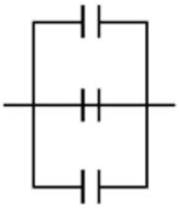
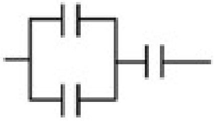
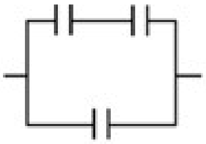
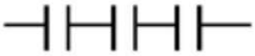
SCORE-JM1-2025-AIOT-PS2-JA
ESJMRT13PH052
Section-I(SC)

Q1. In four options below, all the four circuits are arranged in order of equivalent capacitance. Select the correct order. Assume all capacitors are of equal capacitance.

(A) 

(B) 

(C) 

(D) 

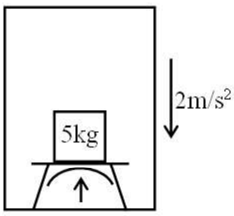
(A) $C_1 > C_2 > C_3 > C_4$ (B) $C_1 > C_3 > C_2 > C_4$

(C) $C_1 < C_2 < C_3 < C_4$ (D) $C_1 < C_3 < C_2 < C_4$

Ans. (B)

Sol. $C_1 = 3C \quad C_2 = \frac{2C}{3} \quad C_3 = \frac{3C}{2} \quad C_4 = \frac{C}{3}$

Q2. If lift is moving downward with acceleration $2\,m/s^2$ then Reading of weighing machine is:



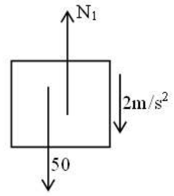
(A) 5 kg (B) 4 kg (C) 3 kg (D) 6 kg

Ans. (B)

Sol. $50 - N_1 = 5(2)$

$N_1 = 40\,N$

Reading $= \frac{N_1}{g} = 4g$



Q3. If the displacement of a particle executing SHM is given by $y = 0.30\sin(220t + 0.64)m$, then the frequency and maximum velocity of the particle is:

(A) $35\,Hz, 66\,m/s$

(B) $45\,Hz, 66\,m/s$

(C) $58\,Hz, 113\,m/s$

(D) $35\,Hz, 132\,m/s$

Ans. (A)

Sol. $\omega = 220$

$2\pi f = 220$

$f = 35\,Hz$

Q4. Two charges 4q and q are placed 30 cm. apart. At what point the value of electric field will be zero

(A) 10 cm away from q and between the charges

(B) 20 cm away from q and between the charges

(C) 10 cm away from q and out side the line joining the charges.

(D) 10 cm away from 4q and out side the line joining them.

Ans. **(A)**

Sol. $k\frac{4q}{(30-x)^2} = \frac{kq}{x^2}$

$\Rightarrow \frac{2}{30-x} = \frac{1}{x}$

$x = 10$

Q5. If one face of a prism of prism angle 30° and $\mu = \sqrt{2}$ is silvered, the incident ray retraces its initial path. What is the angle of incident?

(A) 90°

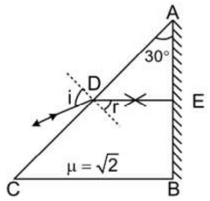
(B) 180°

(C) 45°

(D) 55°

Ans. (C)

Sol.



As in ΔAED

$30^\circ + 90^\circ + \angle D = 180^\circ$

$\angle D = 60^\circ$

Now as by construction

$\angle D + \angle r = 90^\circ$

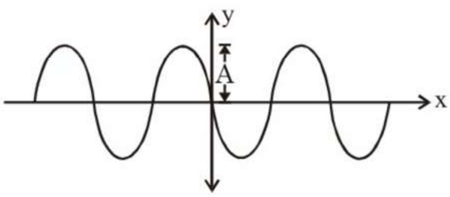
$\angle r = 90^\circ - 60^\circ = 30^\circ$

Applying Snell's law at surface AC

$$\sin i = (\sqrt{2}) \sin 30^\circ = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$i = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Q6. A progressive wave travelling along the positive x -direction is represented by $y(x,t) = A\sin\ (kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure:

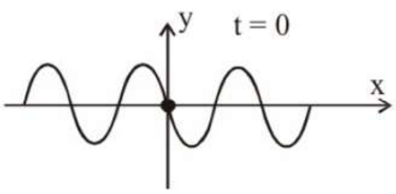


For this wave, the phase ϕ is:

(A) 0 (B) $-\frac{\pi}{2}$ (C) π (D) $\frac{\pi}{2}$

Ans. (C)

Sol.

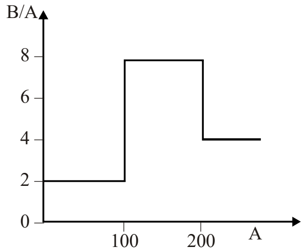


$y = A\sin(kx - wt + \phi)$

at x = 0, t = 0, y = 0 and slope is negative.

$\Rightarrow \phi = \pi$

Q7. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below :



(A) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy

(B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will absorb energy

(C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments

(D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments

Ans. (D)

Sol. Lighter nuclie fuse together, while heavy nuclie perform fission reaction.

Q8. A spherical planet of density d of radius of R. What will be gravitational field at depth h

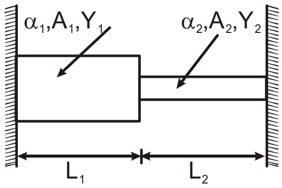
(A) $\dfrac{4\pi Gd(R-h)}{3}$ (B) $\dfrac{\pi Gd(R-h)}{3}$

(C) $\dfrac{4\pi Gd(h)}{3}$ (D) $\dfrac{\pi Gd(h)}{3}$

Ans. (A)

Sol. Gravitational field at distance r from centre is $\dfrac{\rho r}{3\varepsilon_0}$

Q9. Two rods are joined between fixed supports as shown in the figure. Condition for no change in the lengths of individual rods with the increase of temperature will be (α_1, α_2 = line ar expansion co-efficient A_1, A_2 = Area of rods Y_1, Y_2 = Young modulus)



(A) $\frac{A_1}{A_2} = \frac{\alpha_1}{\alpha_2}\frac{Y_1}{Y_2}$ (B) $\frac{A_1}{A_2} = \frac{L_1 \alpha_1 Y_1}{L_2 \alpha_2 Y_2}$ (C) $\frac{A_1}{A_2} = \frac{L_2 \alpha_2 Y_2}{L_1 \alpha_1 Y_1}$ (D) $\frac{A_1}{A_2} = \frac{\alpha_2 Y_2}{\alpha_1 Y_1}$

Ans. **(D)**

Sol. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\Delta\ell/\ell}$

$T = \frac{Y.\Delta\ell}{\ell}A = Y.A\,\alpha\,\Delta T$

In both the rods tension will be same so

$T_1 = T_2$

Hence $Y_1 A_1 \alpha_1 \Delta T = Y_2 A_2 \alpha_2 \Delta T$

$\frac{A_1}{A_2} = \frac{Y_2\alpha_2}{Y_1\alpha_1}$

Q10. A carnot engine works between ice point and steam point. It is desired to increase efficiency by 20%, by changing temperature of sink to –

(A) 253 K (B) 293 K (C) 303 K (D) 243 K

Ans. **[A]**

Sol. Temp. of source $T_1 = 100 + 273 = 373$ K.

