SCORE-JM1-AIOT-FS-1
ESJMRT13PH051
Section-I(SC)

Q1. The electric potential in a region is given as $V=-4ar^2+3b$, where r is distance from the origin, a and b are constants. If the volume charge density in the region is given by $\rho = na\ \varepsilon_0$, then what is the value of n ?

(A) 20 (B) 24 (C) 18 (D) 23

Ans. (B)

Sol. $V=-4ar^2+3b$

$E=\frac{-dV}{dr}=8ar$

The small amount of flux $d\phi$ associated with the closed surface bounded by two spheres of radius r and $r+dr$ is

$d\phi=(E+dE)(A+dA)-EA$

$d\phi=EdA+dEA$

dE=8 adr and dA $=8\pi rdr$

Using Gauss's law

$\frac{\rho\times 4\pi r^2 dr}{\varepsilon_0}=8adr\times 4\pi r^2+8ar\times 8\pi rdr$

$\Rightarrow \rho=24a\varepsilon_0$

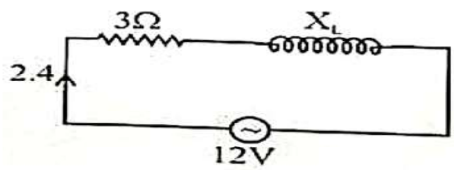
n = 24

Q2. A current of 4 A flows in a coil when connected to a 12V d.c. source. If the same coils is connected to a 12V, 50 rad/s, a.c. source, a current of 2.4 A flows in the circuit. Determine the reactance of coil.

(A) 5Ω (B) 3Ω (C) 4Ω (4) 2Ω

Ans. (C)

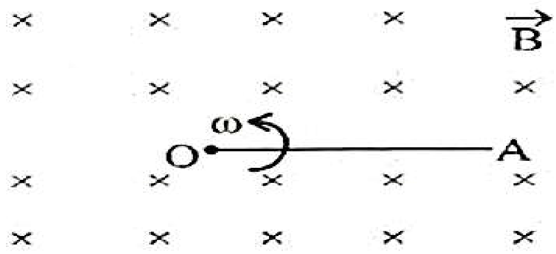
Sol. $\therefore R=\frac{12}{4}=3\Omega$ (When connected to DC source)



$12=2.4\sqrt{3^2+x_L^2}$

$5=\sqrt{3^2+x_L^2}$

Q3. A conducting rod OA of length $2l$ is rotated with constant angular velocity ω about a point O in a uniform magnetic field $\vec{B}$ directed into the paper. Then



(A) $V_A - V_O = \frac{B\omega l^2}{2}$

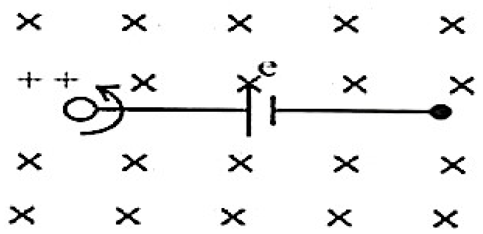
(B) $V_A - V_O = -B\omega l^2$

(C) $V_A - V_O = 2B\omega l^2$

(D) $V_A - V_O = -2B\omega l^2$

Ans. (D)

Sol.



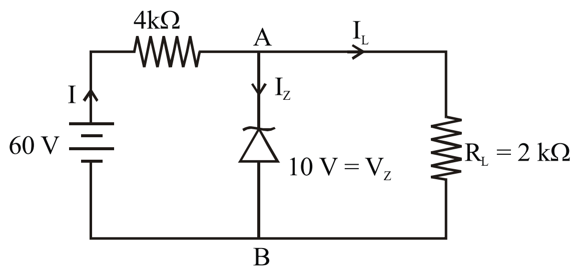
$e = V_O - V_A = \frac{B\omega(2\ell)^2}{2}$

$V_O - V_A = 2B\omega\ell^2$

$V_A - V_O = -2B\omega\ell^2$

Q4 A Zener diode is connected to a battery and a load as shown below :-

The currents I, I_Z and I_L are respectively.



(A)12.5 mA, 5 mA, 7.5 mA

(B) 15 mA, 7.5 mA, 7.5 mA

(C) 12.5 mA, 7.5 mA, 5 mA

(D) 15 mA, 5 mA, 10 mA

Ans. (C)

Sol. $I_L = \frac{10V}{2k\Omega} = 5mA$

$I = \frac{(60-10)V}{4k\Omega} = \frac{50V}{4k\Omega} = 12.5\ mA$

$I_Z = I - I_L = (12.5 - 5)\ mA = 7.5\ mA$

Q5. The displacement y of a wave travelling in the x-direction is given by

$$y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right) metre,$$

where x is expressed in metre and t in second. The speed of the wave in ms^{-1} is

(A) 300 (B) 600 (C) 1200 (D) 200

Ans. (A)

Sol. $V = \dfrac{\text{coefficient of } t}{\text{coefficient of } x}$

$= \dfrac{600}{2} = 300\,\text{m/s}$

Q6. According to the Bohr theory of the hydrogen atom, electrons starting in the 4^{th} energy level and eventually ending in the ground state could produce a total of how many lines in the hydrogen spectra?

(A) 7 (B) 6 (C) 5 (D) 4

Ans. (B)

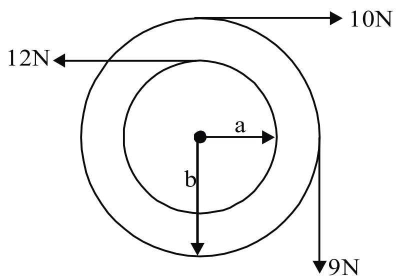
Sol. No. of spectral lines $= \dfrac{n(n-1)}{2}$

$n = 4 \rightarrow n = 1$

$= \dfrac{4(4-1)}{2} = 6$

$\therefore$ No of spectral lines are 6.

Q7. In the figure a = 5cm and b = 20cm. If the M.I. of the wheel is 3200 kg-m^2, the angular acceleration would be



(A) 10^{-1} rad/s^2 (B) 10^{-2}rad/s^2 (C) 10^{-3} rad/s^2 (D) 10^{-4} rad/s^2

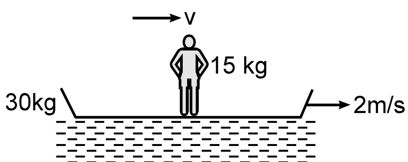
Ans. (C)

Sol. $\tau = I\alpha$

$3200\alpha = 10 \times 0.2 + 9 \times 0.2 - 12 \times 0.05$

$= 2 + 1.8 - 0.6 \Rightarrow \alpha = 10^{-3}$ rad/s^2

Q8. In the figure shown the initial velocity of (boat + person) is 2 m/s. Find velocity of person w.r.t. boat (v) so that velocity of boat will be 1 m/s in right.



