \text{ cm}^2$. If a force of 100 N is applied normally on the area A_1 , then the force on the area A_2 is _____ N.

Answer (400)

Sol. From Pascal's law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ or } \frac{100 \text{ N}}{25 \text{ cm}^2} = \frac{F_2}{100 \text{ cm}^2}$$

$$\Rightarrow F_2 = 400 \text{ N}$$

22. Find magnitude of component of torque about origin in z-direction when force $\vec{F} = \hat{i} - \hat{j} + \hat{k}$ acts at (1, 1, 1).

Answer (2)

Sol. $\vec{\tau}_z = \hat{k}(-1, -1) = -2\hat{k} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & +1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

23.

24.

25.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. Which of the following is animal starch?
 (1) Glycogen
 (2) Lactose
 (3) Amylopectin
 (4) Amylose

Answer (1)

Sol. Lactose is present in milk.

Amylopectin and amylose are part of starch.

Glycogen is animal starch.

2. **Statement 1 :** Correct order of ionic radius for Mg^{2+} ,

Na^+ , O^{2-} , & F^- is $F^- > O^{2-} > Na^+ > Mg^{2+}$

Statement 2 : Correct order of electron gain enthalpy

for 17th group elements follows order $Cl > F > Br > I$

(Magnitude only)

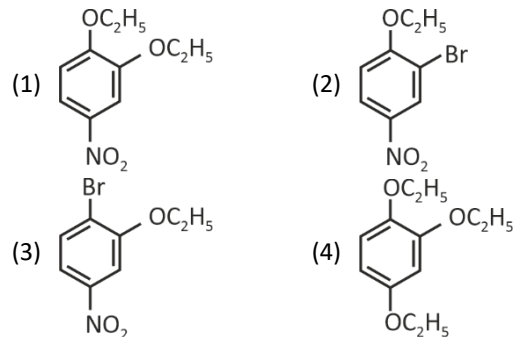
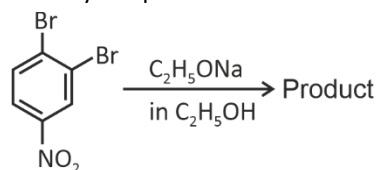
- (1) Statement-1 & Statement-2 are correct
 (2) Statement-1 is correct Statement-2 is incorrect
 (3) Statement-1 & Statement-2 are incorrect
 (4) Statement-1 is incorrect Statement-2 is correct

Answer (4)

Sol.: Correct order of ionic radius $O^{2-} > F^- > Na^+ > Mg^{2+}$

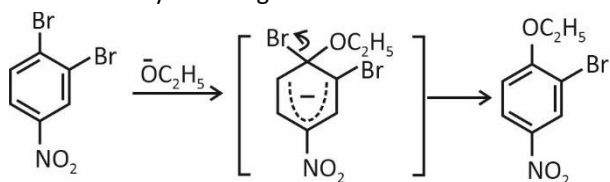
Correct order for electron gain enthalpy (Magnitude)
 $Cl > F > Br > I$

3. Identify the product formed in the following reaction



Answer (2)

Sol. Aryl halides having strong electron withdrawing group like NO_2 either at the ortho or para position undergo SNAR reaction easily involving carbanion intermediate



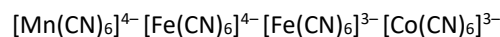
4. Which of the following is steam volatile

- (1) Ortho nitrophenol (2) Para nitrophenol
 (3) Para aminophenol (4) Para nitroaniline

Answer (1)

Sol. Ortho nitrophenol is steam volatile due to intramolecular H-bonding. Its B.P is less. p-nitrophenol, p-amino phenol, paranitro aniline show intermolecular H-bonding

5. Consider the following complexes



- (1) (2) (3) (4)

Correct order of CFSE (Δ) will be

- (1) $3 > 4 > 2 > 1$ (2) $4 > 3 > 2 > 1$
 (3) $4 > 3 > 1 > 2$ (4) $3 > 4 > 1 > 2$

Answer (2)

- Sol.** (1) $[Mn(CN)_6]^{4-}$, Mn^{2+}
 (2) $[Fe(CN)_6]^{4-}$, Fe^{2+}
 (3) $[Fe(CN)_6]^{3-}$, Fe^{3+}
 (4) $[Co(CN)_6]^{3+}$, Co^{3+}