Temp of sink $T_2 = 0 + 273 = 273$ K.

Efficiency of carnot engine

$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{373}$

$\eta = \frac{100}{373}$

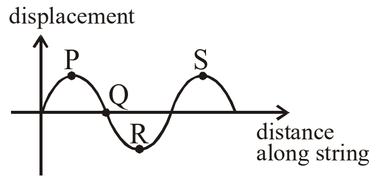
To increase $\eta' = \frac{100}{373} + \frac{100}{373} \times \frac{1}{5} = \frac{120}{373}$

$\eta' = 1 - \frac{T_2}{373} = \frac{120}{373} \Rightarrow T_2 = 373 - 120$

$\Rightarrow T_2 = 253$ K.

$\boxed{\text{New sink temp} = 253\text{K.}}$

Q11. The given graph illustrates a transverse wave travelling on a string at a particular instant, and the points P, Q, R and S represent elements of the string. Which of the following statements about the motion of the elements is correct ?



(A) The speed of the element at P is maximum

(B) The displacement of the element at Q is always zero

(C) The energy of the element at R is entirely kinetic

(D) The acceleration of the element at S is maximum

Ans. (D)

So. $v_P = 0$

Displacement of element at Q is zero at this instant only.

Kinetic energy of element at R is zero.

Acceleration of element at S is maximum.

Q12. Given below are two statements:

Statement-I: Light can travel in vacuum but sound cannot do so.

Statement-II: Light is an electromagnetic wave and sound is a mechanical wave.

Then,

(A) Both Statement-I and Statement-II are correct.

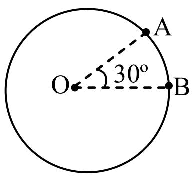
(B) Both Statement-I and Statement-II are incorrect.

(C) Statement-I is correct & Statement-II is incorrect.

(D) Statement-I is incorrect & Statement-II is correct.

Ans. (A)

Q13. A uniform wire of resistance 36 Ω is bent in the form of a circle. The effective resistance between A and B is (O is the centre of circle):

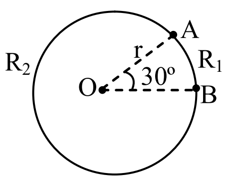


(A) 2.75 Ω (B) 3 Ω (C) 33 Ω (D) 36 Ω

Ans. **[A]**

Sol. $2\pi r$ length of wire has resistance = 36 Ω

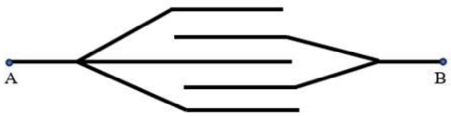
$\therefore$ $r\pi/6$ length of wire has resistance $= \frac{36}{2\pi r} \times \frac{r\pi}{6} = 3\Omega$



$\therefore R_1 = 3\Omega$ and $R_2 = 33\Omega$

$R_{eq} = \frac{3 \times 33}{3 + 33} = \frac{3 \times 33}{36} \simeq 2.75$

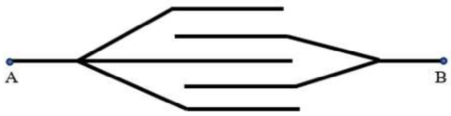
Q14. Five identical plates of equal area A are placed parallel to and at equal distance d from each other as shown in figure. The effective capacity of the system between the terminals A and B is-

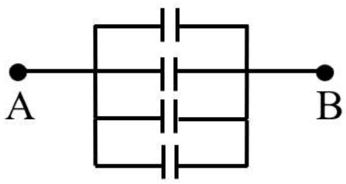


(A) $\frac{3}{5}\frac{\epsilon_0 A}{d}$ (B) $\frac{5}{4}\frac{\epsilon_0 A}{d}$ (C) $\frac{4\epsilon_0 A}{d}$ (D) $\frac{4}{5}\frac{\epsilon_0 A}{d}$

Ans. **[C]**

Sol.





The capacity of each capacitor is, $C_0 = \frac{\epsilon_0 A}{d}$

From fig. it is clear that $C_{eq} = \frac{5}{3}C_0 = \frac{5}{3}\frac{\epsilon_0 A}{d}$

Q15. A body starting from rest is moving under a constant acceleration in straight line. Suppose S_1 is the displacement in first 10 sec., S_2 is the displacement in the next 10 sec. Find $\frac{S_2}{S_1}$.

(A) 3 (B) 6 (C) 9 (D) 12

Ans. **(A)**

Sol. U = 0 (given)

$\therefore\ S_1 = \frac{1}{2}at_1^2 = \frac{1}{2}a(10)^2 = 50a$

$[S = ut + \frac{1}{2}\ at^2]$ ....(1)

$v_1 = u + at_1 = 0 + a(10) = 10a$

$\therefore$ $S_2 = v_1t + \frac{1}{2}at^2$ $[S = ut + \frac{1}{2}at^2]$

$= (10a) \cdot 10 + \frac{1}{2}a \cdot (10)^2$

$= 100\ a + 50\ a = 150\ a$ ....(2)

From (1) & (2)

$\frac{S_2}{S_1} = \frac{150a}{50a} = 3$

$\therefore \frac{S_2}{S_1} = 3$

Q16. Sixty four spherical rain drops of equal size are falling vertically through air with terminal velocity 1.5 m/s. If these drops coallese to form a big spherical drop, then terminal velocity of big drop is (in m/s) :-

(A) 19 (B) 24 (C) 29 (D) 34

Ans. **(B)**

Sol. For one small drop

$\rho_{water}\frac{4}{3}\pi r^3 g - \rho_{air}\frac{4}{3}\pi r^3 g = 6\pi\eta r v_{T_1}$ ...(1)

for are, big drop

$\rho_{water}\frac{4}{3}\pi R^3 g - \rho_{air}\frac{4}{3}\pi R^3 g = 6\pi\eta R v_{T_2}$ ...(2)

(1) ÷ (2)

