PHYSICS

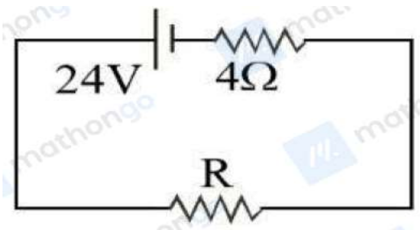
Section-I(SC)

Q1. A 24 V battery of internal resistance 4 Ω is connected to a variable resistance. If the rate of heat production in the resistor is maximum, then the current in the circuit is

(A) 2A (B) 3A (C) 4A (D) 6A

1. (B)

1. When R = r (power dissipated is maximum)



$\therefore R = 4\Omega$

$\therefore I = \frac{V}{R_{net}} = \frac{24}{8} = 3\,A$

Q2. A wire of length L carrying a current I is bent into a circle. The magnitude of the magnetic field at the centre of the circle is

(A) $\frac{\pi\mu_0 I}{L}$ (B) $\frac{\mu_0 I}{2L}$ (C) $\frac{2\pi\mu_0 I}{L}$ (D) $\frac{\mu_0 I}{2\pi L}$

2. (A)

2. $\frac{\pi\mu_0 I}{L}$

Length L = circumference of circle = $2\pi R$

$\Rightarrow \quad R = \frac{L}{2\pi}$

$\therefore$ Magnetic field at the centre of circular current loop is

$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2L} \times 2\pi$

$B = \frac{\mu_0 \pi I}{L}$

Q3. A uniform metal rod is moving with a uniform velocity v parallel to a long straight wire carrying a current I. The rod is perpendicular to the wire with its ends at distances r1 and r2 (with $r_2 > r_1$) from it. The E.M.F. induced in the rod at that instant is

(A) zero (B) $\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$

(C) $\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{r_1}{r_2}\right)$ (D) $\frac{\mu_0 Iv}{4\pi}\left(1-\frac{r_1}{r_2}\right)$

3. (B)

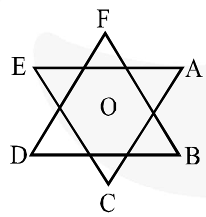
3. $\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$

$$B = \frac{\mu_0 i}{2\pi x}$$

$$d\varepsilon = Bv(dx)$$

$$\varepsilon = \frac{\mu_0 iv}{2\pi}\int_{r_2}^{r_1}\frac{dx}{x} = \left(\frac{\mu_0 iv}{2\pi}\ln\frac{r_2}{r_1}\right)$$

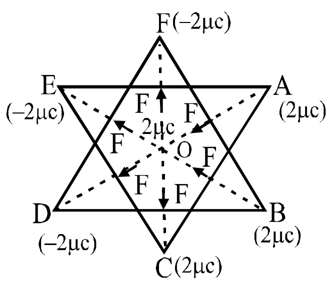
Q4. The magnitude of electric force on 2μC charge placed at the centre O of two equilateral triangles each of side 10 cm, as shown in figure is P. If charge A, B, C, D, E & F are 2μc, 2μc, 2μc, –2μc, –2μc, –2μc respectively, then P is:-



(1) 21.6 N (2) 64.8 N (3) 0 (4) 43.2 N

4. (4)

4.



The given figure shows force diagram for charge at O due to all other charges with $r = \frac{10}{\sqrt{3}}$ cm

$\therefore\ F_{net} = 2F + 4F\cos 60^\circ = 4F$

$$= \frac{4k(2\mu c)(2\mu c)}{\left(\frac{10}{\sqrt{3100}}\right)^2}$$

$$= \frac{4\times 9\times 10^9 \times 2\times 2\times 10^{-12}}{\left(\frac{1}{300}\right)}$$

$= 36 \times 4 \times 300 \times 10^{-3}$ N

$= 43.2$N

Q5. In Young's double slit experiment, the intensity of light at a point on the screen where the path difference is λ is I_0. The intensity of light at a point where the path difference becomes $\frac{\lambda}{3}$ is :-

(1) I_0 (2) $\frac{I_0}{4}$ (3) $\frac{I_0}{3}$ (4) $\frac{I_0}{2}$

5. (2)

5. $I_0 = I + I + 2\sqrt{I}\,I\cos\frac{2\pi}{\lambda}\cdot\lambda$

$I_0 = 4I$

$I = \frac{I_0}{4}$

Resultant $= I + I + 2\sqrt{I}\,I\cos\frac{2\pi}{\lambda}\cdot\frac{\lambda}{3}$

$= I$

$= \frac{I_0}{4}$

Q6. A motorcylist going around a circular track of radius 50 m with a speed of 25 m/s, is at a point X. A static siren at Y, which is outside the circular track is emitting sound of frequency n. How many times (approximately) in an hour will the motor cyclist hear the sound of actual frequency Y ?

(1) 24 (2) 287 (3) 600 (4) 574

6. **Ans.(4)**

Actual frequency will be heard when velocity of motor cyclist is perpendicular to line joining the source and motor cyclist is perpendicular to line joining the source and motor cyclist. If will happen 2 times per round.