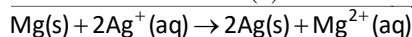
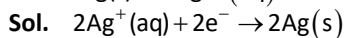
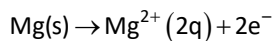
order of CFSE will be $4 > 3 > 2 > 1$



$$(3) E_{\text{cell}} = E_{\text{cell}}^{\circ} + \frac{RT}{F} \ln \frac{[\text{Mg}^{2+}]}{[\text{Ag}^+]^2}$$

$$(4) E_{\text{cell}} = E_{\text{cell}}^{\circ} + \frac{RT}{2F} \ln \frac{[\text{Ag}^+]^2}{[\text{Mg}^{2+}]}$$

Answer (1)



$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{RT}{2F} \ln \frac{[\text{Mg}^{2+}]}{[\text{Ag}^+]^2}$$

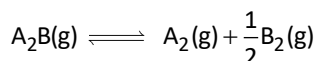
11. The correct order of melting point of d-block elements is :

- (1) Fe > Mn (2) Tc > Ru
 (3) Os > Re (4) Ta > W

Answer (1)

Sol. Melting point order is Fe > Mn, Ru > Tc, Re > Os, W > Ta

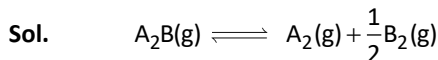
12. Consider the following reaction



If P is total pressure at equilibrium & K_P is equilibrium constant. Then α in terms of K_P & P is (Assume $\alpha \ll 1$)

- (1) $\sqrt{\frac{K_P}{P}}$ (2) $\sqrt[4]{\frac{K_P}{P}}$
 (3) $\sqrt{\frac{2K_P}{P}}$ (4) $\sqrt[3]{\frac{2K_P^2}{P}}$

Answer (4)



$$t = 0 \quad p_0$$

$$t = t_{\text{eq}} \quad p_0(1 - \alpha) \quad p_0\alpha \quad p_0\frac{\alpha}{2}$$

$$P = p_0 + p_0\frac{\alpha}{2}$$

$$P = p_0\left(1 + \frac{\alpha}{2}\right) \quad (P \approx p_0)$$

At equilibrium $K_P = \frac{(p_{\text{A}_2})\left(\frac{p_{\text{B}_2}}{2}\right)}{(p_{\text{A}_2\text{B}})} = (\alpha \ll 1)$

$$k_p = \frac{(p_0\alpha)\left(p_0\frac{\alpha}{2}\right)^{\frac{1}{2}}}{p_0(1 - \alpha)} = k_p = \alpha\left(p\frac{\alpha}{2}\right)^{\frac{1}{2}}$$

$$\frac{K_P}{\frac{1}{P^2}} = \frac{\alpha^{3/2}}{2^{1/2}}$$

$$\frac{2K_P^2}{P} = \alpha^3$$

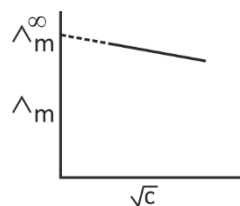
$$\sqrt[3]{\frac{2K_P^2}{P}} = \alpha$$

13. \wedge_m is linearly dependent to \sqrt{c} for an electrolyte, then molar conductance for the same electrolyte at infinite dilution shows

- (1) Small increase (2) Small decrease
 (3) Sharp increase (4) Sharp decrease

Answer (1)

Sol. \wedge_m decreases linearly with \sqrt{c} for strong electrolytes having small -ve slope. It can be extrapolated to \wedge_m^{∞} as $c \rightarrow 0$.



The molar conductance of the same electrolyte at infinite dilution or as $c \rightarrow 0$ shows small increase.

14. Given ionisation enthalpy of element E(g) is 300 kJ/mol and electron gain enthalpy of A, B, C and D gaseous atoms are -320 kJ/mol, -340 kJ/mol, -200 kJ/mol and -250 kJ/mol, then what will be the correct order of ionic nature of compounds?