$\frac{r^3}{R^3} = \frac{r}{R}.\frac{(1.5m/s)}{v_{T_2}}$

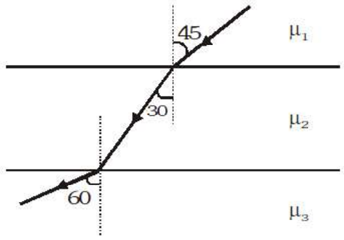
$\frac{r^2}{R^2} = \frac{1.5m/s}{v_{T_2}}$

as $64\ \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\therefore \frac{r}{R} = \frac{1}{4}$ $\therefore v_{T_2} = 16 \times 1.5$

$\Rightarrow v_{T_2} = 24$ m/s

Q17. For shown situation the value of $\frac{\mu_3}{\mu_1}$ is equal to



(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{\frac{2}{3}}$ (C) $\sqrt{\frac{3}{2}}$ (D) $\frac{2}{\sqrt{3}}$

Ans. (B)

Sol. $\mu_1 \sin 45^\circ = \mu_2 \sin 30^\circ = \mu_3 \sin 60^\circ \Rightarrow \frac{\mu_3}{\mu_1} = \frac{\sin 45^\circ}{\sin 60^\circ} = \sqrt{\frac{2}{3}}$

Q18. Consider Fraunhofer diffraction pattern obtained with a single slit illuminated at normal incidence. At the angular position of the first diffraction minimum, the phase difference (in radians) between the wavelets from the opposite edges of the slit is :-

(A) π (B) 2π (C) $\pi/4$ (D) $\pi/2$

Ans. (B)

Sol. Phase diff between top & middle is π.
& middle & bottom is π.
$\therefore$ between top & bottom is 2π.

Q19. Length of rod A is 3.25 ± 0.01 cm and rod B is 4.19 ± 0.01 cm then find the difference of length of rods.

(A) 0.94 ± 0.00 (B) 0.94
(C) 0.94 ± 0.02 (D) 0.95

Ans. (C)

Sol. $x = B - A = 0.94$
$\Delta x = \Delta A + \Delta B = 0.02$
$x \pm \Delta x = 0.94 \pm 0.02$

Q20. Young's double slit experiment is first performed in air and then in a medium other than air. It is found that 8^{th} bright fringe in the medium lies where 5^{th} dark fringe lies in air. The refractive index of the medium is nearly :-

(A) 1.59 (B) 1.69 (C) 1.78 (D) 1.25

Ans. (C)

Sol. $(y_8)_{\text{Bright, medium}} = (y_5)_{\text{Dark, air}}$

$$\frac{8\lambda_m D}{d} = \left(\frac{2(5)-1}{2}\right)\frac{\lambda D}{d}$$

SECTION II (NT)

Q21. Moment of inertia of a rod of length L and mass M about axis of rotation perpendicular to rod and passing through centre is $\frac{ML^2}{n}$ then n is:

Ans. (12)

Sol. Not required

Q22. A transverse periodic wave on a string with a linear mass density of $0.200\,kg/m$ is described by the following equation $y = 0.05\sin(420t - 21.0x)$ where x and y are in metres and t is in seconds. The tension in the string is equal to T Newton then $\frac{T}{16}$ is

Ans. (5)

Sol. $V = \frac{\omega}{k} = \frac{420}{21}$

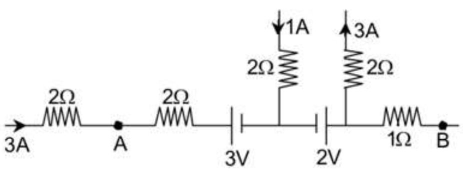
$V = 20\,m/sec$

$\because V = \sqrt{\frac{T}{\mu}}$

$T = \mu V^2 = 0.2(20)^2$

$T = 80$

Q23. The potential difference between point $A \& B (V_A - V_B)$ in the section of the circuit shown is (in volt):

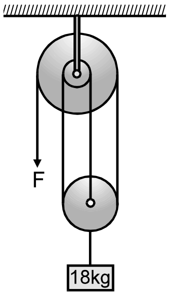


Ans. (8)

Sol. $V_A - 2\times3 - 3 + 2 - 1 = V_B$

$V_A - V_B = 8\text{volt}$

Q24. In the figure, at the free end of the light string, a force F is applied to keep the suspended mass of 18 kg at rest. Then the force (in N) exerted by the ceiling on the system (assume that the string segments are vertical and the pulleys are light and smooth) is: (g= 10 m/s^2)



Ans. **(240)**

Sol. To keep block at rest

$3F = 180$ and

force exerted by the ceiling on the system is

$4F = 240$

Q25. A mass M attached to a spring oscillates with a period of 2s . If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M (in kg) assuming that Hooke's Law is obeyed.

Ans. **(1.6)**

Sol. $T = 2\pi\sqrt{\frac{M}{K}}$ or $T \propto \sqrt{M}$,

$$\frac{T_2}{T_1} = \sqrt{\frac{M_2}{M_1}}$$

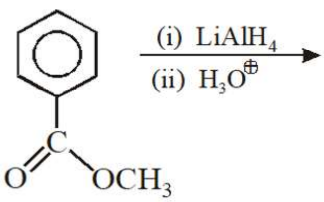
$$\therefore \frac{3}{2} = \sqrt{\frac{M+2}{M}}$$

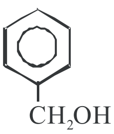
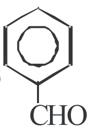
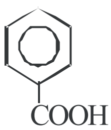
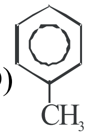
Solving this equation we get, $M = 1.6\,kg$.

CHEMISTRY

Section-I(SC)

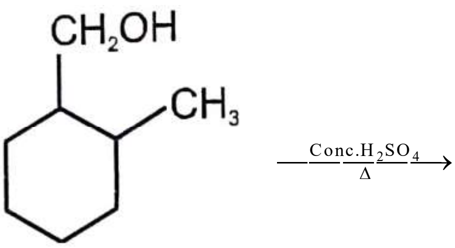
Q1. Major product obtained during reaction.



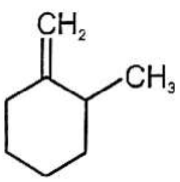
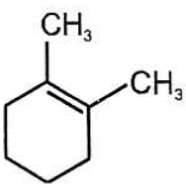
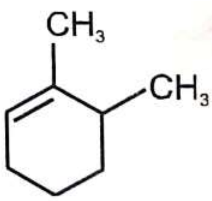
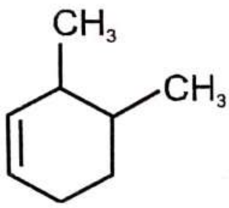
(A)  (B)  (C)  (D) 

Ans. (A)

Q2. In the given reaction:

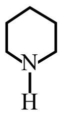
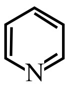
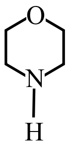
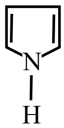


as major product [X] will be:

(A)  (B)  (C)  (D) 

Ans. (B)

Q3. In the following compounds–

(I) (II) (III) (IV)

The order of basicity is -

(A) IV > I > III > II

(B) III > I > IV > II

(C) II > I > III > IV

(D) I > III > II > IV

Ans. **[D]**

Q4. How many alcoholic acyclic structural isomers are possible for $C_4H_{10}O$?