No. of rounds in a hour $= \frac{3600}{4\pi}$ (where 4π is time for one round)

≈ 287

Answer is double of 287 ie 574

Q7. An object is projected from height h at angle 45° with horizontal. If lands at a horizontal distance R from the point of projection. The velocity of projection should be

(1) $\sqrt{\frac{R^2 g}{R+h}}$ (2) $\sqrt{\frac{hRg}{R+h}}$ (3) $\sqrt{\frac{h^2 g}{R+h}}$ (4) $\sqrt{\frac{R^2 g}{R+2h}}$

7. **Ans. (1)**

Taking point of projection as origin and x & y as horizontal and vertical,

Equation of trajectory

$$y = x - \frac{1}{2} g \frac{x^2}{u^2} \times 2$$

$$y = x - g\frac{x^2}{u^2}$$

As it passes through (– h, R)

$$-h = R - g\frac{R^2}{u^2} \Rightarrow u = \sqrt{\frac{gR^2}{h+R}}$$

Q8. A laser beam of power 27 mW has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by [Given permittivity of space $\varepsilon_0 \approx 9\times10^{-12} C^2N^{-1} m^{-2}$, speed of light $c = 3\times10^8 ms^{-1}$]

(1) $1kV \quad m^{-1}$ (2) $1.4kVm^{-1}$

(3) $0.7kVm^{-1}$ (4) $2kVm^{-1}$

8. (B)

8. $1.4kVm^{-1}$

$$I = \frac{P}{A} = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$\therefore E_0 = \sqrt{\frac{2P}{\varepsilon_0 CA}} = \sqrt{\frac{2\times27\times10^{-3}\times36\pi\times10^9}{3\times10^8\times10\times10^{-6}}}$$

$$E_0 = 1.4kVm^{-1}$$

Q9. Choose the **INCORRECT** statement for a fluid flow :-

(1) there is no flow across the streamline.

(2) $\oint \vec{v}.d\vec{\ell} = 0$ for irrotational flow.

(3) For flow to be irrotational viscosity should be absent.

(4) tangent to the streamline at a point does not give flow field direction.

9. Ans. (4)

Q10. A projectile is fired with a speed of $\sqrt{\frac{GM}{r}}$ at certain angle (θ ≠ 0°, 90°, 180°) with the radius vector. Here r is distance of projectile from center of Earth and M is mass of Earth. Choose the **INCORRECT** statement.

(1) The projectile will follow elliptical path.

(2) The projectile is at semi-minor axis of its elliptical path.

(3) Earth is at nearer focus if the projectile is initially approaching the Earth.

(4) Earth could be at any focus of the ellipse.

10. Ans. (3)

Sol. The projectile is initially at semi-minor axis

∴ Satellite is at equi distance from both focuses.

Q11. A block was observed to move on a straight line uniformly. It displaces 10 cm in 2 second. The straight line has calibrations of 1mm. and the stop watch used to measure time has least count of 0.1s. Using this data its speed is calculated. Now the displacement of the block is calculated for next 10s using same stop watch. The absolute error in the calculated value of displacement will be :

(1) 3.5 cm (2) 3.0 cm (3) 1.5 cm (4) 0.5 cm

11. Ans. (1)

Error in calculated value of speed

$= \left(\frac{\Delta \ell}{\ell} + \frac{\Delta t}{t}\right) \times 100 = \left(\frac{0.1}{10} + \frac{0.1}{2}\right) \times 100 = 6\%$

Error in calculated value of distance $= \left(\frac{\Delta v}{v} + \frac{\Delta t}{t}\right) \times 100$

$= 6\% + \frac{0.1}{10} \times 100 = 7\%$

Absolute error ⇒ distance × 7%

⇒ 50 cm × 7% = 3.5 cm

Q12. One mole of a diatomic gas undergoes a process $P = \frac{P_0}{\left(1+\frac{V}{V_0}\right)}$, where P_0, V_0 are constants. The translational kinetic energy of the gas when $V = V_0$ is given by

(1) $\frac{3P_0V_0}{2}$ (2) P_0V_0 (3) $\frac{3P_0V_0}{4}$ (4) $\frac{5P_0V_0}{2}$

12. Ans. (3)

Transational k.E of a molecule = $\frac{3}{2}kT$

Transational k.E of a mole = $\frac{3}{2}RT$

$= \frac{3}{2}PV = \frac{3}{2}\frac{P_0}{2}V_0$

Q13. The dimensions of $\frac{a}{b}$, in the equation $P = \frac{a-t^2}{bx}$, where P is pressure, x is distance and t is time, are

(1) $\left[M^2LT^{-3}\right]$ (2) $\left[MT^{-2}\right]$ (3) $\left[ML^3T^{-1}\right]$ (4) $\left[LT^{-3}\right]$

13. (B)

$P = \frac{a-t^2}{bx}$

$\Rightarrow \quad Pbx = a - t^2$

$\Rightarrow \quad [Pbx] = [a] = \left[t^2\right]$

or $[b] = \frac{\left[t^2\right]}{[P][x]} = \frac{\left[T^2\right]}{\left[ML^{-1}T^{-2}\right][L]} = \left[M^{-1}T^4\right]$

$\therefore \quad \left[\frac{a}{b}\right] = \frac{\left[T^2\right]}{\left[M^{-1}T^4\right]} = \left[MT^{-2}\right]$

$V' = \frac{mV}{M}$

Energy balance

$\frac{1}{2}MV'^2 = \mu Mg\cos\alpha\left(\frac{h}{\sin\alpha}\right) + Mgh$

$$\frac{V'^2}{2g} = (\mu\cot\alpha + 1)h$$

$$h = \frac{V'^2}{2g(\mu\cot\alpha + 1)} = \frac{m^2V^2\sin\alpha}{2M^2g(\mu\cos\alpha + \sin\alpha)}$$

Q14. A cannon of mass M, located at the base of inclined plane, shoots a shell of mass m in a horizontal direction with velocity v. To what vertical height does the cannon ascend the inclined plane as a result of recoil, if angle of inclination of plane is α and coefficient of friction between the cannon and the plane is μ

(1) $\frac{m^2v^2\sin\alpha}{2M^2g(\sin\alpha + \mu\cos\alpha)}$

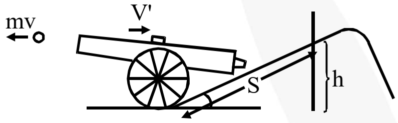
(2) $\frac{M^2v^2\sin\alpha}{2m^2g(\cos\alpha + \mu\sin\alpha)}$

(3) $\frac{mv^2\sin\alpha}{2Mg(\sin\alpha + \mu\cos\alpha)}$

(4) None of above

14. (1)

14.