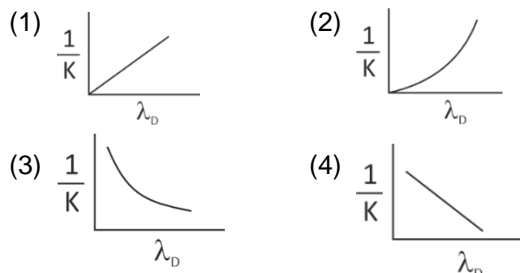
- (1) EB > EA > ED > EC (2) EB > EA > EC > ED
 (3) EC > ED > EA > EB (4) EC > ED > EB > EA

Answer (1)



Sol. Since ionic strength depends on IE of electropositive atom; E.G.E. of electronegative element and lattice energy, more the negative value of electron gain enthalpy, more will be ionic nature.

15. Graph between de Broglie wavelength (λ_D) and kinetic energy (K) of an electron is



Answer (2)

Sol. de Broglie wavelength (λ_D) of an electron of mass (m), moving with velocity (v) is given by

$$\lambda_D = \frac{h}{mv}$$

Where h is planck's constant.

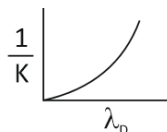
$$\text{Kinetic energy (K)} = \frac{1}{2}mv^2$$

$$mv = \sqrt{2mK}$$

$$\lambda_D = \frac{h}{\sqrt{2mK}}$$

$$\frac{1}{K} = \frac{2m\lambda_D^2}{h^2}$$

Plot of $\frac{1}{K}$ vs λ_D is



16. Which of the following ions is strongest oxidising agent

$$\text{Given : } E_{\text{Al}^{3+}/\text{Al}}^\circ = -2.7\text{V}$$

$$E_{\text{Cu}^{2+}/\text{Cu}}^\circ = 0.34\text{V}$$

$$E_{\text{Pb}^{4+}/\text{Pb}^{2+}}^\circ = 1.8\text{V}$$

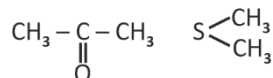
$$E_{\text{Tl}^{3+}/\text{Tl}^{2+}}^\circ = -0.37\text{V}$$

- (1) Al^{3+} (2) Cu^{2+}
 (3) Pb^{4+} (4) Tl^{3+}

Answer (3)

Sol. Reduction potential of $\text{Pb}^{4+} \rightarrow \text{Pb}^{2+}$ is most positive, Hence Pb^{4+} is strongest oxidising agent.

17. Total number of nucleophiles among the following are Ph-SH , OH^- , $\text{CH}_2=\text{CH}_2$, >N-CH_3 , H_3O^+ ,



- (1) 5 (2) 6
 (3) 7 (4) 4

Answer (2)

Sol. Species having atom containing lone pair available for donation can act as nucleophile

18. Radius of 1st orbit of hydrogen atom is $a_0 \text{ \AA}$, then find de-Broglie wavelength of 2nd orbit of hydrogen atom.

- (1) $4\pi a_0$ (2) $\frac{4}{\pi a_0}$
 (3) $8\pi a_0$ (4) $2\pi a_0$

Answer (1)

$$\text{Sol. } r_n = a_0 \frac{n^2}{Z}$$

for $n = 1, Z = 1$

$$r_1 = a_0$$

$$r_2 = a_0 \frac{4}{1} = 4a_0$$

$$2\pi r_n = n\lambda$$

$$\lambda = \frac{2\pi r_2}{2} = \frac{2\pi \times 4a_0}{2} = 4\pi a_0$$

19.

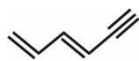
20.

SECTION - B

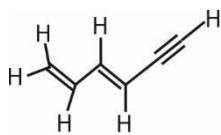
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.



21. Calculate the total number of sigma and π -bonds in the given molecule?



Answer (15)



Sol.

Number of sigma bonds = 11 σ

Number of π -bonds = 4 π

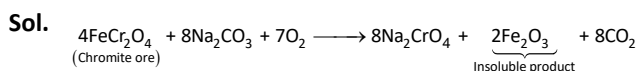
Total = 15

22. Chromite ore + $\text{Na}_2\text{CO}_3 + \text{O}_2 \rightarrow$ Insoluble product

Calculate the molar mass of insoluble product formed.