(A) 4 (B) 5 (C) 7 (D) 8

Ans. [A]

Sol. $C_4H_{10}O$ DBE = O

$CH_3 - CH_2 - CH - CH_3$
　　　　　　　|
　　　　　　OH

$CH_3 - CH_2 - CH_2 - CH_2 - OH$

　　　　CH_3
　　　　|
$CH_3 - C - CH_3$
　　　　|
　　　OH

　　　　　CH_3
　　　　　|
$CH_2 - CH - CH_3$
|
OH

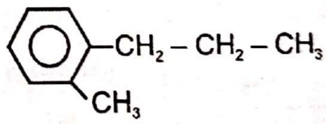
Q5. $(Ph)_2Cd + (CH_3)_2\,CH-\overset{O}{\overset{||}{C}}-Cl \rightarrow [X] \xrightarrow{Zn-Hg/HCl} [Y]$

Identify strucutre of [Y].

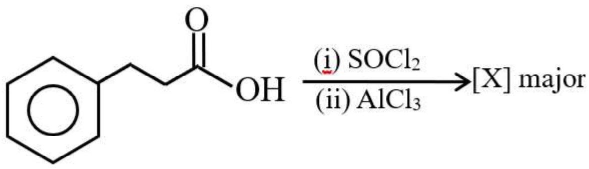
(A) $Ph - CH_2 - CH - CH_3$ with CH_3 on CH

(B) $Ph - CH - CH_2 - CH_3$ with CH_3 on CH

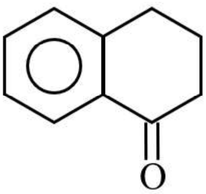
(C) $Ph - CH - CH_3$ with CH_3 above and CH_3 below CH

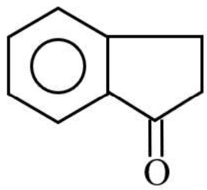
(D) 

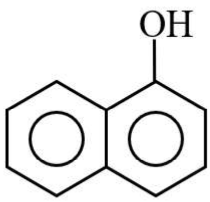
Ans. (A)

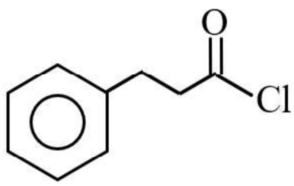
Q6. 

Compound (x) is :

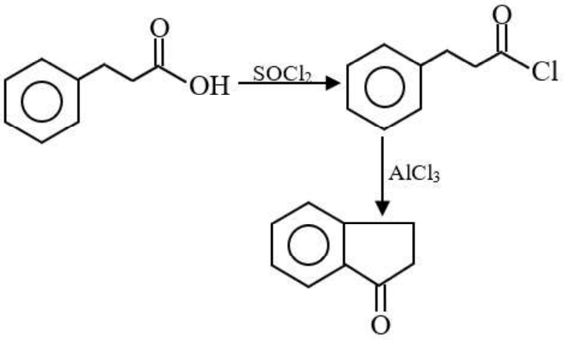
(A) 

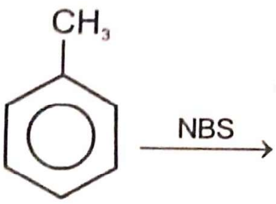
(B) 

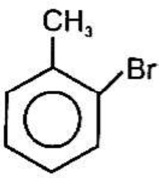
(C) 

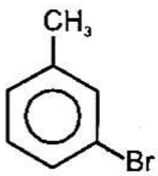
(D) 

Ans. (B)

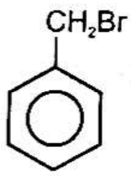
Sol. 

Q7.  Product is :

(A) 

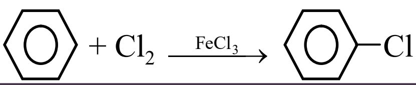
(B) 

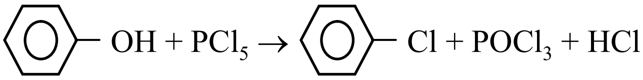
(C) 

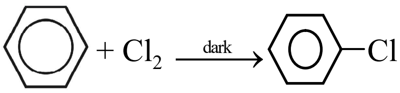
(D) 

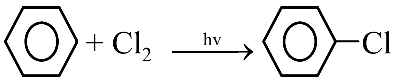
Ans. (D)

Q8. The best method for the prepration of chlorobenzene is-

(A) 

(B) 

(C) 

(D) 

Ans. **[A]**

Q9. A molal solution is one that contains one mole of a solute in

(A) 1000 g of the solvent

(B) One litre of the solution

(C) One litre of the solvent

(D) 22.4 litres of the solution.

Ans. (A)

Sol. One molal solution contains 1 mole of solute in 1 kg of solvent.

Q.10 sp^3 hybridisation is found in -

(A) C_2H_6 (B) C_2H_2 (C) C_2H_4 (D) SO_3

Ans. **[A]**

Q11. Calculate pH of 0.1 M aqueous solution of weak base $BOH\left(K_b = 10^{-7}\right)$ at $25^\circ C$

(A) 4 (B) 10 (C) 7 (D) 5

Ans. (B)

Sol. For weak base

$$pOH = \frac{1}{2}\left(pK_b - \log C\right)$$

$$= \frac{1}{2}(7 - \log 0.1) = \frac{7+1}{2} = 4$$

$$pH = 14 - pOH = 14 - 4 = 10$$

Q12. Among the complex ions given below which is inner-orbital complex –

(A) $[Co(CN)_6]^{3-}$

(B) $\left[Fe\left(H_2O\right)_6\right]^{2+}$

(C) $[FeF_6]^{3-}$ (D) $[CoF_6]^{3-}$

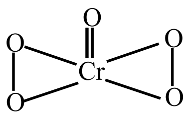
Ans. **[A]**

Q13. Which of the following species has O–O bond ?

(A) $Cr_2O_7^{-2}$ (B) MnO_4^- (C) CrO_5 (D) CrO_4^{-2}

Ans. **[C]**

Sol.



Q14. Which has unequal bond length?

(A) BF_4^- (B) XeF_4 (C) SF_4 (D) SiF_4

Ans. (C)

Sol. SF_4 has equitorial and axial bonds.

Q15. Two moles of an ideal monoatomic gas is heated from $27^\circ C$ to $627^\circ C$, reversibly and isochorically, the entropy of gas

(A) increases by $2R\ln 3$ (B) Increases by $3R\ln 3$

(C) decreases by $2R\ln 3$ (D) decreases by $3R\ln 3$.