$mV - MV' = 0$

$$V' = \frac{mV}{M}$$

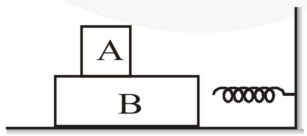
Energy balance

$$\frac{1}{2}MV'^2 = \mu Mg\cos\alpha\left(\frac{h}{\sin\alpha}\right) + Mgh$$

$$\frac{V'^2}{2g} = (\mu\cot\alpha + 1)h$$

$$h = \frac{V'^2}{2g(\mu\cot\alpha + 1)} = \frac{m^2V^2\sin\alpha}{2M^2g(\mu\cos\alpha + \sin\alpha)}$$

Q15. A block A is placed over block B having mass m & 2m respectively. Block B is resting on a frictionless surface and there is friction between block A and B . The system of blocks is pushed towards a spring with a velocity v_0 such that A doesn't slip on B by the time the system comes to momentary rest. The correct statement is



(1) Work done by friction on A is zero

(2) Work done by friction on B is $-\frac{1}{2}mv_0^2$

(3) Work done by spring on B is $-\frac{3}{2}mv_0^2$

(4) Work done by friction on A&B is zero

15. Ans. (3)

By Work-energy theorem on A $W_f = -\frac{1}{2}mv_0^2$

W_f on A + W_f on B = 0

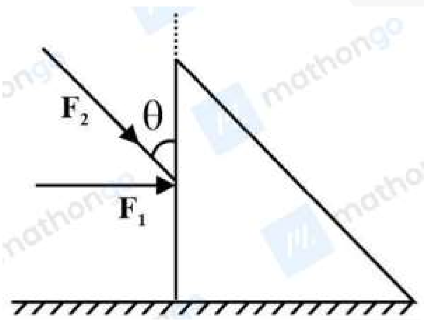
So W_f on $B = \frac{1}{2}mv_0^2$

By work-energy theorem on B

$$W_{spring} + W_f = \frac{1}{2} \times 2m \times v_0^2$$

$$W_{spring} = -\frac{3}{2}mv_0^2$$

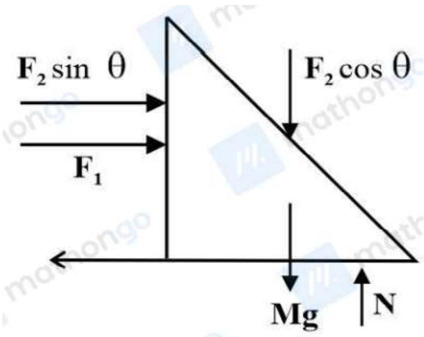
Q16. A wedge of mass m, lying on a rough horizontal plane, is acted upon by a horizontal force F_1 and another force F_2, inclined at an angle θ to the vertical. The block is in equilibrium, then the minimum coefficient of friction between it and the surface is



(1) $\frac{F_2 \sin\theta}{(mg + F_2 \cos\theta)}$ (2) $\frac{(F_1 \cos\theta + F_2)}{mg - F_2 \sin\theta}$ (3) $\frac{(F_1 + F_2 \sin\theta)}{(mg + F_2 \cos\theta)}$ (4) $\frac{(F_1 \sin\theta - F_2)}{(mg - F_2 \cos\theta)}$

16. (3)

$$\frac{(F_1 + F_2 \sin\theta)}{(mg + F_2 \cos\theta)}$$



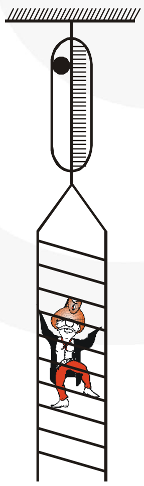
We know $f = \mu N$ and $N = mg + F_2 \cos\theta$

Now, since wedge is in equilibrium,

$$\left(F_1 + F_2 \sin\theta\right) = \mu\left(mg + F_2 \cos\theta\right)$$

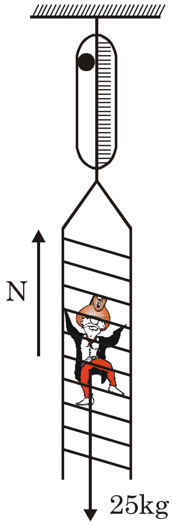
$$\mu = \frac{F_1 + F_2 \sin\theta}{mg + F_2 \cos\theta}$$

Q17. Figure shows a 5 kg ladder hanging from a string that is connected with a ceiling and is having a spring balance connected in between. A man of mass 25 kg is climbing up the ladderatacceleration 1 m/s^2. Assuming the spring balance and the string to be massless, the reading of the spring balance is :-



(1) 30 kg (2) 32.5 kg (3) 35 kg (4) 37.5 kg

17. Ans. (2)

Sol. 