(Given : Molar mass of Cr = 52 g/mol, Na = 23 g/mol, Fe = 56 g/mol, O = 16 g/mol)

Answer (160)

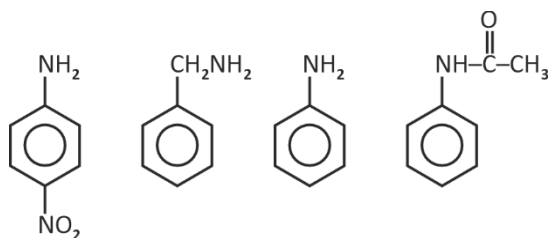


Molar mass of Fe_2O_3

$$\Rightarrow 2(56) + 3(16)$$

$$\Rightarrow 160$$

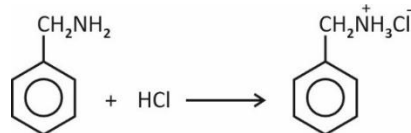
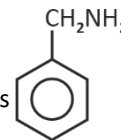
23. Consider the following amines



1 gram of most basic compound reacts with x mg of HCl, calculate value of x.

Answer (341)

Sol. Most basic compound is



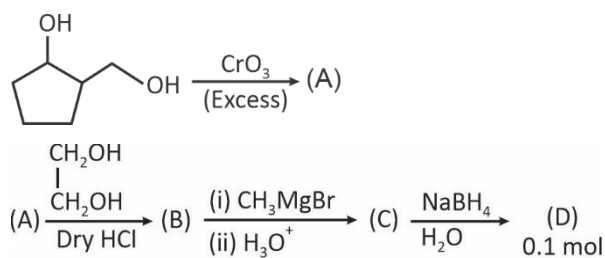
$$\frac{1}{107} \text{ mol} \quad \frac{1}{107} \text{ mol}$$

mass of HCl required to react with Benzyl amine

$$= \frac{1}{107} \times 36.5 \text{ g}$$

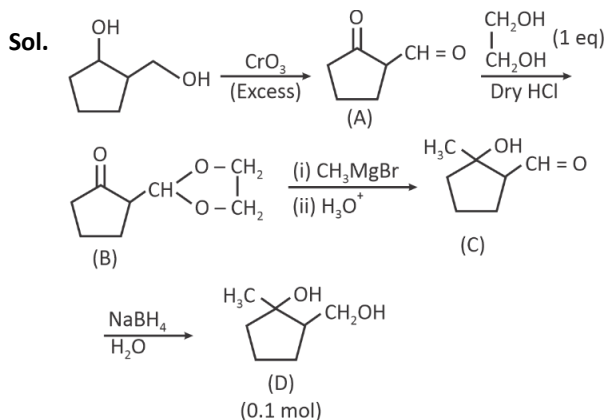
$$= 0.341 \text{ g} = 341 \text{ mg}$$

24. Consider the following reaction



Find the mass of final product(D) formed in g

Answer (13)



Molar mass of D = 130 g mol⁻¹

Mass of 0.1 mol of (D) formed = 13g

25.



MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}$ is equal to

- | | |
|-------------------|-------------------|
| (1) $\frac{5}{3}$ | (2) $\frac{8}{3}$ |
| (3) 3 | (4) $\frac{7}{3}$ |

Answer (1)

Sol.
$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^3 + 6k^2 + 11k + 6 - 1}{(k+3)!}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{(k+1)(k+2)(k+3) - 1}{(k+3)!}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k!} - \frac{1}{(k+3)!}$$

$$= \left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \right) - \left(\frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \dots \right)$$

$$= (e-1) - \left(e - 1 - \frac{1}{1!} - \frac{1}{2!} - \frac{1}{3!} \right)$$

$$= 1 + \frac{1}{2} + \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$$

2. Sum of first three terms of an AP with integer common difference is 54 and sum of first twenty terms lies between 1600 to 1800, find a_{11}

- | | |
|---------|---------|
| (1) 108 | (2) 90 |
| (3) 111 | (4) 115 |

Answer (2)

Sol. Let AP be $a, a + d, a + 2d \dots$

$$3a + 3d = 54$$

$$a + d = 18 \quad \dots(i)$$

$$1600 < \frac{20}{2} [2a + 19d] < 1800$$

$$160 < 2a + 19d < 180$$

$$160 < 18 \times 2 + 17d < 180$$

$$\frac{124}{7} < d < \frac{144}{17}$$

$$\therefore d \in \text{Integer} \Rightarrow d = 8$$

$$a + d = 18$$

$$\Rightarrow a = 10$$

$$\text{Now } a_{11} = a + 10d$$

$$= 10 + 10 \times 8$$

$$= 90$$

3. Evaluate $I = 80 \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{(9 \sin x + 16 \cos x)} dx$

(1) $\frac{80}{327} \left[\frac{25\pi}{2} + 7 \ln \left(\frac{9}{16} \right) \right]$