(A) 3 m/s towards right (B) 3 m/s towards left

(C) 4 m/s towards right (D) 4 m/s towards left

Ans. (A)

Sol. 15(2) + 30(2) = 15(v + 1) + 30(1)

v = 3 m/s towards right

Q9. A bucket tied to a string is lowered at a constant acceleration of g/4. If the mass of the bucket is M and is lowered by a distance d, the work done by the string will be (assume the string to be massless)

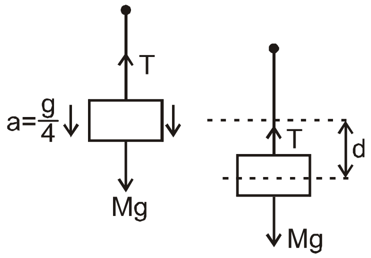
(A) 1/4 Mg d (B) −3/4 Mgd (C) −4/3Mgd (D) 4/3 Mgd

Ans. (B)

Sol. Let tension in string be T, then work done by tension T = – Td

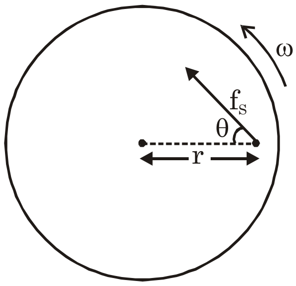
Applying Newton's second law on the bucket.

$$Mg - T = M\left(\frac{g}{4}\right) \text{ or } T = \frac{3}{4}Mg$$



∴ required work done = $-\frac{3}{4}$ Mg d

Q10. A small particle of mass m is at rest on a horizontal circular platform that is free to rotate about a vertical axis through its center. The particle is located at a radius r from the axis, as shown in the figure. The platform begins to rotate with constant angular acceleration α. Because of friction between the particle and the platform, the particle remains at rest with respect to the platform. When the platform has reached angular speed ω, the angle θ between the static frictional force f_s and the inward radial direction is :-



(A) $\theta = \frac{\omega^2 r}{g}$ (B) $\theta = \frac{\alpha}{\omega^2}$

(C) $\theta = \tan^{-1}\left(\frac{\omega^2}{\alpha}\right)$ (D) $\theta = \tan^{-1}\left(\frac{\alpha}{\omega^2}\right)$

Ans. (D)

Sol. $f_r = m\omega^2 r$

$f_s = m\alpha r$

$\tan\theta = \frac{f_s}{f_r} = \frac{\alpha}{\omega^2}$

Q11. In order to achive maximum horizontal range, a particle of mass m is projected with linear momentum P from the ground. What is the minimum kinetic energy of a particle during its flight is

(A) $\frac{P^2}{m}$ (B) $\frac{P^2}{4m}$ (C) $\frac{P^2}{2m}$ (D) $\frac{P^2}{3m}$

Ans. **(B)**

Sol. Kinetic energy of a particle will be minimum at maximum height of its irajectory.

$$K.E_{min} = \frac{1}{2}m(u^2\cos^2 45)$$

$$K.E_{min} = \frac{1}{2}mu^2 \times \frac{1}{2}$$

$$KE_{min} = \frac{P^2}{4m}$$

Q12. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A): Even when monochromatic light is incident on a metal, the kinetic energies of emitted photoelectrons are different.

Reason (R): Kinetic energies of photoelectrons emitted from inside the metallic surface varies due to their collisions with the other atoms in the metal.

(A) Both A and R are correct and R is NOT the correct explanation of A.

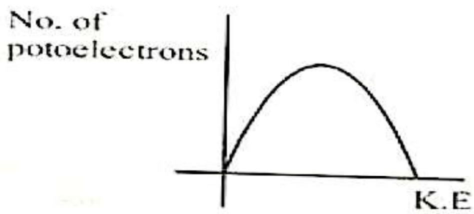
(B) A is correct but R is not correct.

(C) Both A and R are correct and R is the correct explanation of A.

(D) A is not correct but R is correct.

Ans. (C)

Sol.



∴ Kinetic energy of emitted photoelectrons are different. $K.E_{max}$ = hν – work funtion – Energy loss in collision

∴ Kinetic energy of photoelectrons emitted from metallic surface varies due to collision with other atoms in the metal.

Q13. In Young's double slit experiment, if intensity of the interfering waves from two slits are in the ratio 4 : 9, ratio of intensity of maxima to intensity of minima will be

(A) 25 : 1 (B) 9 : 4 (C) 3 : 2 (D) 81 : 16

Ans. (A)

Sol. As ratio of slit widths = Ratio of intensities

$$\therefore \frac{I_1}{I_2}=\frac{4}{9} or \frac{a_1^2}{a_2^2}=\frac{4}{9}\Rightarrow\frac{a_1}{a_2}=\frac{2}{3}$$

$$a_{max}=a_1+a_2=3+2=5; a_{min}=3-2=1$$

$$\frac{I_{max}}{I_{min}}=\frac{(a_1+a_2)^2}{(a_1-a_2)^2}=\frac{(3+2)^2}{(3-2)^2}=\frac{25}{1}$$

Q14. Which of the following has smallest number of significant digits :-

(A) 0.00145 cm (B) 14.50 cm (C) 145.00 cm (D) 145.0×10^{-6} cm

Ans. (A)

Sol. Before non-zero digit trailing zeros are nonsignificant figure if number is less than one

$\therefore$ (A) 0.00145 sinnificant digits

$\therefore$ No. of significant digits = 3

(B) 14.50 : 4 significant digits (Number > 1)

(C) 14.500 : 5 significant digits

(D) $145.0\times10^{-6}:4$ significant digits

Q15. Given that $\vec{A}+\vec{B}=\vec{R}$ and $\vec{A}+2\vec{B}$ is perpendicular to $\vec{A}$ then

(A) 2B = R (B) B = 2R (C) B = R (D) $B^2 = 2R^2$

Ans. (C)

Sol. $(\vec{A}+2\vec{B})\cdot\vec{A}=0$

$\vec{A}.\vec{A}+2\vec{A}.\vec{B}=0$

$A^2 + 2AB\cos\theta = 0$ .....(i)

Now $|\vec{A}+\vec{B}|=|\vec{R}|$

or $A^2 + B^2 + 2AB\cos\theta = R^2$

By equation (i)

$B^2=R^2\Rightarrow|\vec{B}|=|\vec{R}|$

Q16. Dimensional formula for capacitance $\times$ (potential)^2 is

(A) $[M^1L^1T^{-2}]$ (B) $[M^0L^1T^{-2}]$ (C) $[M^1L^2T^{-2}]$ (D) $[M^1L^2T^{-1}]$

Ans. (C)

Sol. $C=\frac{Q}{V}$ so $CV^2=\frac{Q}{V}V^2=QV\Rightarrow[m^1L^2T^{-2}]$

Q17. Four cubes of ice at – 10°C each of one gm is taken out from the refrigerator and are put in 150 gm of water at 20°C. The temperature of water when thermal equilibrium is attained. Assume that no heat is lost to the outside and water equivalent of container is 46 gm. (Specific heat capacity of water = 1 cal/gm-°C, Specific heat capacity of ice = 0.5 cal/gm-°C, Latent heat of fusion of ice = 80 cal/gm-°C)

(A) 0°C (B) – 10°C (C) 17.9°C (D) None

Ans. (C)

Sol. Heat gained by ice = Heat lost by water + Heat lost by container

Initial Temperature of container = 20ºC

$4 \times \frac{1}{2} \times 10 + 4 \times 80 + 4 \times 1 \times (T-0) = 196 \times 1 \times (20-T)$