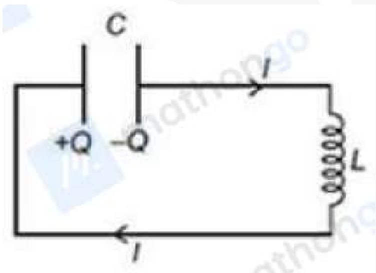
N – 25g = 25 × 1

N = 275

∴ W = 275 + 50 = 325 N

Q18. In the LC circuit shown below, the current is in direction as shown and the charges on the capacitor

plates have the sign as shown. At this instant



(A) I is increasing Q is increasing (B) I is increasing Q is decreasing

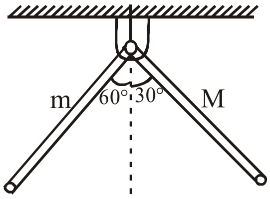
(C) I is decreasing Q is increasing (D) I is decreasing and Q is decreasing

18. (C)

18. As clearly 'Q' is increasing so electric field energy in capacitor must be increasing. And hence, magnetic field energy in inductor must be decreasing. So I is decreasing.

Q19. Two uniform rods of same length but masses m and M are joind to form a L–shape figure shows equillibrium

position then $\frac{M}{m}$ is :-



(1) 2 (2) 3 (3) $\sqrt{2}$ (4) $\sqrt{3}$

19. (4)

19. $L_{net} = 0$

$$mg\frac{L}{2}\sin 60° = Mg\frac{L}{2}\sin 30°$$

$$\frac{M}{m} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

Q20. A paramagnetic substance, in the form of a cube with sides 1 cm, has a magnetic dipole moment of $20\times10^{-6}\,JT^{-1}$, when a magnetic intensity of $60\times10^{3}\,A\,m^{-1}$ is applied. Its magnetic susceptibility is

(A) 3.3×10^{-4} (B) 2.3×10^{-2} (C) 4.3×10^{-2} (D) 3.3×10^{-2}

20. (A)

20. 3.3×10^{-4}

M= Magnetic moment per unit volume = $\frac{20\times10^{-6}}{\left(1\times10^{-2}\right)^{3}}JT^{-1}\,m^{-3}$

$= 20\,JT^{-1}\,m^{-3}$

$H = 60\times10^{3}\,A\,m^{-1}$

$\chi = \frac{M}{H}$

$= \frac{20}{60\times10^{3}} = 3.3\times10^{-4}$

Section-II(NV)

Q21. A transistor having β equal to 80 has a change in base current of 250 μA, then the change in collector current (in μA) is -

21. Ans. (20,000)

Using $\beta = \frac{I_c}{I_b}$

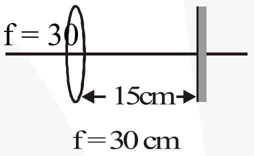
we get $\Rightarrow I_c = I_b \times \beta = 250 \times 80\ \mu A = 20{,}000\ \mu A$

Q22. The optical system consist of a thin convex lens of focal length 30 cm and a plane mirror 15 cm behind the lens. An object is placed 15 cm in front of lens. The distance of final image from lens is (in cm).

22. Ans. (60)

For lens

$u = -15$



$$\frac{1}{V} - \frac{1}{60} = -\frac{1}{30}$$

$$\frac{1}{V} = \frac{1}{60} - \frac{1}{30} = \frac{1-2}{60}$$

$V = -60$ cm

So, final image at 60 left of lens.

Q23. A beam of light consists of two wavelengths, 6300Å and 5600Å. This beam of light is used to obtain an interference pattern in YDSE. If 4^{th} bright fringe of 6300Å coincides with the n^{th} dark fringe of 5600Å from the central line, then find the value of n.

23. (5)

23. Fringe width $\Delta w_1 = \frac{(6300\text{Å})D}{d}$ and for $5600\text{Å}; \Delta_{W2} = \frac{(5600\text{Å})D}{d}$

So 4^{th} bright fringe position of 6300Å is at $4 \times \frac{(6300\text{Å})D}{d}$.

So for nth dark fringe of 5600Å is at $\left(n - \frac{1}{2}\right)\frac{5600 \times D}{d}$

$$\Rightarrow 4 \times \frac{6300 \times D}{d} = \left(n - \frac{1}{2}\right) \times \frac{5600D}{d}$$

$$\Rightarrow \frac{4 \times 63}{56} = n - \frac{1}{2}$$

$\Rightarrow n-\frac{1}{2}=\frac{9}{2}$

$\Rightarrow n=5$

Q24. A metallic rod of cross-sectional area 20 cm^2, with the lateral surface insulated to prevent heat loss, has one end immersed in boiling water and the other in ice water mixture. The heat conducted through the rod melts the ice at the rate of 1 gm for every 84 sec. The thermal conductivity of the rod is 160 $Wm^{-1}K^{-1}$. Latent heat of ice = 80 cal/gm, 1 cal = 4.2 joule. What is the length (in m) of the rod?

(1) 4 (2) 8 (3) 12 (4) 16

24. (2)

24. $\frac{dx}{dt}=\frac{KA(T_1-T_2)}{\ell}$

$$\frac{1\times10^{-3}\times80\times4200}{84}=\frac{160\times20\times10^{-4}\times100}{\ell}$$

l = 8 m

Q25. The efficiency of a Carnot's engine at a particular source and sink temperature is $\frac{1}{2}$. When the sink temperature is reduced by 100°C, the engine efficiency, becomes $\frac{2}{3}$. Find the source temperature (in K)

25. **(600)**

25. $\eta=1-\frac{T_2}{T_1}=\frac{1}{2}$

$1-\frac{T_2-100}{T_1}=\frac{2}{3}$

On solving,

T_1 = 600K

26. In a common emitter transistor amplifier, signal voltage across the collector resistance of $2k\Omega$ is 2 volt. What should be the value of resistance R_B (in $k\Omega$) in series with V_{BB} voltage supply of 2 volt in the input circuit, if the D.C. base current is 10 times the signal current in R_B? (Given: base emitter voltage $V_{BE}=0.6\,V$, current amplification factor $\beta_{ac}\approx\beta_{dc}=100$)