(2) $\frac{80}{337} \left[\frac{25\pi}{2} - 7 \ln \left(\frac{9}{16} \right) \right]$

(3) $\frac{40}{327} \left[\frac{25\pi}{2} + 7 \ln \left(\frac{9}{16} \right) \right]$

(4) $\frac{40}{327} \left[\frac{25\pi}{2} - 7 \ln \left(\frac{9}{16} \right) \right]$

Answer (2)

Sol. $\sin x \cos x = A[9 \sin x + 16 \cos x] + B[9 \cos x - 16 \sin x]$

$$= \sin x [9A - 16B] + \cos x [16A + 9B]$$

$$\Rightarrow 9A - 16B = 16A + 9B = 1$$

$$\Rightarrow -7A = 25B \Rightarrow B = \frac{-7A}{25}$$

$$9A - 16 \left(\frac{-7A}{25} \right) = 1 \Rightarrow 337A = 25, B = \frac{-7}{337}$$

$$I = 80 \int_0^{\frac{\pi}{2}} \frac{\frac{25}{337} (9 \sin x - 16 \cos x) - \frac{7}{337} [9 \cos x - 16 \sin x]}{(9 \sin x + 16 \cos x)} dx$$



$$I = 80 \int_0^{\frac{\pi}{2}} \frac{25}{337} dx - 80 \int_0^{\frac{\pi}{2}} \frac{7}{337} \frac{d(9\sin x + 16\cos x)}{(9\sin x + 16\cos x)}$$

$$I = 80 \left(\frac{25x}{337} \right) \Big|_0^{\frac{\pi}{2}} - \frac{80 \cdot 7}{337} \ln(9\sin x + 16\cos x) \Big|_0^{\frac{\pi}{2}}$$

$$I = \frac{80 \cdot 25}{337} \left(\frac{\pi}{2} \right) - \frac{80 \cdot 7}{337} \ln \left(\frac{9}{16} \right)$$

4. If R be a relation defined on $(0, \pi/2)$ such that $xRy \Rightarrow \sec^2 x - \tan^2 y = 1$, then the relation R is

- (1) Equivalence relation
- (2) Reflexive and transitive only
- (3) Symmetric and transitive only
- (4) Neither reflexive nor transitive

Answer (1)

Sol. $xRy \Rightarrow \sec^2 x - \tan^2 y = 1$

- $xRx \Rightarrow \sec^2 x - \tan^2 x = 1$

$\Rightarrow R$ is reflexive

- $xRy \Rightarrow yRx$

$\Rightarrow \sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 x = (1 + \tan^2 y) - (\sec^2 x - 1)$

$= 2\sec^2 x + \tan^2 y$

$= 2 - (\sec^2 x - \tan^2 y) = 2 - 1 = 1$

$\Rightarrow R$ is symmetric

- $xRy \Rightarrow yRz$

$\Rightarrow \sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 z = 1$

Add $\Rightarrow \sec^2 x + \sec^2 y - \tan^2 y - \tan^2 z = 2$

$\Rightarrow \sec^2 x + (1) - \tan^2 z = 2$

$\Rightarrow \sec^2 x - \tan^2 z = 1$

$\Rightarrow xRz$

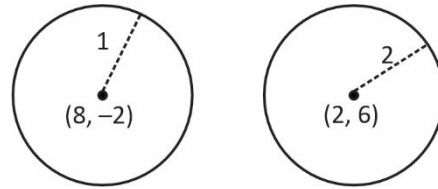
$\Rightarrow R$ is transitive.