$20+320+4T=196 \times 20-196\ T$

$200\ T = 196 \times 20 - 340$

$T = \frac{3580}{200} = 17.9ºC$

Q18. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is: (R = Gas constant)

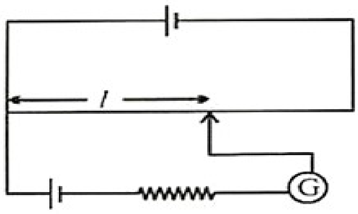
(A) 4 R T (B) 5 R T (C) 15 R T (D) 11 R T

Ans. (D)

Sol. In an ideal gas internal energy = $\frac{f}{2}$nRT

$U = \frac{5}{2} \times 2 \times RT + 4 \times \frac{3}{2} RT = 11\ RT.$

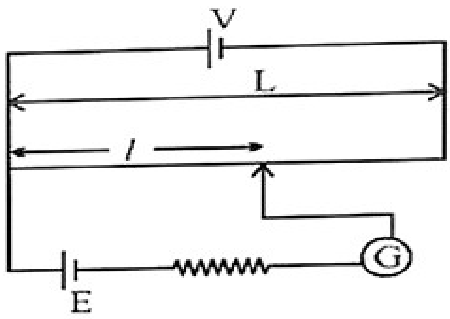
Q19. In a potentiometer (see figure) a balance is obtained at a length of 400 mm when using a known battery of emf 1.6 volts. After removing this battery, another battery of unknown emf is used and balance is obtained at a length of 650 mm. The emf of unknown battery is



(A) 2.6 volt (B) 1.6 volt (C) 3.4 volt (D) 4.7 volt

Ans. (A)

Sol.



Voltage drop in length L is = V

Voltage drop in unit length $= \frac{V}{L}$

Voltage drop in length $l = \frac{V}{L} \times l$

$\therefore E = \frac{V}{L} . l$

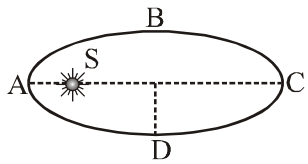
Now $1.6 = \frac{V}{L} \times 400$ ..... (1)

$$E = \frac{V}{L} \times 650 \qquad \text{..... (2)}$$

From $\frac{(1)}{(2)} \Rightarrow \frac{1.6}{E} = \frac{400}{650}$

E = 2.6 volt

Q20. A planet revolves in elliptical orbit around the sun. The linear speed of the planet will be maximum at :-



(A) A (B) B (C) C (D) D

Ans. **(A)**

Sol. here, angular momentum is conserved. i.e.,

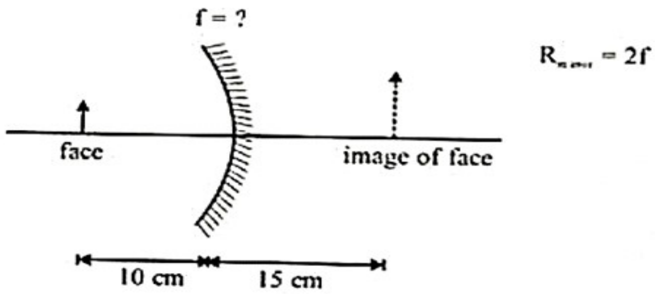
L = Iω = constant. At A, the moment of inertia I is least, so angular speed and therefore the linear speed of planet at A is maximum.

Section-II(NV)

Q21. You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm from himself. The magnitude of radius of curvature (in cm) of the mirror would then be __________.

Ans. 60

Sol.



For mirror formula we use

$u = -10\,\text{cm}$

$v = +15\,\text{cm}$

$$\Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{f} = -\frac{1}{10} + \frac{1}{15} = -\frac{5}{150} = -\frac{1}{30}$$

$$\Rightarrow \text{f} = -30\,\text{cm} \Rightarrow R = 2\text{f} = 2 \times -30 = -60\,\text{cm}.$$

Q22. If the wavelenth of K_α radiation emitted by an atom of atomic number $Z = 41 \text{ is } \lambda$, then the atomic number for an atom that emits K_α radiation with wavelength 4λ, is

Ans. (21)

Sol. $\frac{1}{\lambda} \propto (Z-1)^2$

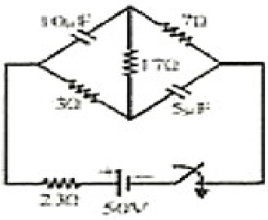
$\therefore \frac{\lambda_1}{\lambda_2} = \frac{(Z_2-1)^2}{(Z_1-1)^2}$

$\frac{1}{4} = \left(\frac{Z_2-1}{41-1}\right)^2$

$\frac{1}{2} = \frac{Z_2-1}{40}$

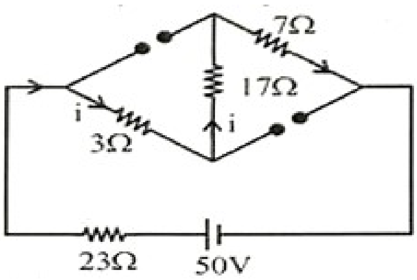
Solving this, we get, $Z_2 = 21$

Q23. In the given circuit if switch is closed, write down the charge in μC on $10\mu F$ capacitor in steady state.



Ans. 200

Sol. In steady state, capacitors behave like an open circuit.



Effective resistance = 23 + 3 + 17 + 7

$= 50\ \Omega$

$\therefore$ Current through the battery.

$i = \frac{50}{50}$

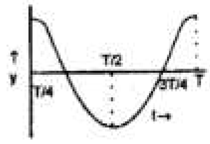
$i = 1A$

the Charge on $10\mu C$ capacitor, $Q = CV$

$= (10\mu F)(1 \times (17+3))$

$= 200\mu C$

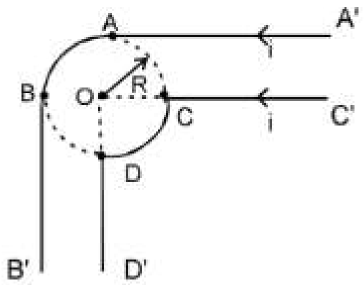
Q24. The graph in the figure shows how the displacement of a particle describing S.H.M. varies with time. The force is zero at time $\frac{T}{m}$ and $\frac{nT}{4}$. Find the value of $m+n$.



Ans. 7

Sol. Force is zero at $\frac{T}{4}$ and $\frac{3T}{4}$ $m + n = 4 + 3 = 7$

Q25. All straight wires are very long. Both AB and CD are arcs of the same circle, both subtending right angles at the centre O . Then the magnetic field at O is $\frac{\mu_0 i}{N\pi R}$. Find the value of N .



Ans. 2

Sol. field due to $AA' = \frac{\mu_0 i}{4\pi R}$

= field due to BB' field due to CC' = field due to $DD' = 0$

field due to $BA = \frac{\mu_0 i}{8R}$

field due to $CD = \frac{-\mu_0 i}{8R}$

$\therefore$ net field at O=$\frac{\mu_0 i}{2\pi R}$.

CHEMISTRY

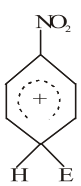
Q1. Which type of isomerism is not shown by C_3H_8O.