26. 14

Sol. $(i_c)_{AC} = \frac{(V_{out})_{AC}}{R_C} = \frac{2\text{volt}}{2k\Omega} = 1\text{mA}$

$$(i_B)_{AC} = \frac{(i_c)_{AC}}{\beta_{ac}} = \frac{1\text{mA}}{100} = 0.01\text{mA}$$

$$(i_B)_{DC} = 10(i_B)_{AC} = 0.1\text{mA} = 10^{-4}\text{ A}$$

$$R_B = \frac{V}{i_B} = \frac{2-0.6}{10^{-4}} = 14k\Omega$$

27. The largest magnitude the electric field on the axis of a uniformly charged ring of radius 3 m is at a distance h from its centre. What is the value of h ?

$\left(\text{Take } \frac{1}{\sqrt{2}} = 0.7\right)$

27. 2.1

Sol. $E_{axial} = \frac{Qh}{4\pi\varepsilon_0 \left(h^2 + a^2\right)^{\frac{3}{2}}}$

where,
a = radius of the ring,
h = distance of point from the centre.or the maximum value,

$$\frac{dE}{dh} = 0$$

$$\therefore \frac{d}{dh}\left[\frac{Qx}{4\pi\varepsilon_0 \left(h^2 + a^2\right)^{\frac{3}{2}}}\right] = 0$$

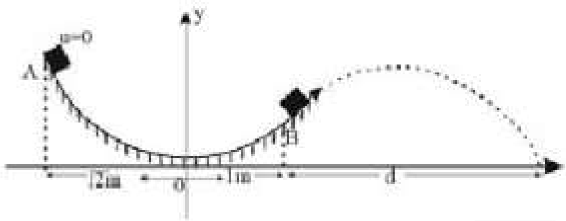
$$\therefore \frac{\left(h^2 + a^2\right)^{\frac{3}{2}} - h\left(\frac{3}{2}\right)\left(h^2 + a^2\right)^{\frac{1}{2}} \cdot (2h)}{\left(h^2 + a^2\right)^3} = 0$$

$$\therefore \left(h^2 + a^2\right) - 3h^2 = 0$$

$$\therefore h = \pm\frac{a}{\sqrt{2}}$$

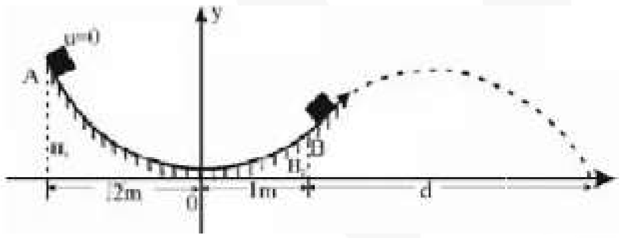
$= \frac{3}{\sqrt{2}} m \quad ...(\because a = 3m)$ $= 3 \times 0.7 = 2.1 m$

28. A small body is released from point A of smooth parabolic path $y = x^2$, where y is vertical axis and x is horizontal axis at ground, as shown. The body leaves the surface from point B . If g = 10 ms^{-2} then what is the value of d (in m) ?



28. 2

Sol.



$H_1 = (\sqrt{2})^2 = 2m$

$H_2 = (1)^2 = 1m$

$\therefore v = \sqrt{2 \times g \times (H_2 - H_1)}$

$v = \sqrt{20 ms^{-1}}$

$\tan\theta = \left.\frac{dy}{dx}\right|_{x-1} = 2x|_{x-1} = 2$

$-H_2 = d\tan\theta - \frac{gd^2}{2u^2\cos^2\theta}$

$-1 = 2d - \frac{10d^2}{2(\sqrt{20})^2(1/\sqrt{5})^2}$

$\frac{5}{4}d^2 - 2d - 1 = 0$

On solving we get $d = 2m$

29. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars

is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is $330\,ms^{-1}$

29. 7

Sol. Let v_1 and v_2 be the velocities of sources, then apparent frequencies are

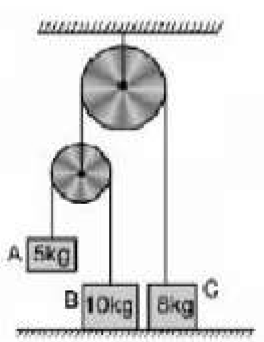
$$f_1 = \left(\frac{c+v_1}{c-v_1}\right)f_0 \text{ and } f_2 = \left(\frac{c+v_2}{c-v_2}\right)f_0$$

Then difference between frequencies is,

$$\Delta f = \frac{2\Delta v}{C} f_0 \qquad \because c >> V_1 \ \& c > V_2$$

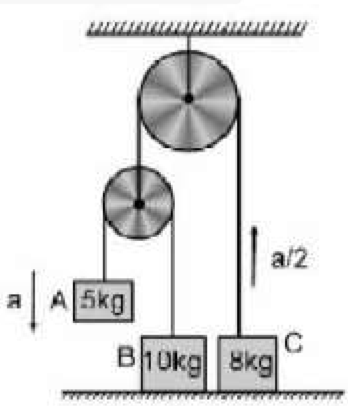
$\therefore \Delta v = 7\text{kmph}$

30. In the following arrangement, the system is initially held at rest by an external agent. At t =0 , the 5 kg block is released. If the acceleration of block C is $\frac{x}{10}ms^{-2}$, then find the value of x . Assume that the pulleys and strings are massless and smooth. $\left[g = 9.8\,ms^{-2}\right]$