Ans. (B)

Sol. $\Delta S = nC_{v,m}\ln\frac{T_2}{T_1}$

For monoaomic gas $C_{v,m} = \frac{3R}{2}$

$T_1 = 300\,K, T_2 = 900\,K$

$\Delta S = 2\times\frac{3R}{2}\ln\left(\frac{900}{300}\right) = 3R\ln 3$

Q.16 The formation of cyanohydrin from a ketone is an example of –

(A) Electrophilic addition (B) Nucleophilic addition

(C) Nucleophilic substitution (D) Electrophilic substitution

Ans. **[B]**

Q17. The equivalent mass of H_2SO_4 (M = molecular mass of H_2SO_4) in its reaction with NaOH to form $NaHSO_4$ equal to -

(A) M (B) M/2 (C) M/4 (D) M/6

Ans. **[A]**

Sol. $H_2SO_4 + NaOH \rightarrow NaHSO_4 + H_2O$

$n_f = 1$

$$\text{Equivalent mass} = \frac{\text{Molecular mass}}{\text{n-factor}}$$

$$= \frac{M}{1}$$

Q18. $\frac{K_P}{K_C}$ for the gaseous reaction –
$2A + 3B \rightleftharpoons 2C$

would be respectively -

(A) $(RT)^{-3}$ (B) $(RT)^{+3}$ (C) $(RT)^{1}$ (D) $(RT)^{-1}$

Ans. **[A]**

Sol. $K_p = K_c(RT)^{\Delta ng}$

$\Delta n_g = 2 - (2+3) = -3$

$So, K_p = K_c(RT)^{-3}$

$$\frac{K_p}{K_c} = (RT)^{-3}$$

Q19. In which compound carbon atom is present in its least oxidation state.

(A) CO_2 (B) CH_4 (C) CH_2Cl_2 (D) $CHCl_3$

Ans. (B)

Sol. $\overset{+4}{C}O_2, \overset{-4}{C}H_4, \overset{0}{C}H_2Cl_2, \overset{+2}{C}HCl_3$

Q20. NaCl is 40 % ionized in aqueous solution. The value of van't Hoff factor is

(A) 1.4 (B) 1.2 C) 2.0 (D) 1.6

Ans. (A)

Sol. $i = 1 + (n-1)\alpha$

Given, $n = 2, \alpha = 0.4$

$i = 1 + (2-1) \times 0.4 = 1.4$

Section-II(NV)

Q21. The rate of decomposition of a substance increases by a factor of 2.25 for 1.5 times increase in concentration of substance at same temperature. Find order of the reaction?

Ans. (2)

Sol. $2.25 = (1.5)^n$

$n = 2$

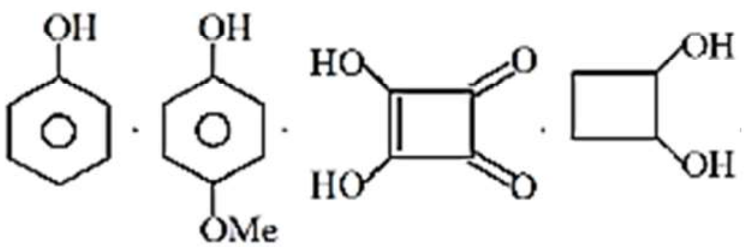
Q22. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litres, the change in internal energy is __________J.

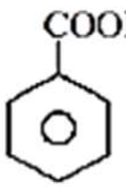
Ans. (0)

Sol. For ideal gas U = f(T) and for isothermal process

$\Delta T = 0 \Rightarrow \Delta U = 0$

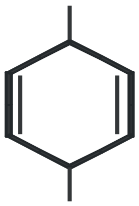
Q23. Total no. of species which will give effervesce of CO_2 with $NaHCO_3$



HCOOH, 

Ans. (3)

Q24. The number of all the possible Geometrical isomers formed by the given compound is:



Ans. (2)

Q25. The molar conductivity of 0.1 M weak acid is 100 times lesser than that at infinite dilution. The degree of dissociation of weak electrolyte at 0.1 M is 10^{-x}. The value of x is ?

Ans. **(2)**

Sol. The molar conductivity of 0.1 M weak acid is 100 times lesser than that at infinite dilution.

$$\Lambda_m = \frac{\Lambda^0}{100}$$

$$\therefore \text{ Degree of dissociation } (\alpha) = \frac{\Lambda_m^c}{\Lambda_m^0} = \frac{\Lambda^0}{100\Lambda^0} = 0.01$$

MATH

Section-I (SC)

1. A plane passes through the point (–2, –2, 2) and contains the line joining the points (1, –1, 2) and (1, 1, 1). Then the image of (–7, 2, 3) in the plane is

(A) (1, –1, 5) (B) (–5, –4, –2) (C) (–6, –1, –3) (D) $\left(\frac{13}{23}, \frac{7}{23}, \frac{6}{23}\right)$

1. (C)

1. The plane passes through the points (–2, –2, 2), (1, –1, 2), (1, 1, 1)

$$\text{Its equationis } \begin{vmatrix} x+2 & y+2 & z-2 \\ 3 & 1 & 0 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow x - 3y - 6z + 8 = 0$

Image of (–7, 2, 3) in the plane is

$$\frac{x+7}{1}=\frac{y-2}{-3}=\frac{z-3}{-6}=-2\left(\frac{-7-6-18+8}{1+9+36}\right)$$

$$\frac{x+7}{1}=\frac{y-2}{-3}=\frac{z-3}{-6}=\left(\frac{23}{23}\right)=1$$

$$x=-6, y=-1, z=-3$$

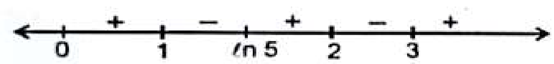
2. If A is domain of $f(x)=\ell n \tan^{-1}\left(\left(x^3-6x^2+11x-6\right)(x)\left(e^x-5\right)\right)$ and B is the range of RELATION $g(x)=\sin^2\frac{x}{4}+\cos\frac{x}{4}$ .Then find $A\cap B$

(A) $\left[\frac{\pi}{3},\frac{2\pi}{3}\right]$ (B) $\left[\frac{\pi}{3},\frac{2\pi}{3}\right]$ (C) (0, 1) (D) (0, 2)

Ans. (C)

Sol. $f(x)=\ell n \tan^{-1}\left((x-1)(x-2)(x-3)x\left(e^x-5\right)\right)$

for defined $(x-1)(x-2)(x-3)x\left(e^x-5\right)>0$



A = Domain $(0,1)\cup(\ell n 5,2)\cup(3,\infty)$ and $g(x)=1-\cos^2\frac{x}{4}+\cos\frac{x}{4}=\frac{5}{4}-\left(\cos\frac{x}{4}-\frac{1}{2}\right)^2$

B = Range $\left[-1, \frac{5}{4}\right]$ $A\cap B=(0,1)$.