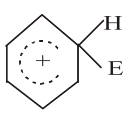
(A) Position isomerism (B) Functional isomerism

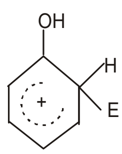
(C) Constitutional isomerism (D) Stereo isomerism

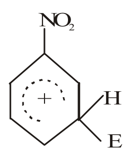
Ans. (D)

Q2. The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ–complex. Of the following, which σ–complex is of lowest energy?

(A) 

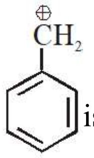
(B) 

(C) 

(D) 

Ans. (C)

Q3. The order of stability of the following carbocations:

$CH_2 = CH - \overset{\oplus}{C}H_2$ (I); $CH_3 - CH_2 - \overset{\oplus}{C}H_2$ (I);  (III) is :-

(A) III > II > I (B) II > III > I

(C) I > II > III (D) III > I > II

Ans. **(D)**

Q4. The most suitable reagent for the conversion of
$R - CH_2 - OH \rightarrow R - CHO$ is :-

(A) CrO_3 (B) PCC (Pyridinium chlorochromate)

(C) $KMnO_4$ (D) $K_2Cr_2O_7$

Ans. **(B)**

Sol. PCC is used to form aldehyde/ketone from alcohol.

Q5. Which of the following will not undergo aldol condensation :-

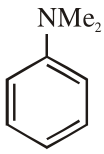
(A) Acetaldehyde (B) Propanaldehyde

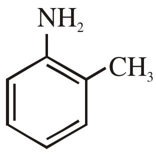
(C) Benzaldehyde (D) Acetone

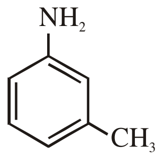
Ans. **(C)**

Sol. Due to absence of H_α

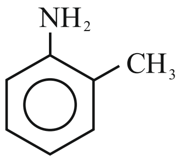
Q6. Identify the compound which is less basic than aniline?

(A) CH_3NH_2 (B) 

(C) 

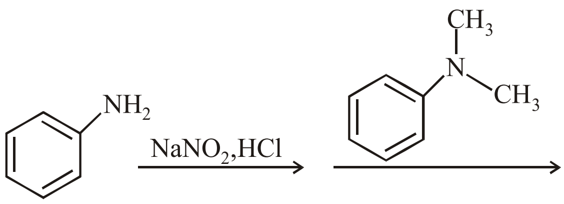
(D) 

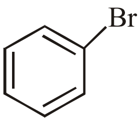
Ans. **(C)**

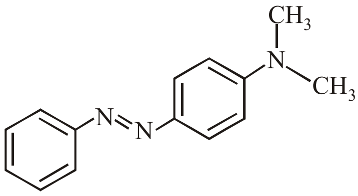
Sol. 

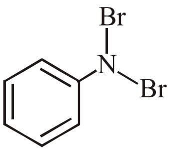
is less basic than ailine due to SIP.

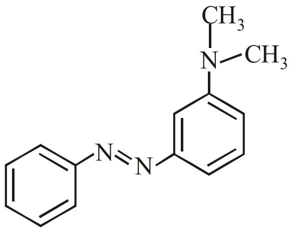
Q7. What could be the product for the following reaction ?



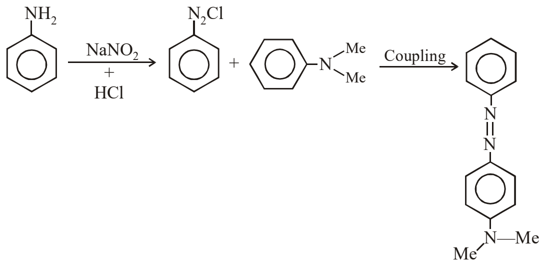
(A) 

(B) 

(C) 

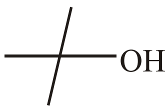
(D) 

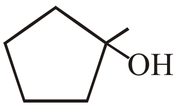
Ans. (B)

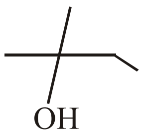


Q8. Which of the following will change the colour of acidic dichromate solution?

(A) 

(B) 

(C) 

(D) 

Ans. (B)

Sol. 3° alcohol does not get oxidise.

Q9. Highest ionisation potential in a period is shown by?

(A) Alkali metals (B) Noble gases

(C) Halogens (D) Alkali Earth Metals

Ans. (B)

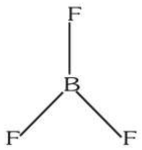
Q.10 Which of the following molecule has zero dipole moment?

(A) BF_3 (B) CH_2Cl_2

(C) NF_3 (D) SO_2

Ans. (A)

Sol.



Dipole moment of BF_3 is zero due to symmatrical structure.

Q11. One mole of magnesium nitride on reaction with excess of water gives -

(A) Two mole of HNO_3 (B) Two mole of NH_3

(C) 1 mole of NH_3 (D) 1mole of HNO_3

Ans. (B)

Sol. $Mg_3N_2 + 6H_2O \longrightarrow 3Mg(OH)_2 + 2NH_3$

Q.12 The formula of sodium nitroprusside is -

(A) $Na_4[Fe(CN)_5NOS]$ (B) $Na_2[Fe(CN)_5NO]$

(C) $NaFe[Fe(CN)_6]$ (D) $Na_2[Fe(CN)_6NO_2]$

Ans. (B)

Sol. Sodium nitroprusside = Na_2 $[Fe(CN)_5NO]$

Q.13 Among the following, the compound that is both paramagnetic and coloured is -

(A) $K_2Cr_2O_7$ (B) $(NH_4)_2[TiCl_6]$

(C) $CuSO_4$ (D) $K_3[Cu(CN)_4]$

Ans. (C)

Sol. $CoSO_4$ have d^7 configuration because $Co^{2+} = 3d^7\ 4s^o$), thus it both paramagnetic as well as coloured.

Q14. Which of the following metals has maximum unpaired electrons-

(A) Aluminium (B) Magnesium (C) Iron (D) None of these

Ans. (C)

Sol. Iron has four unpaired electrons

Q15. $(Me)_2$ $SiCl_2$ on hydrolysis followed by condensation will produce -

(A) $(Me)_2Si(OH)_2$ (B) $(Me)_2Si=O$

(C) $-[-O-(Me)_2Si-O-]_n-$ (D) $Me_2SiCl(OH)$

Ans. (C)

Sol. $(Me)_2SiCl_2 \xrightarrow{H_2O} [-O-(Me)_2Si-O-]_n$ Silicone

Q16. The average molar mass of chlorine is 35.5 g mol^{-1}. The ratio of ^{35}CL to ^{37}Cl in naturally occuring chlorine is close to

(A) 4:1 (B) 3:1 (C) 2:1 (D) 1:1

Ans. (B)

Sol. Let total moles = 1

moles of ^{35}Cl = x

moles of ^{37}Cl = 1-x

$$M_{arg} = \frac{\text{Toatl mass}}{\text{Total moles}}$$

$$35.5 = \frac{x(35)+(1-x)37}{1}$$

x = 0.75

$$\text{Molar ratio} = \frac{x}{1-x} = \frac{0.75}{0.25} = \frac{3}{1}$$

Q17. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

What is value of x ?