5. If z_1 lies on $|z - 8 + 2i| = 1$ and z_2 lies on $|z - 2 - 6i| = 2$, then $|z_1 - z_2|_{\min}$ is

- (1) 8
- (2) 10
- (3) 7
- (4) 9

Answer (3)

Sol.



$$|Z_1 - Z_2|_{\min} = \sqrt{(8-2)^2 + (-2-6)^2} - 3$$

$$= \sqrt{36 + 64} - 3$$

$$= 10 - 3 = 7$$

6. If $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x - 1)$, then find the sum of all values of 'x'.

- (1) 1
- (2) $\frac{1}{2}$
- (3) 0
- (4) $\frac{3}{2}$

Answer (3)

Sol. $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x - 1)$

Now $-1 \leq 2x - 1 \leq 1$

$0 \leq x \leq 1$

$\Rightarrow \pi + \sin^{-1} x + \sin^{-1}(2x - 1) \geq \frac{\pi}{2}$

and $\cos^{-1} x$ for $x \in [0, 1]$ always lies in $\left[0, \frac{\pi}{2}\right]$

$\Rightarrow \text{LHS} = \text{RHS} = \frac{\pi}{2}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{2} \Rightarrow \boxed{x=0}$

Hence only $x = 0$ is the possible solution.

Sun of all solution = 0.

7. If $\begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \sin 4x \\ 1 + \sin^2 x & \cos^2 x & \sin 4x \\ \sin^2 x & \cos^2 x & 1 + \sin 4x \end{vmatrix} = L$

and $L_{\min} = m$ and $L_{\max} = M$, then $|M^4 - m^4|$ is

- (1) 79
- (2) 78
- (3) 80
- (4) 76

Answer (3)



11. If two lines $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$;

$L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$. Let the line L_3 passes through the

point (α, β, γ) such that L_3 is perpendicular to L_1 to L_2 and L_3 intersects L_1 . Then $|5\alpha - 11\beta - 8\gamma|$ is equal to

- (1) 18
- (2) 25
- (3) 16
- (4) 20

Answer (2)

Sol. Let the L_3 be

$$\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c}, (a\hat{i} + b\hat{j} + c\hat{k}) \text{ is parallel to}$$

$$(\hat{i} - \hat{j} + 2\hat{k}) \times (-\hat{i} + 2\hat{j} + \hat{k})$$

$$(a, b, c) \equiv (5, 3, 1)$$

$$\Rightarrow \frac{x-\alpha}{5} = \frac{y-\beta}{3} = \frac{z-\gamma}{-1}$$

\Rightarrow Let the point of intersection be P .

$$\Rightarrow 5\lambda + \alpha = P + 1, 3\lambda + \beta = P + 2, -\lambda + \gamma = 2P + 1$$

$$\Rightarrow \alpha = (P + 1 - 5\lambda), \beta = (-P + 2 - 3\lambda), \gamma = (2P + 1 + \lambda)$$

$$\Rightarrow |5\alpha - 11\beta - 8\gamma| = |-25| = 25$$

- 12.
- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The minimum value of n for which the number of integer terms in the binomial expansion $\left(7^{\frac{1}{3}} + 11^{\frac{1}{12}}\right)^n$ is 183, is

Answer (2184)

Sol. $T_{k+1} = {}^nC_k \cdot \left(11^{\frac{1}{12}}\right)^k \cdot 7^{\frac{1}{3}(n-k)}$

$$12|k \text{ and } 3|(n-k) \Rightarrow 3|n$$

For integer terms.

\Rightarrow Multiples of 12 for k would work.

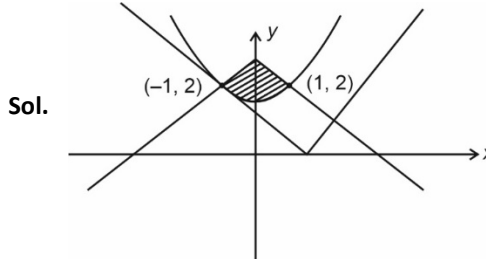
$\Rightarrow k = 0, 12, 24, \dots$

$$\Rightarrow k_{\max} = 12 \times 182 = 2184$$

\Rightarrow Minimum value of n will be 2184 as $3|2184$.