3. If $f:R\to R$ be defined as $f(x)=\frac{e^{2x}-e^{-2x}}{2}$, then

(A) f is many-one (B) f is into

(C) $f^{-1}(x)=\frac{1}{2}\left[\log\left(x-\sqrt{x^2+1}\right)\right]$ (D) $f^{-1}(x)=\frac{1}{2}\left[\log\left(x+\sqrt{x^2+1}\right)\right]$

3. (D)

3. Let us check for invertibility of f(x)

(A) one-one : we have, $f(x)=\frac{e^{2x}-e^{-2x}}{2}$

$\Rightarrow f'(x)=\frac{e^{4x}+1}{e^{2x}}$, which is strictly increasing as $e^{4x}>0$ for all x

Thus, f is one-one

(B) Onto : Let y = f(x)

$\Rightarrow \frac{dy}{dx} = e^{2x} + e^{-2x}$, where y is strictly monotonic

Hence, the range of f(x) = $(f(-\infty), f(\infty))$

$\Rightarrow$ range of $f(x) = (-\infty, \infty)$

So, the range of f(x) = co-domain

Hence, f(x) is one-one and onto

(C) To find $f^{-1} : y = \frac{e^{4x} - 1}{2e^{2x}}$

$\Rightarrow e^{4x} - 2e^{2x}y - 1 = 0$

$\Rightarrow e^{2x} = \frac{2y \pm \sqrt{4y^2 + 4}}{2}$

$\Rightarrow 2x = \log\left(y \pm \sqrt{y^2 + 1}\right)$

$\Rightarrow f^{-1}(y) = \frac{\log\left(y \pm \sqrt{y^2 + 1}\right)}{2}$

since, $e^{f^{-1}(x)}$ is always positive, so neglecting negative sign.

Hence, $f^{-1}(x) = \frac{\log\left(x + \sqrt{x^2 + 1}\right)}{2}$

4. Let Z be the set of integers, if $A = \left\{x \in Z : |x - 3|^{(x^2 - 5x + 6)} = 1\right\}$ and $B = \{x \in Z : 10 < 3x + 1 < 22\}$, then the number of subsets of the set $A \times B$ is

(A) 2^6 (B) 2^8 (C) 2^{15} (D) 2^9

4. (A)

4. For A,

$|x - 3| = 1 \Rightarrow x = 2, 4$ or

$x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$ but $x \neq 3$

$\therefore A = \{2, 4\}$

For B,

$B = \{4, 5, 6\}$

$n(A \times B) = 6$

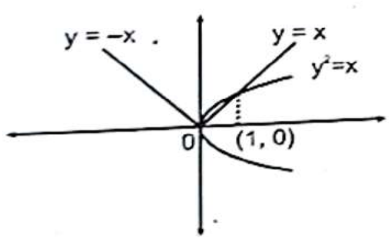
$\therefore$ number of subsets $= 2^6$

5. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(A) $\frac{2}{3}$ sq unit (B) 1 sq unit (C) $\frac{1}{6}$ sq unit (D) $\frac{1}{3}$ sq unit

5. (C)

5.



Required area $A = \int_0^1 (\sqrt{x} - x)dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right|_0^1$

$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ sq unit

6. For a complex number Z, if one root of the equation $Z^2 - aZ + a = 0$ is $(1+i)$ and its other root is α then the value of $\frac{a}{\alpha^4}$ is equal to

(A) 4 (B) $-\frac{1}{2}$ (C) 2 (D) –2

6. (B)

6. $\alpha + 1 + i = a$

$\alpha(1+i) = a$

$\alpha + \alpha i = \alpha + 1 + i$

$\Rightarrow \alpha = \frac{1+i}{i} = 1 - i$

$a = 1 - i + 1 + i = 2$

$\alpha^4 = (\alpha^2)^2 = (1 + (-1) - 2i)^2$

$= (-2i)^2 = 4i^2 = -4$

7. The value of k for which the sum of the squares of the roots of $2x^2 - 2(k-2)x - k - 1 = 0$ is least is

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$

7. (B)

7. $\alpha+\beta=k-2$

$\alpha\beta=-\frac{(k+1)}{2}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(k-2)^2+(k+1)$

$=k^2-4k+4+k+1$

$=k^2-3k+5$

$\Rightarrow \alpha^2+\beta^2=\left(k-\frac{3}{2}\right)^2+\frac{11}{4}$

Which is least for $k=\frac{3}{2}$

8. The value of $\lim_{x\to\infty} y(x)$ obtained from the differential equation $\frac{dy}{dx}=y-y^2$, where $y(0)=2$ is

(A) zero (B) 1 (C) ∞ (D) none of these

8. (B)

8. $\frac{dy}{dx}=y-y^2 \Rightarrow \int\frac{dy}{y-y^2}=\int dx$

$\int\frac{1}{1-y}+\frac{1}{y}dy=x+c \Rightarrow \quad \ell n\frac{y}{1-y}=x+c$

$\frac{y}{1-y}=ke^x \Rightarrow y=ke^x-kye^x \Rightarrow y=\frac{ke^x}{1+ke^x}$

$x=0, y=2; 2=\frac{k}{1+k} \quad \Rightarrow 2+2k=k \Rightarrow k=-2, y=\frac{-2e^x}{1-2e^x} \Rightarrow y=\frac{-2}{e^{-x}-2}$

$\lim_{x\to\infty}(y(x))=\lim_{x\to\infty}\frac{-2}{e^{-x}-2}=1$

9. 2. Let $I_n=\int_1^e(\log x)^n dx$, where n is a non-negative integer. Then $I_{2011}+2011\, I_{2010}$ is equal to

(A) 1 (B) –1 (C) e (D) –e

9. (C)

9. $I_n=\int_1^e(\log x)^n dx$

$$I_n = x(\log x)^n\Big|_1^e - \int_1^e \frac{n(\log x)^{n-1}}{x}\cdot x dx$$

I_n=(e-0)-n I_{n-1}

I_n+n I_{n-1}=e

I_{2011}+2011 I_{2010}=e

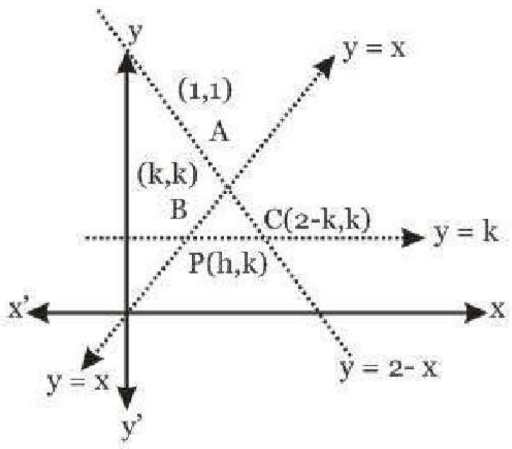
$\therefore I_{100} + 100\ I_{99} = e$

10. If the area (in sq. units) of the triangle formed by the intersection of a line parallel to the x-axis and passing through the point P(h, k) with the line y = x and x + y = 2 is $4h^2$, then the locus of the point P is

(A) $2x = \pm(y-1)$ (B) $3x = \pm(y-1)$ (C) $5x = \pm(y-1)$ (D) $7x = \pm(y-1)$

10. (A)

10. Here, the triangle formed by a line parallel to the x-axis and passing through P(h, k) and the straight line y = x and y = 2 – x is shown below.