(A) 10 (B) 2 (C) 6 (D) 12

Ans. (A)

Sol. $2IO_3^- + 10I^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

Q18. The molar conductance at infinite dilute of $AgNO_3$, $AgCl$ and NaCl are 115,120 and 110 respectively. The molar conductance of $NaNO_3$ at infinite dilution is:

(A) 110 (B) 105 (C) 130 (D) 150

Ans. (B)

Sol. $\Lambda_m^{\infty}\left(NaNO_3\right)=\lambda_m^{\infty}\left(Na^+\right)+\lambda_m^{\infty}\left(NO_3^-\right)$

$=\Lambda_m^{\infty}(NaCl)+\Lambda_m^{\infty}\left(AgNO_3\right)$

$-\Lambda_m^{\infty}(AgCl)$

= 110 + 115 - 120

= 105

Q19. The solubility of A_2X_3 is Y mol/L. Its solubility product is

(A) $6Y^2$ (B) $64Y^4$ (C) $36Y^5$ (D) $108Y^5$

Ans. (D)

Sol. $A_2X_3(s) \rightleftharpoons 2A^{3+}+3x^{2-}$

y - -

\- 2y 3y

$k_{sp}=\left[A^{3+}\right]^2[X^{2-}]^3$

$=(2y)^2(3y)^3=108y^5$

Q20. The vapour pressure at a pure liquid solvent (x) is decreased to 0.60 atm from 0.80 atm on addition of non volatile substance (y). The mole fraction of (y) in the solution is :

(A) 0.12 (B) 0.25 (C) 0.5 (D) 0.75

Ans. (B)

Sol. $P_S=P^{\circ}x_{solvent}$

$x_{solvent}=\frac{P_S}{P^{\circ}}=\frac{0.60}{0.80}=\frac{3}{4}$

$x_{Solute}=1-x_{Solvont}=1-\frac{3}{4}=\frac{1}{4}=0.25$

Section-II(NV)

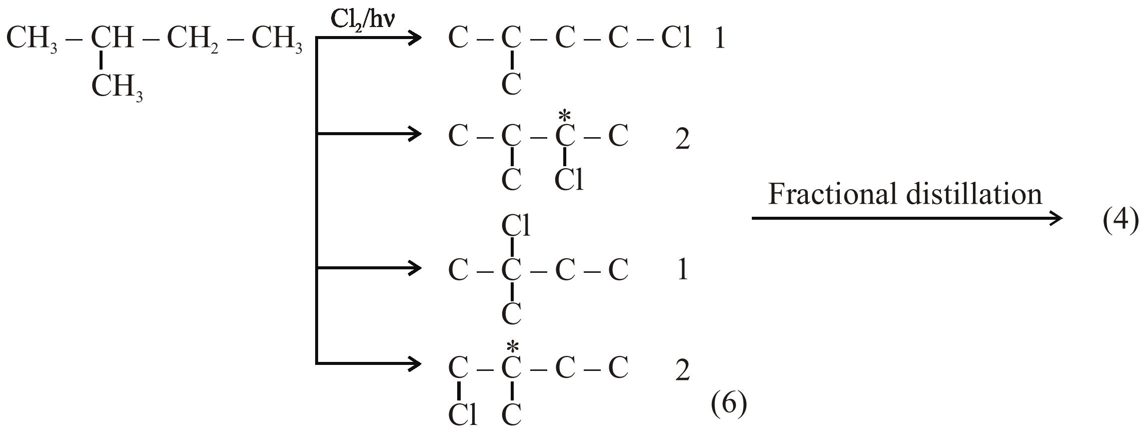
Q21. $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_3 \xrightarrow{Cl_2/h\nu}$ [x] Number of Total mono-chloro product(s).

[x] $\xrightarrow[\text{distilation}]{\text{Fractional}}$ [y] number of Total fraction(s)

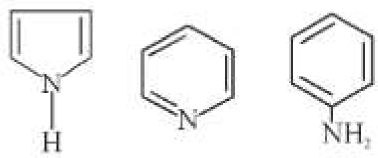
Value of (X+Y) is

Ans. (10)

Sol.



Q22.



How many statement are correct

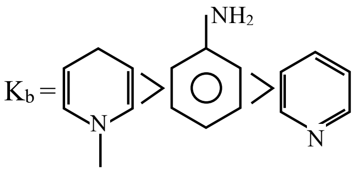
(i) Pyrrole is more basic than pyridine

(ii) Pyridine is more basic than pyrrole and aniline.

(iii) Aniline is more basic than pyridine

(iv) All are aromatic bases

Ans. (2)

Sol. 

Q23. What is the change in the oxidation no. of nitrogen when concentrated HNO_3 reacts with Zn metal?

Ans. (1)

Sol. $\underset{\text{(Verydil)}}{HNO_3} \xrightarrow{Zn} NO_2$

Q24. The ratio of wave length of photon corresponding to α-line of Lyman series in H -atom to β-line of Balmer series in He^+ is x:1, the value of x is ?

Ans. (1)

Sol. $$\frac{1}{\lambda_\alpha} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \times 1^2 \quad \text{...........(i)}$$

$$\frac{1}{\lambda_\beta} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \times 2^2 \quad \text{...........(ii)}$$

Equation(ii) ÷ Equation(i)

$$\frac{\lambda_\alpha}{\lambda_\beta} = 1$$

Q25. In a chemical equilibrium, the rate constant of forward reaction (K_f) is 8×10^{-4} and rate constant of backward reaction (K_b) is 4×10^{-4}. The value of equilibrium constant is ?

Ans. (2)

Sol. $$k_{eq} = \frac{k_f}{k_b} = \frac{8\times10^{-4}}{4\times10^{-4}} = 2$$

MATH

Score JEE Main - 1 2025 AIOT 1

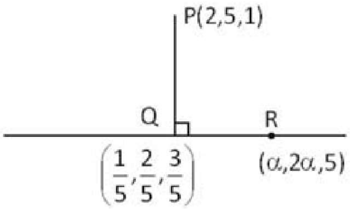
ESJMRT13MT051

Section-I(SC)

Q1. If the foot of perpendicular drawn from the point (2,5,1) on a line passing through (α, 2α, 5) is $\left(\frac{1}{5},\frac{2}{5},\frac{3}{5}\right)$, then α is equal to

(A) $\frac{19}{9}$ (B) $\frac{11}{570}$ (C) $\frac{57}{54}$ (D) $\frac{1}{25}$

1. (D)

1. 