30. 7

Sol. Block B will not move



$5g - T = 5a$ .....(i)

$2T - 8g = 8\frac{a}{2}$ ...(ii)

$2g = 14a$

$a = \frac{g}{7}$

$$\therefore \frac{a}{2} = \frac{g}{14} = \frac{9.8}{14} = \frac{7}{10} ms^{-2} = 0.7$$

$\therefore x = 7$

CHEMISTRY

Section-I(SC)

Q1. Half life of a reaction becomes half when initial concentration of reactants are made double. The order of reaction will be

(A) 1 (B) 2 (C) 0 (D) 3

Ans. **[B]**

Sol. $$\frac{t_1}{t_2} = \left(\frac{a_2}{a_1}\right)^{n-1}$$

$2 = (2)^{n-1}$

$n - 1 = 1 \Rightarrow n = 2$

Q2. The correct order of electron affinity is-

(A) F > Cl > Br (B) F > Cl < Br

(C) F < Cl > Br (D) F < Cl < Br

Ans. **[C]**

Sol. F has lesser electron affinity than that of Cl because of its compact electronic configuration.

Q3. The hybridisation of atomic orbitals of nitrogen in NO_2^+, NO_3^- and NH_4^+ are -

(A) sp, sp^3 and sp^2 respectively (B) sp, sp^2 and sp^3 respectively

(C) sp^2, sp and sp^3 respectively (D) sp^2, sp^3 and sp respectively

Ans. **[B]**

Q4. Which out of the following is potash alum -

(A) K_2SO_4. $Al_2(SO_4)_3$. $24H_2O$ (B) K_2SO_4. $Cr_2(SO_4)_3$. $24H_2O$

(C) K_2SO_4. $Fe_2(SO_4)_3$. $24H_2O$ (D) $(NH_4)_2SO_4$. $FeSO_4.6H_2O$.

Ans. **[A]**

Q5. Which statement is incorrect –

(A) $Ni(CO)_4$ – Tetrahedral, paramagnetic

(B) $[Ni(CN_4)]^{-2}$ – Square planar, diamagnetic

(C) $Ni(CO)_4$ – Tetrahedral, diamagnetic

(D) $[NiCl_4]^{-2}$ – Tetrahedral, paramagnetic

Ans. **[A]**

Sol. $Ni(CO)_4$ = sp^3, Tetrahedral, Paramagnetic

Q6. Which of the following statements is incorrect ?

(A) On heating potassium dichromate , the gas evolved is oxygen

(B) Compounds of La^{+3} are diamagnetic

(C) Nd^{+3} has greater tendency to form complex compounds than Eu^{+3}

(D) Sm^{+2} acts as reducing agent

Ans. **[C]**

Sol. Nd^{+3} has more zeff.

Eu^{+3} has smaller size than Nd^{+3}. Hence, Eu^{+3} has greater tendency to form complex than Nd^{+3}.

Q7. Which anion is smallest

(A) Cl^- (B) O^{-2} (C) F^- (D) I^-

Ans. (C)

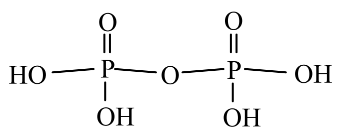
Sol. F^- is smallest anion.

Q8. The number of P–O–P and P – O – H bonds present respectively in pyrophosphoric acid molecule are -

(A) 1, 2 (B) 2, 2 (C) 1, 4 (D) 1, 8

Ans. **[C]**

Sol. Pyrophosphoric acid is $H_4P_2O_7$, The structure is



Thus it contains 1 P – O – P and 4 P – O – H bonds.

Q9. In following acid, the correct increasing order of acidic strength acid

HCOOH, CH_3COOH, C_6H_5COOH

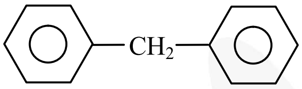
I II III

(A) I < II < III (B) II < I < III

(C) III < II < I (D) II < III < I

Ans. **[D]**

Q10. The molecular formula of diphenylmethane



is $C_{13}H_{12}$

How many structural isomers are possible when one of the hydrogen atom is replaced by achlorine atom ?

(A) 6 (B) 4 (C) 8 (D) 7

Ans. **[B]**

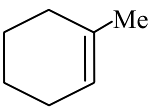
Q11. $(CH_3)_3COH + CH_3MgBr \rightarrow$ Hydrocarbon (A); (A) is -

(A) $(CH_3)_3\ CCH_3$ (B) $(CH_3)_3CH$

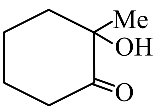
(C) CH_4 (D) none of these

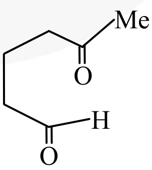
Ans. **[C]**

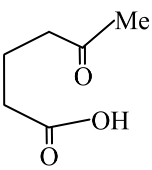
Sol. $ROH + CH_3MgBr \longrightarrow CH_4 + R-O^{-}\overset{+}{M}gBr$

Q12. 

$\xrightarrow{OsO_4} \xrightarrow{H_2O} [X] \xrightarrow[AcOH]{CrO_3} [Y] \xrightarrow[KMnO_4]{NaIO_4} [Z]$ [Z] is -

(A) 

(B) 

(C) 

(D) None of these

Ans. **[C]**

Q13. An alkyl halide react with reagent (x) to give an alkanenitrile as a major product. The reagent (x) is-

(A) AgCN (B) KCN (C) HCN (D) All of these

Ans. **[B]**

Q14. Phenol on heating with $NaNO_2$ and a few drops of conc. H_2SO_4 gives –

(A) *p*-Nitrophenol (B) *p*-Nitrosophenol

(C) *o*-Nitrophenol (D) *m*-Nitrosophenol

Ans. **[B]**

Q15. When chloroform reacts with acetone, the product is-

(A) Ethylidene dichloride (B) Mesitylene

(C) Chloretone (D) chloral

Ans. **[C]**

Q16. Which of the following may be prepared by Gabriel phthalimide synthesis ?