Since, area of $\Delta ABC = 4h^2$

$\therefore \frac{1}{2}\cdot AB\cdot AC = 4h^2$

Where $AB = \sqrt{2}\,|k-1|$ and $AC = \sqrt{2}(|k-1|)$

$\Rightarrow \frac{1}{2}\cdot 2(k-1)^2 = 4h^2$

$\Rightarrow 4h^2 = (k-1)^2$

$\Rightarrow 2h = \pm(k-1)$

Hence, the locus of the point P is $2x = \pm(y-1)$

11. A fair coin is flipped 'n' times. Let 'E' be the event "a head is obtained on the 1^{st} flip", & let F_k be the event "exactly k heads are obtained". For which one of the following pairs (n, k) are E & F_k independent ?

(A) (12,4) (B) (40,10) (C) (100,51) (D) (20,10)

11. (D)

Sol. $P(E)=\frac{1}{2};P\left(F_k\right)=\frac{{}^nC_k}{2^n}\Rightarrow P\left(E\cap F_k\right)=\frac{1}{2}\frac{{}^{n-1}C_{k-1}}{2^{n-1}}$

$\therefore\ \frac{1}{2^n}{}^{n-1}C_{k-1}=\frac{1}{2}{}^nC_k\cdot\frac{1}{2^n}\Rightarrow n=2k$

12. Let A and B are two non-singular square matrices of order n with real entries such that $\operatorname{adj}A=\operatorname{adj}B$, then which of the following is necessarily true-

(1) A = B if n is even (2) A = –B if n is even

(3) $A=-B$ if n is odd (4) A = B if n is odd

12. (1)

Sol. $\operatorname{adj}A=\operatorname{adj}B\Rightarrow|A|^{n-1}=|B|^{n-1}$

$\Rightarrow\begin{cases}|A|=|B| & \text{if } n \text{ is even}\\ |A|=\pm|B| & \text{if } n \text{ is odd}\end{cases}$

If n is even $\operatorname{adj}A=\operatorname{adj}B$

$\Rightarrow|A|A^{-1}=|B|B^{-1}$

$\Rightarrow\ A^{-1}=B^{-1}=A=B$

If n is odd $\operatorname{adj}A=\operatorname{adj}B$

$\Rightarrow|A|A^{-1}=|B|B^{-1}$

$\Rightarrow A^{-1}=\pm B^{-1}\Rightarrow A=B$ or $A=-B$

13. The product of all values of x satisfying the equation $\sqrt{\log_{10}\sqrt{x^2}}=\log_{10}\left(-x^3\right)$ is-

(A) 1 (B) –1 (C) -10^9 (D) $10^{1/9}$

13. (D)

Sol. $\sqrt{\log_{10}|x|}=\log_{10}\left(-x^3\right)$ here x < 0

$\log_{10}\left(-x^3\right)=\sqrt{\log_{10}(-x)}$

Let $\log_{10}(-x)=t$

$3t=\sqrt{t}$

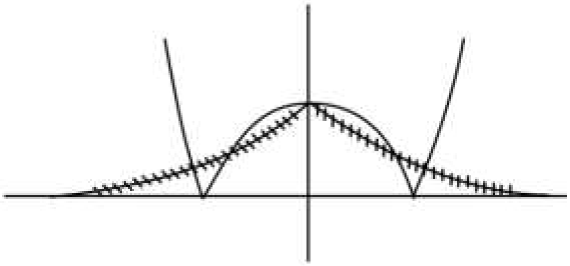
14. Let $f: R \to [0,1]$ be a function defined by $f(x) = \min\left(e^{-|x|}, \left|x^2 - 1\right|\right)$. Identify which of the following statement(s) is/are INCORRECT?

(A) f is surjective but not injective (B) f is neither injective nor surjective

(C) f is non-derivable at exactly seven points (D) f is continuous every where

14. (B)

Sol.



Look at the graph and we can check options.

15. Given $\vec{a}, \vec{b}$ and $\vec{c}$ are 3 vectors such that $\vec{b}, \vec{c}$ are parallel unit vectors and $|\vec{a}| = 3$. If $\vec{a} + \lambda\vec{c} = 4\vec{b}$, then the sum of all the possible positive values of λ is

(A) 2 (B) 4 (C) 6 (D) 8

15. (D)

15. $|\vec{a}| = |4\vec{b} - \lambda\vec{c}|$

$|\vec{a}|^2 = 16|\vec{b}|^2 + \lambda^2|\vec{c}|^2 - 8\lambda\vec{b}\cdot\vec{c}$

$9 = 16 + \lambda^2 - 8\lambda\vec{b}\cdot\vec{c}$

$\Rightarrow \lambda^2 - 8\lambda + 7 = 0 \quad (\text{ as } \vec{b}\cdot\vec{c} = 1)$

Sum of all the values of $\lambda = 8$

16. The order of the differential equation of the family of curves $y = a3^{bx+c} + d\sin(x+e)$ is (where, a, b, c, d, e are arbitrary constants)

(A) 5 (B) 4 (C) 3 (D) 2

16. (B)

16. We can re-write the equation of the curve as

$y = a\cdot 3^c \cdot 3^{bx} + d\sin(x+e)$

or $y = k\cdot 3^{bx} + d\sin(x+e)$ (where, k is an arbitrary constant)

As there are '4' such constants, hence the order of the differential equation will be '4'

17. The value of $\int \frac{e^{\sqrt{x}}}{\sqrt{x}\left(1+e^{2\sqrt{x}}\right)}dx$ is equal to (where, C is the constant of integration)

(A) $\tan^{-1}\left(2e^{\sqrt{x}}\right)+C$

(B) $\ln\left(\frac{1+e\sqrt{x}}{1-e\sqrt{x}}\right)+C$

(C) $2\tan^{-1}\left(e^{\sqrt{x}}\right)+C$

(D) $\left(\tan^{-1}x\right)e^{\sqrt{x}}+C$

17. (C)