From diagram $\overrightarrow{PQ}\cdot\overrightarrow{QR}=0$

$$\left[\left(2-\frac{1}{5}\right)\hat{i}+\left(5-\frac{2}{5}\right)\hat{j}+\left(1-\frac{3}{5}\right)\hat{k}\right]\cdot\left[\left(\alpha-\frac{1}{5}\right)\hat{i}+\left(2\alpha-\frac{2}{5}\right)\hat{j}+\left(5-\frac{3}{5}\right)\hat{k}\right]=0$$

$$\Rightarrow\frac{9\alpha}{5}-\frac{9}{25}+\frac{46\alpha}{5}-\frac{46}{25}+\frac{44}{25}=0$$

$$\Rightarrow\alpha=\frac{1}{25}$$

Q2. If $\sum_{i=1}^{n}(x_i-5)=n$ and $\sum_{i=1}^{n}(x_i-5)^2=5n$, then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

(1) 2n (2) 2 (3) 4 (4) $2\sqrt{n}$

2. (2)

Sol. S.D $=\sqrt{\frac{\sum_{i=1}^{n}(x_i-5)^2}{n}-\left(\frac{\sum_{i=1}^{n}(x_i-5)}{n}\right)^2}$

$=\sqrt{5-1}=2$

Q3. The function $f(x)=\sin^{-1}\left(2x-x^2\right)+\sqrt{2-\frac{1}{|x|}}+\frac{1}{\left[x^2\right]}$ is defined in the interval (where [.] is the greatest integer function)

(A) $x \in (1-\sqrt{2},1)$ (B) $x \in [1,1+\sqrt{2}]$

(C) $x \in [1-\sqrt{2},1+\sqrt{2}]$ (D) $x \in [1-\sqrt{2},2]$

3. (B)

3. (i) $-1 \le 2x - x^2 \le 1$ (for $\sin^{-1}$ to be defined)

$\Rightarrow -1 \le x^2 - 2x \le 1$

i.e. $x^2 - 2x + 1 \ge 0$ and $x^2 - 2x - 1 \le 0$

$(x-1)^2 \ge 0$ and $(x-1)^2 - (\sqrt{2})^2 \le 0$

$x \in R$ and $(x-1-\sqrt{2})(x-1+\sqrt{2}) \le 0$

$\Rightarrow x \in [1-\sqrt{2},1+\sqrt{2}] \ldots (1)$

(ii) $2 - \frac{1}{|x|} \ge 0 \Rightarrow \frac{1}{|x|} \le 2 \Rightarrow |x| \ge \frac{1}{2} \Rightarrow x \in \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right) \ldots (2)$

(iii) $\left[x^2\right] \ne 0 \Rightarrow x^2 \notin [0,1)$

$\Rightarrow x \notin (-1,1) \Rightarrow x \in (-\infty,-1] \cup [1,\infty) \ldots (3)$

Hence, $(1) \cap (2) \cap (3)$

$\Rightarrow x \in [1,1+\sqrt{2}]$

Q4. If set $A = \{x : \tan x + \sec x = 2, x \in [0,4\pi]\}$

and set $B = \left\{x : \sin^2 x = 1, x \in [0,4\pi]\right\}$, then

(A) $B \subset A$

(B) $A \subset B$

(C) number of relations defiend from set A to set B = 2^{16}

(D) number of relations defiend from set A to set B = 256

4. (D)

For set A, $\sec x + \tan x = 2$

4. $\Rightarrow \sec x - \tan x = \frac{1}{2}$

$\Rightarrow \sec x = \frac{5}{4}$ *and* $\tan x = \frac{3}{4}$

$x = \theta,\ 2\pi + \theta$, *where* $\theta = \tan^{-1}\frac{3}{4}$

$\therefore A = \{\theta, 2\pi + \theta\}$

for B, $\sin x = 1$, $\sin x = -1$

$x = \frac{\pi}{2}, \frac{5\pi}{2}$ or $x = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore B = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}\right\}$

$\therefore n(A \times B) = 8$

$\therefore$ number of relations defiend from set A to set B = 2^8

Q5. Let $\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{c} = -8\hat{i} - \hat{j} + \gamma\hat{k}$. Find γ such that $\vec{a}, \vec{b}, \vec{c}$ lie in the same plane.

(A) 2 (B) –3 (C) 3 (D) 4

5. (C)

5. $\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$

$$\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & \gamma \end{vmatrix} = 0$$

$\Rightarrow \gamma = 3$

Q6. For a complex number Z, if all the roots of the equation $Z^3 + aZ^2 + bZ + c = 0$ are unimodular, then

(A) $|a| > 3$ and $|c| = 1$ (B) $|a| \le 3$ and $|c| = 3$

(C) $|a| > 3$ and $|c| = \frac{1}{3}$ (D) $|a| \le 3$ and $|c| = 1$

6. (D)

6. Given, $|Z_1| = |Z_2| = |Z_3| = 1$

Now, $|Z_1 + Z_2 + Z_3| \le |Z_1| + |Z_2| + |Z_3|$

$\Rightarrow |-a| \le 1 + 1 + 1$

$\Rightarrow |a| \le 3$

Also, $|Z_1 Z_2 Z_3| = |Z_1| \times |Z_2| \times |Z_3|$

$\Rightarrow |-c| = 1 \times 1 \times 1$

$\Rightarrow |c| = 1$

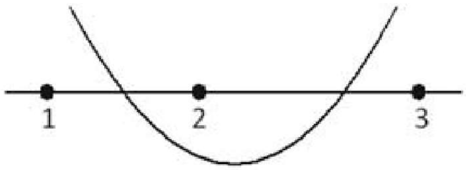
Q7. If α and β are the roots of $4x^2 - 16x + t = 0$, $\forall\, t > 0$ such that $1 < \alpha < 2 < \beta < 3$, then the number of integral values of t are

(A) 2 (B) 3 (D) 4 (D) 5

7. (B)

7. $f(x) = 4x^2 - 16x + t$

Opening upward parabola



$f(1) > 0, f(2) < 0, f(3) > 0$

$f(1) > 0 \Rightarrow 4 - 16 + t > 0 \Rightarrow t > 12$

$f(2) < 0 \Rightarrow 16 - 32 + t < 0 \Rightarrow t < 16$

$f(3) > 0 \Rightarrow 36 - 48 + t > 0 \Rightarrow t > 12$

$\Rightarrow 12 < t < 16 \Rightarrow t = 13, 14, 15$

Hence, the number of integral values (t = 13,14,15) is 3.

Q8. The number of ways in which 10 boys can be divided into 2 groups of 5, such that two tallest boys are in two different groups, is equal to

(A) 70 (B) 35 (C) 252 (D) 126

8. (A)

8. Excluding the 2 tallest boys, we can divide the 8 boys into 2 groups of 4 each by

$\frac{8!}{4!4!2!}$ ways.

The tallest boys can be assigned to the groups in 2 ways.