(A) Aliphatic amines (B) Aromatic amines

(C) Aliphatic amides (D) Aromatic amides

Ans. **[A]**

Q17. A gaseous hydrocarbon gives upon combustion 0.72 gm water and 3.08 gm of CO_2. The empirical formula of the hydrocarbon is :

(A) C_2H_4 (B) C_6H_5 (C) C_3H_4 (D) C_7H_8

Ans. (D)

Sol. Moles of $CO_2 = \frac{3.08}{44} = 0.07$

$= $ moles of carbon

Moles of $H_2O = \frac{0.72}{18} = 0.04$

Moles of Hydrogen $= 2 \times moles\ of\ H_2O$

$= 2 \times 0.04 = 0.08$

Moles Ratio of carbon to hydrogen = 0.07 : 0.08

= 7 : 8

So, hydrocarbon will be C_7H_8

Q18. What happens to freezing point of benzene when small amount of napthalene is added to it

(A) increases (B) No change

(C) decreases (D) first decreases then increases

Ans. (C)

Sol. Depression in freezing point occurs

Q19. The ground state energy of hydrogen atom is –13.6 eV, the energy of second excited state of Li^{2+} ion in eV is :

(A) –3.4 (B) –13.6 (C) –1.51 (D) –54.4

Ans. (B)

Sol. $E = -13.6\frac{Z^2}{n^2}eV$

$= -13.6 \times \frac{3^2}{3^2}eV$

$= -13.6 \text{ eV}$

Q20. $\Delta_r H = \Delta_r U$ is valid for

(A) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ (B) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

(C) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$. (D) $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

Ans. (C)

Sol: $\Delta H = \Delta U + \Delta n_g RT$

$\Delta_r H = \Delta U + (\Delta n_g)RT$

when $\Delta n_g = 0 \Rightarrow \Delta_r H = \Delta_r U$

Section-II(NV)

Q21. 80 Cal heat is needed to raise the temperature of 4 mol ideal gas at $5^\circ C$ at constant volume. Heat needed to raise temperature of 8 mol of same ideal gas by 5K at constant pressure is _____ cal?

Ans. (240)

Sol. $q_v = nC_{V.m}\Delta T$

$80 = 4 \times C_{V,m} \times 5$

$C_{V,m} = 4cal$

$C_{p,m} = C_{V,m.} + R = 4 + 2 = 6 \text{ Cal}$

$q_p = C_{P.m.}\Delta T$

$= 8 \times 6 \times 5 = 240 \text{ Cal}$

Q22. The enthalpy of formation of H_2O (ℓ) is – 290 kJ/mol and enthalpy of neutralisation of strong acid with strong alkali is – 56 kJ/equiv. The magnitude of enthalpy of fromation of OH^- (aq) is __________KJ/mol ? Given $\Delta_f H$ (H^+, aq) = 0

Ans. (234)

Sol. $H^+(aq) + OH^-(aq) \rightarrow H_2O_{(\ell)}$

$$\Delta_r H = -56 KJ/eq$$

$$\Delta_r H = \Delta_f H(H_2O, \ell) - \Delta_f H(H^+, aq) - \Delta_f H(OH^-, aq)$$

$$-56 = -290 - 0 - \Delta_f H(OH^-, aq)$$

$$\Delta_f H(OH^-, aq) = -290 + 56$$

$$= -234 \text{ KJ / mol}$$

Q23. A compound MX_2 has observed and normal molar masses 65.6 and 164 respectively. The apparent percentage ionization of MX_2 is __________% ?

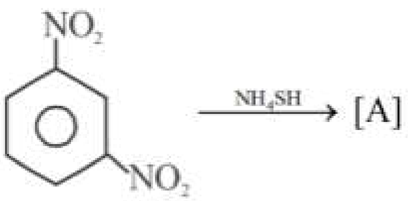
Ans. **(75)**

Sol.

$$i = \frac{\text{Theo. Mw.}}{\text{exp. Mw}} = \frac{164}{65.6} = 2.5$$

$$\alpha = \frac{1-i}{n-1} = \frac{2.5-1}{3-1} = 0.75$$

$$\alpha = 75\ \%$$

Q24. (i) 

(ii) $CH_2 = CH - CH_2 - Cl \xrightarrow{NaN_3} [B] \xrightarrow{H_2/Pt} [C]$

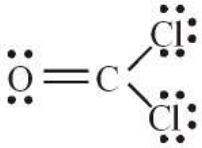
$[B] \xrightarrow{LiAlH_4} [D]$

Find the sum of total number of nitrogen atoms, which are present in products A, B, C and D

Ans. (7)

Q25. Number of lone pairs(s) in $COCl_2$ molecule is :

Ans. (8)

Sol. 

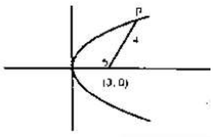
MATH

Section-I(SC)

Q1. The points on the parabola $y^2 = 12x$ whose focal distance is 4, are

(A) $(1, \pm 2\sqrt{3})$ (B) $(2, \pm 2\sqrt{3})$ (C) $(3, \pm 2\sqrt{3})$ (D) $(3, \pm 6)$

Ans. (A)

Sol. 