17. Let, $I=\int\frac{e\sqrt{x}}{\sqrt{x}\left(1+e^{2\sqrt{x}}\right)}dx$

Substituting $e^{\sqrt{x}}=t$

$\frac{e^{\sqrt{x}}}{2\sqrt{x}}dx=dt$

$I=2\int\frac{dt}{1+t^2}$

$=2\left(\tan^{-1}t\right)+C$

$I=2\tan^{-1}\left(e^{\sqrt{x}}\right)+C$

18. If $f(x)=\frac{x}{1+x\tan x}, x\in\left(0,\frac{\pi}{2}\right)$, then

(A) f(x) has exactly one point of minima

(B) f(x) has exactly one point of maxima

(C) f(x) is increasing in $\left(0,\frac{\pi}{2}\right)$

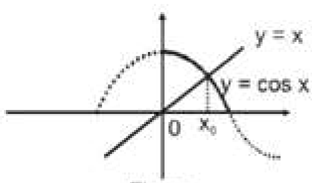
(D) minima occurs at x_0 where $x_0=\cos x_0$

18. (B)

Sol. $f'(x)=\frac{\sec^2 x(\cos x+x)(\cos x-x)}{(1+x\tan x)^2}$

The only factor in $f'(x)$ which changes sign is $\cos x-x$.

Let us consider graph of $y=\cos x$ and $y = x$



Figure

It is clear from figure that for $x\in\left(0,x_0\right), \cos x-x>0$ and for $x\in\left(x_0,\frac{\pi}{2}\right)$ $\cos x-x<0$,

$\Rightarrow$ $f'(x)$ has maxima at x_0 where $x_0=\cos x_0$

19. If $f(x)=\prod_{r=1}^{2025}(x-r)$, then the value of $f'(2025)$ is equal to

(A) 0 (B) 2024 (C) 2024! (D) 2025!

19. (C)

19. $f(x)=(x-1)(x-2).........(x-2024)(x-2025)$

$\Rightarrow f(x)=(1)(x-2)(x-3)...(x-2024)(x-2025)$

$+(x-1).1(x-3)...(x-2025)+....+(x-1)(x-2)...(x-2024).1$

Putting x = 2025, we get,

$f'(2025)=2024!$

20. If a, b, c, $\lambda \in N$, then the least possible value of $\begin{vmatrix} a^2+\lambda & ab & ac \\ ba & b^2+\lambda & bc \\ ca & cb & c^2+\lambda \end{vmatrix}$ is

(A) 2 (B) 3 (C) 4 (D) 5

20. (C)

20. Multiplying first, second, third rows by a, b, c and then taking a, b, c common from first, second, third columns, we get,

$$D=\begin{vmatrix} a^2+\lambda & a^2 & a^2 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix}$$

Now, $R_1 \leftrightarrow R_1+R_2+R_3$

$$=\left(a^2+b^2+c^2+\lambda\right)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix}$$

Now, $C_2 \leftrightarrow C_2-C_1, C_3 \rightarrow C_3-C_1$

$$=\left(a^2+b^2+c^2+\lambda\right)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & \lambda & 0 \\ c^2 & 0 & \lambda \end{vmatrix}=\lambda^2\left(a^2+b^2+c^2+\lambda\right)$$

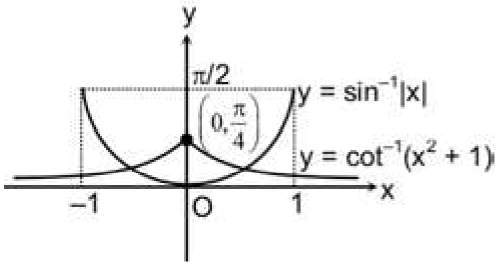
Hence, $D_{min}=4$ when $a=b=c=\lambda=1$

Section-II(NV)

22. Find the number of solutions of the equation, $\cot^{-1}\left(x^2+1\right)=\sin^{-1}|x|$

22. (2)

Sol.



Two solutions

23. Consider the data on x taking the values $0, 2, 4, 8, \ldots, 2^n$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to_______

23. 6

Sol.

x	0	2	4	8		2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

Mean $= \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum_{r=1}^{n} 2^r\, {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$

Mean $= \frac{(1+2)^n - {}^nC_0}{2^n} = \frac{728}{2^n}$

$\Rightarrow \frac{3^n - 1}{2^n} = \frac{728}{2^n}$

$\Rightarrow 3^n = 729$

$\Rightarrow n = 6$

25. If the value of the integral $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \max(\sin x, \tan x)dx$ is equal to ℓnk, then the value of k^2 is equal to

25. 2

25. For $x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$, $\tan x > \sin x (\text{as } \cos x < 1)$

$\therefore \max(\tan x, \sin x) = \tan x$

Thus, $I=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x\, dx$

$=\left(-\ln|\cos x|\right)_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $\quad = -\ln\left(\frac{1}{2}\right)+\ln\left(\frac{1}{\sqrt{2}}\right)$ $=\ln\sqrt{2}$

26. Let normals to the parabola $y^2=4x$ at variable points $P\left(t_1^2,2t_1\right)$ and $Q\left(t_2^2,2t_2\right)$ meet at the point $R\left(t^2,2t\right)$, then the line joining P and Q always passes through a fixed point (α,β), then the value of $|\alpha+\beta|$ is equal to

26. 2

Sol. $t=-t_1-\frac{2}{t_1}$ and $t=-t_2-\frac{2}{t_2}$

$\Rightarrow t_1,t_2$ are the roots of the equation $t=-x-\frac{2}{x}$

$\Rightarrow x^2+tx+2=0$ has roots t_1 & t_2

$\Rightarrow t_1+t_2=-t$ and $t_1t_2=2$

Now, the equation of PQ is

$\left(t_1+t_2\right)y=2x+2t_1t_2$

$\Rightarrow -ty=2x+4 \Rightarrow 2(x+2)+ty=0$

$\Rightarrow$ PQ always passes through the common point of x + 2 = 0 and y = 0

$\Rightarrow (\alpha,\beta)=(-2,0) \Rightarrow \alpha\beta=0$ & $\alpha+\beta=-2$

28. The absolute value of coefficient of x^9 in the expansion of $\left(1+x+x^2+x^3\right)^3(1-x)^6$ is

28. 9

Sol. Since,

$1+x+x^2+x^3=\frac{1-x^4}{1-x} \Rightarrow \left(1+x+x^2+x^3\right)(1-x)=1-x^4$

$\Rightarrow \left(1+x+x^2+x^3\right)^3(1-x)^3(1-x)^3$

$=\left(1-x^4\right)^3(1-x)^3$

$=\left(1-3x^4+3x^8-x^{12}\right)\left(1-3x+3x^2-x^3\right)$

Coefficient of x^9 in this product is

$=3(-3)=-9$