The desired number of ways are

$$\frac{8!}{4!4!2!} \times 2! = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2} = 70$$

Q9. The coefficient of x^4 in the expansion of $\left(1 - x - 2x^2\right)^8$ is

(A) 144 (B) -144 (C) 154 (D) -154

9. (D)

9. $(1 - x(1 + 2x))^8$

$= 1 - {}^8C_1 \times (1 + 2x) + {}^8C_2\, x^2 (1 + 2x)^2$

$- {}^8C_3\, x^3 (1 + 2x)^3 + {}^8C_4\, x^4 (1 + 2x)^4 \cdots\cdots$

The coefficient of x^4 is

$$^{8}C_2\cdot 2^2-{}^{8}C_3\cdot 2\cdot 3+{}^{8}C_4$$
$$=\frac{8\times 7}{2}\times 2^2-\frac{8\times 7\times 6}{3\times 2}\cdot 3\cdot 2+\frac{8\times 7\times 6\times 5}{4\times 3\times 2}$$
$$=112-336+70$$
$$=-154$$

Q10. Let S be the set of all real numbers. Then the relation $R=\{(a,b):1+ab>0\}$ on S is

(A) Reflexive and symmetric but not transitive

(B) Reflexive, transitive and symmetric

(C) Symmetric, transitive but not reflexive

(D) Reflexive, transitive but not symmetric

10. (A)

Sol. $1+a\cdot a=1+a^2>0, \forall a\in S, \quad \therefore (a,a)\in R$

$\therefore$ R is reflexive

$(a,b)\in R \Rightarrow 1+ab>0 \Rightarrow \quad 1+ba>0 \Rightarrow \quad (b,a)\in R$

$\therefore$ R is symmetric.

because $(a,b)\in R$ and $(b,c)\in R$ need not imply $(a,c)\in R$

Hence, R is not transitive.

Q11. Let D is a point on the line $l_1: x+y-2=0$ and $S(3,3)$ is a fixed point. The line l_2 is perpendicular to DS and passes through S. If M is another point on the line l_1 (other than D), then the locus of the point of intersection of l_2 and the angle bisector of the angle MDS is

(A) Straight line (B) Circle

(C) Parabola (D) Hyperbola

11. (C)

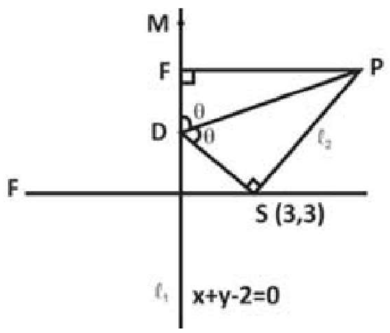
11. Let the point of intersection of l_2 and the angle bisector of angle MDS is P(h, k)

Draw perpendicular PF to l_1 from P.

Now, ΔPFD & ΔPSD are congruent triangles.

Hence PF = PS

$\therefore$ Locus of P is Parabola



Q12. Two tangents are drawn from a point (–4, 3) to the parabola $y^2 = 16x$. If α is the angle between them, then the value of $\cos\alpha$ is

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$

12. (A)

12. $\therefore$ (–4, 3) lies on the directrix of $y^2 = 16x$.

$\therefore$ Angle between tangents is 90°.

$\therefore$ $\cos 90° = 0$

Q13. The area bounded by $y \le 2 - |2 - x|$ and $y \ge \frac{3}{|x|}$ is:

(A) $\frac{4 + 3\ln 3}{2}$ (B*) $\frac{4 - 3\ln 3}{2}$ (C) $\frac{3}{2} + \ln 3$ (D) $\frac{1}{2} + \ln 3$

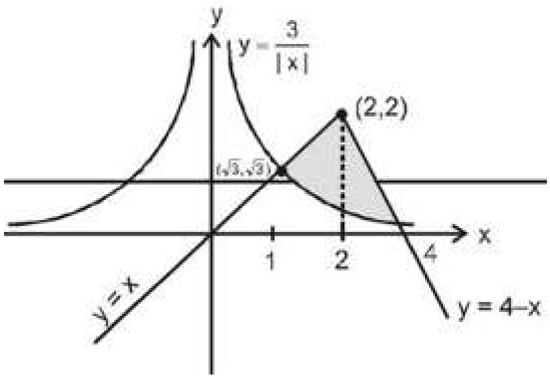
13. (B)

Sol. When $x < 2$

$$2 - 2 + x = \frac{3}{x} \Rightarrow x = \sqrt{3}$$

when $x > 2$

$$= 2 + 2 - x = \frac{3}{x} \Rightarrow X = 3, 1$$



$$A = \int_{\sqrt{3}}^{2}\left(x - \frac{3}{x}\right)dx + \int_{2}^{3}\left(4 - x - \frac{3}{x}\right)dx$$

$$= \left(\frac{x^2}{2} - 3\ell nx\right)_{\sqrt{3}}^{2} + \left(4x - \frac{x^2}{2} - 3\ell nx\right)_{2}^{3}$$

$$= \frac{4 - 3\ell n3}{2}$$

Q14. If the 6th term of a geometric progression is 5 then the product of first 11 terms of G.P. is equal to

(A) 5^5 (B) 5^{11} (C) 5^7 (D) 5^{10}

14. (B)

Sol. $ar^5 = 5$

$$T_1T_2T_3.......T_{11} = a.ar.ar^2........ar^{10}$$

$$= a^{11}r^{\left(\frac{10\times 11}{2}\right)} = (ar^5)^{11} = 5^{11}$$

Q15. The value of $10+2\log_{6/5}\left(\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}....}}}\right)$ is a proper diviser of:-

(A) 12 (B) 6 (C) 15 (D) 20

15. (A)

Sol. Let $x=\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+\frac{\sqrt{7}}{6}......}}$

$\Rightarrow x=\frac{\sqrt{7}}{6}\sqrt{\frac{25}{14}+x}$

$\Rightarrow x^2=\frac{7}{36}\left(\frac{25}{14}+x\right)$

$\Rightarrow x=\frac{25}{36}$

Now $10+2\log_{\frac{6}{5}} x=10+2\log_{(5/6)^{-1}}(5/6)^2$

$= 10-4$

$= 6$

Q16. The value of the integral $I=\int_0^{\pi}[|\sin x|+|\cos x|]dx$,

(where [.] denotes the greatest integer function) is equal to

(A) 1 (B) 2 (C) π (D) 2π

16. (C)

16. Let, $y=|\sin x|+|\cos x|$

$\Rightarrow y^2=1+|\sin(2x)|\in[1,2]$

$\Rightarrow y\in[1,\sqrt{2}]$

$\Rightarrow [|\sin x|+|\cos x|]=1$

Thus, $I=\int_0^{\pi}1\cdot dx=\pi-0$

$=\pi$

Q17. If $f(x)=\begin{cases}2x^2+3 & ; \quad x>3\\ ax^2+bx+1 & ; \quad x\le 3\end{cases}$ is differentiable everywhere, then $\frac{a}{b^2}$ is equal to

(A) 5 (B) $\frac{7}{3}$ (C) 1 (D) $\frac{16}{9}$

17. (C)

17. $\because$ f(x) is continuous at x = 3

$\therefore \lim_{x\to 3} f(x) = f(3)$

$2(3^2)+3 = a(3^2)+b(3)+1$

$9a+3b = 20\ldots(1)$

Now, $f'(x) = \begin{cases} 4x & ; \quad x > 3 \\ 2ax+b & ; \quad x \le 3 \end{cases}$

$\because$ f(x) is also differentiable at x = 3

$\therefore 4(3) = 2a(3)+b$

$6a+b = 12\ldots(2)$

From (1) & (2), we get,

$9a = 16 \Rightarrow a = \frac{16}{9}, b = \frac{4}{3}$

Q18. Consider a skew-symmetric matrix $A = [a_{ij}]_{2\times 2}$ such that a_{ij} is selected from the set S = {0, 1, 2, 3,.........12} for all i and j. If |A| is divisible by 3, then the number of such possible matrices is