22. Area enclosed by $y \geq |x - 1|$, $y + |x| \leq 3$, $x^2 \leq 2y - 3$ is A , then $6A$ is (in sq. units)

Answer (10)



Sol.

$$\text{Area} = 2 \left[\int_0^1 (3-x) - \left(\frac{x^2+3}{2} \right) dx \right]$$

$$= 2 \left[3x - \frac{x^2}{2} - \frac{1}{2} \left[\frac{x^3}{3} + 3x \right] \right]_0^1$$

$$= 2 \left(3 - \frac{1}{2} - \frac{1}{2} \left[\frac{1}{3} + 3 \right] \right)$$

$$= 2 \left(\frac{5}{6} - \frac{1}{6} - \frac{3}{2} \right) = 2 \left(\frac{5}{6} \right) = A$$

$$6A = 10$$



23. Number of 7 digit numbers made with the digits 1, 2, 3 such that sum of the digits is 11 is equal to

Answer (161)

Sol. Case-I : 3 2 2 1 1 1 1

$$n_1 = \frac{7!}{4!2!} = 105$$

Case II: 2 2 2 2 1 1 1

$$\Rightarrow n_2 = \frac{7!}{4!3!} = 35$$

Case III : 3 3 1 1 1 1 1

$$\Rightarrow n_3 = \frac{7!}{5!2!} = 21$$

$$\begin{aligned} \text{Total numbers } n_1 + n_2 + n_3 \\ = 105 + 35 + 21 \\ = 161 \end{aligned}$$

24. The minimum value of p such that

$$\lim_{x \rightarrow 0^+} x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{p}{x} \right] \right) - x^2 \left(\left[\frac{1}{x^2} \right] + \left[\frac{2}{x^2} \right] + \dots + \left[\frac{9}{x^2} \right] \right) \geq 1,$$

is equal to (where $[.]$ represents greatest integer function)

Answer (24)

Sol. Since $x^2 \left[\frac{r^2}{x^2} \right] = x^2 \left(\frac{r^2}{x^2} - \left\{ \frac{r^2}{x^2} \right\} \right)$

$$= r^2 - x^2 \left\{ \frac{r^2}{x^2} \right\}$$

$$\lim_{x \rightarrow 0^+} x^2 \left[\frac{r^2}{x^2} \right] = \lim_{x \rightarrow 0^+} r^2 - x^2 \left\{ \frac{r^2}{x^2} \right\} = r^2$$

Also,

$$\lim_{x \rightarrow 0^+} x \left[\frac{k}{x} \right] = \lim_{x \rightarrow 0^+} x \left(\frac{k}{x} - \left\{ \frac{k}{x} \right\} \right) = \lim_{x \rightarrow 0^+} k - x \left\{ \frac{k}{x} \right\} = k$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \left(\sum_{k=1}^p x \left[\frac{k}{x} \right] - \sum_{k=1}^9 x^2 \left[\frac{k^2}{x^2} \right] \right)$$

$$= \sum_{k=1}^p \lim_{x \rightarrow 0^+} x \left[\frac{k}{x} \right] - \sum_{k=1}^9 \lim_{x \rightarrow 0^+} x^2 \left[\frac{k^2}{x^2} \right]$$

$$= \sum_{k=1}^p k - \sum_{k=1}^9 k^2$$

$$= \frac{p(p+1)}{2} - \frac{(9)(10)(19)}{6} \geq 1$$

$$\Rightarrow \frac{p(p+1)}{2} - 285 \geq 1$$

$$\Rightarrow p(p+1) \geq 2.286$$

$$\Rightarrow p(p+1) \geq 572$$

Clearly $p = 23$ doesn't satisfy

$$\Rightarrow \text{Minimum value is } p = 24, \text{ as } 24^2 = 576 > 572$$

25. Two parabolas having common focus at $(4, 3)$ intersect at points A and B . Find the value of $(AB)^2$, given that directrices of these parabolas are along X -axis and Y -axis respectively.

Answer (192)

Sol. Equation of parabolas:

$$(x - y)^2 + (y - 3)^2 = x^2$$

$$(x - y)^2 + (y - 3)^2 = y^2$$

Let them intersect at (x_1, y_1) and (x_2, y_2)

$$\therefore x_1^2 = y_1^2 \Rightarrow x_1 = y_1 \quad (x_1 > 0, y_1 > 0)$$

$$\therefore (x_1 - 4)^2 + (x_1 - 3)^2 = x_1^2$$

$$\Rightarrow x_1^2 - 14x_1 + 25 = 0$$

$$x_1 + x_2 = 14, x_1 \cdot x_2 = 25$$

$$(AB)^2 = \left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \right)^2$$

$$= 2(x_1 - x_2)^2$$

$$= 2((x_1 + x_2)^2 - 4x_1 x_2)$$

$$= 2(196 - 100)$$

$$= 192$$