(A) 4 (B) 5 (C) 6 (D) 12

18. (B)

18. $\because$ A is skew symmetric $\Rightarrow A = \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$

$|A| = b^2$

$\because |A|$ is divisible by 3

$\Rightarrow b$ can be 0,3,6,9,12 (five possibilities)

Q19. A bag contains 5 balls of unknown colours. A ball is drawn at random from it and is found to be red. Then the probability that all the balls in the bag are red, is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{2}{5}$ (D) $\frac{1}{3}$

19. (D)

19. Let $R_i (i = 1,2,3,4,5)$ denotes 1,2,3,4 or 5 red balls in the bag. Let R represent the ball drawn is red, then

$P(R_1) = P(R_2) = P(R_3) = P(R_4) = P(R_5) = \frac{1}{5}$

and $P\left(\frac{R}{R_1}\right)=\frac{1}{5}, P\left(\frac{R}{R_2}\right)=\frac{2}{5}, \quad P\left(\frac{R}{R_3}\right)=\frac{3}{5}, P\left(\frac{R}{R_4}\right)=\frac{4}{5}$

$P\left(\frac{R}{R_5}\right)=1$

$$\therefore P\left(\frac{R_5}{R}\right)=\frac{P(R_5)\cdot P\left(\frac{R}{R_5}\right)}{\sum_{i=1}^{5} P(R_i)\cdot P\left(\frac{R}{R_i}\right)}$$

$$=\frac{1}{1+\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}}=\frac{5}{15}=\frac{1}{3}$$

Q20. The sum of the roots of the equation cos4x + 6 = 7 cos2x in the interval [0, 314] is $\lambda\pi$, then the numerical value of λ is

(A) 4950 (B) 2475 (C) 9900 (D) 4945

20. (A)

20. $\left(2\cos^2 2x-1\right)+6=7\cos 2x$

On putting cos2x = t, we get,

$2t^2-1+6=7t$

$2t^2-7t+5=0$

$(2t-5)(t-1)=0$

$t=\frac{5}{2},1$

$t=\frac{5}{2}$ (not possible)

$t=1\Rightarrow \cos 2x=1\Rightarrow 2x=2n\pi$

$\Rightarrow x=n\pi$

The roots in [0,314] are

$\pi, 2\pi, 3\pi, \ldots, 99\pi \ \{100\pi>314\}$

Sum of roots $=\pi+2\pi+3\pi+\ldots+99\pi=4950\pi$

$\Rightarrow \lambda=4950$

Section-II(NV)

Q21. Let $f(x)=x^2-4x-3, x>2$ and g(x) be the inverse of f(x). Then the value of $\frac{1}{g'(2)}$ is (here, g' represents the first derivative of g)

21. 6

21. As $g(f(x))=x\left(\because g(x)=f^{-1}(x)\right)$

Differentiating, we get,

$g'(f(x))f'(x)=1$

$\Rightarrow g'(f(x))=\frac{1}{f'(x)}$

when f(x) = 2

$\Rightarrow x^2-4x-3=2$

$\Rightarrow x=5, x=-1$

$\because x>2$

$\therefore x=5$

$g'(2)=\frac{1}{(2x-4)_{x=5}}=\frac{1}{6}$

Q22. Let $y=f(x)$ satisfies $\frac{dy}{dx}=\frac{x+y}{x}$ and $f(e)=e,$ then the value of f(1) is

22. 0

22. $\frac{dy}{dx}=1+\frac{y}{x}$

$\frac{dy}{dx}-\frac{y}{x}=1$

If $=e^{-\int\frac{1}{x}dx}=\frac{1}{x}$

$\frac{y}{x}=\int\frac{1}{x}dx \Rightarrow y=x\ln x+cx=f(x)$

$f(e)=e+ce=e\Rightarrow c=0$

$f(1)=0$

Q23. If $\lim\limits_{x\to 0}\frac{ae^x+b\cos x+c+dx}{x\sin^2 x}=3$, then the value of $272\frac{abd}{c^3}$ is equal to

23. 34

23. Using expansions, we get,

$$\lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+..\right)+b\left(1-\frac{x^2}{2!}+\ldots\right)+c+dx}{x\left(x-\frac{x^3}{3!}+\ldots.\right)^2}=3$$

$$\lim_{x\to 0}\frac{(a+b+c)+(a+d)x+\left(\frac{a-b}{2}\right)x^2+\frac{a}{6}x^3+\cdots}{x^3\left(1-\frac{x^2}{3!}+\ldots.\right)^2}=3$$

$\because$ in the denominator lowest power of x is 3

For the limit to be finite, the numerator should also have the least power of x as 3

$\therefore a+b+c=0 \quad \ldots(1)$

$a+d=0\ldots(2)$

$\frac{a-b}{2}=0\ldots(3)$

Now, $\frac{\left(\frac{a}{6}\right)}{1}=3\Rightarrow a=18$

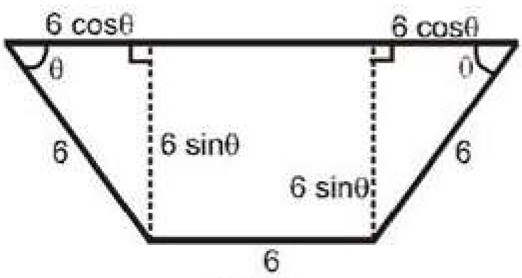
From (1),(2),(3), we get

a = 18, b = 18, c = –36, d = –18

$$\frac{abd}{c^3}=\frac{-(18)^3}{-8(18)^3}=\frac{1}{8}$$

24. The three sides of a trapezium are equal each being 6 cms long. Let $\Delta\,\text{cm}^2$ be the maximum area of the trapezium. The value of $\sqrt{3}\Delta$ is :

24. 81

Sol. 

Figure

Area of trapezium

$=\frac{1}{2}(12+12\cos\theta)\cdot 6\sin\theta$

$=36(1+\cos\theta)\sin\theta$

$f'(\theta)=36\left((1+\cos\theta)\cos\theta-\sin^2\theta\right)$

$=36\left(2\cos^2\theta+\cos\theta-1\right)=0$

$\Rightarrow \theta=\pi/3$

$\therefore \Delta=36\cdot\frac{3}{2}\cdot\frac{\sqrt{3}}{2}$

$=27\sqrt{3}$

25. If $\int\frac{3x^2-4x}{\left(x^2-3x+2\right)^2}dx=\frac{-x^2}{f(x)}+C$ (where C is constant of integration), where $f(x)$ is a quadratic expression, then the value of f(-1) is:

25. 6

Sol. $\int\frac{3x^2-4x}{\left(x^2-3x+2\right)^2}dx=\int\frac{\frac{3}{x^2}-\frac{4}{x^3}}{\left(1-\frac{3}{x}+\frac{2}{x^2}\right)^2}dx$

$1-\frac{3}{x}+\frac{2}{x^2}=t$

$\left(\frac{3}{x^2}-\frac{4}{x^3}\right)dx=dt$

$=\int\frac{dt}{t^2}$

$=-\frac{1}{t}+C$

$=-\frac{x^2}{x^2-3x+2}+c$

$\therefore f(-1)=1+3+2=6$