Let the point P is $\left(3t^2, 6t\right)$ and $PS = 3 + 3t^2 = 4$

$t^2 = 1/3 \Rightarrow t = \pm \frac{1}{\sqrt{3}}$

$\therefore$ Points are $(1, 2\sqrt{3})$ & $(1, -2\sqrt{3})$

Q2. Matrix $A_\lambda = \begin{bmatrix} \lambda & \lambda - 1 \\ \lambda - 1 & \lambda \end{bmatrix}, \lambda \in N$, then value of $|A_1| + |A_2| + \ldots + |A_{300}|$ is

(A) $(299)^2$ (B) $(300)^2$ (C) $(301)^2$ (D) 0

Ans. (B)

Sol. $|A_\lambda| = \lambda^2 - (\lambda - 1)^2 = 2\lambda - 1$

$\therefore E = 1 + 3 + 5 + \ldots 300$ terms (A.P.)

$= \frac{300}{2}[2.1 + 299.2] = (300)^2$

Q3. If $P = \frac{\sin 300^\circ \cdot \tan 330^\circ \cdot \sec 420^\circ}{\tan 135^\circ \cdot \sin 210^\circ \cdot \sec 315^\circ}$ & $Q = \frac{\sec 480^\circ \cdot \operatorname{cosec} 570^\circ \cdot \tan 330^\circ}{\sin 600^\circ \cdot \cos 660^\circ \cdot \cot 405^\circ}$ then P & Q are respectively:

(A) 2,16 (B) $\sqrt{2}, \frac{16}{3}$ (C) $-2, \frac{3}{16}$ (D) none of these

Ans. (B)

Sol. $P = \dfrac{\sin 300 \tan 330 \cdot \sec 420}{\tan 135 \cdot \sin 210 \cdot \sec 315} = \dfrac{\sin\left(2\pi - \frac{\pi}{3}\right) \cdot \tan\left(2\pi - \frac{\pi}{6}\right) \sec\left(2\pi + \frac{\pi}{3}\right)}{\tan\left(\pi - \frac{\pi}{4}\right) \sin\left(\pi + \frac{\pi}{6}\right) \sec\left(2\pi - \frac{\pi}{4}\right)}$

$$= \frac{\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)(2)}{(-1)\left(-\frac{1}{2}\right)\sqrt{2}} = \sqrt{2}$$

$$Q = \frac{\sec 480 \cdot \operatorname{cosec} 570 \cdot \tan 330}{\sin 600 \cos 660 \cot 405} = \frac{\sec\left(2\pi + \frac{2\pi}{3}\right) \cdot \operatorname{cosec}\left(3\pi + \frac{\pi}{6}\right) \tan\left(2\pi - \frac{\pi}{6}\right)}{\sin\left(3\pi + \frac{\pi}{3}\right) \cdot \cos\left(4\pi - \frac{\pi}{3}\right) \cot\left(2\pi + \frac{\pi}{4}\right)}$$

$$= \frac{(-2)(-2)\left(-\frac{1}{\sqrt{3}}\right)}{\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)(1)} = \frac{16}{3}$$

Q4. Equation of line inclined at an angle of 45° with positive x-axis and dividing the line joining the points (3, –1) and (8, 9) in the ratio 2 : 3 internally, is

(A) $x - y - 2 = 0$ (B) $3x - 3y + 1 = 0$

(C) $\sqrt{3}x - \sqrt{3}y + 2 = 0$ (D) None of these

Ans. (A)

Sol. Point M is $\left[\dfrac{2\times 8 + 3\times 3}{2+3}, \dfrac{2\times 9 + 3(-1)}{2+3}\right] \equiv (5, 3)$

$y - 3 \equiv 1(x - 5)$

$x - y - 2 = 0$

Q5. Find the sum of the series $31^3 + 32^3 + \ldots\ldots\ldots\ldots\ldots\ldots 50^3$

(A) 1509400 (B) 1509600 (C) 1409400 (D) 1409600

Ans. (C)

Sol. $\left(1^3 + \ldots\ldots 50^3\right) - \left(1^3 + 2^3 + \ldots 30^3\right)$

$$\left(\frac{50\times 51}{2}\right)^2 - \left(\frac{30\times 31}{2}\right)^2 = \frac{100}{4}\left\{(5\times 51)^2 - (3\times 31)^2\right\}$$

$$= 25\left(255^2 - 93^2\right) = 25\times(255+93)(255-93) = 25\times 348\times 162 = 14,09,400$$

Q6. If letters of the word "PARKAR" are written down in all possible manner as they are in a dictionary, then the rank of the word 'PARKAR' is

(A) 98 (B) 99 (C) 100 (D) 101

Ans. (B)

Sol. AA KP RR

Word begin	A	$\frac{5!}{2}$
Word begin	K	$\frac{5!}{2\times 2}$
Word begin	PAA	$=\frac{3!}{2!}$
	PAK	$=\frac{3!}{2!}$
	PARA	$=2!$
	PARKAR	$=1$

Rank 60 + 30 + 3 + 3 + 2 + 1 = 99

Q7. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of A×B having 3 or more elements is

(A) 256 (B) 220 (C) 219 (D) 211

Ans. (C)

Sol. $n(A)=2$

$n(B)=4$

$n(A\times B)=8$

$^8C_3+{}^8C_4+\ldots\ldots+{}^8C_8=2^8-{}^8C_0-{}^8C_1-{}^8C_2$

$256-1-8-28=219$

Q8. The sum of intercepts made by the circle $x^2+y^2-5x-13y-14=0$ on the x-axis and y-axis is

(A) 12 (B) 24 (C) 36 (D) 48

Ans. (B)

Sol. $x-\text{intercept}=2\sqrt{g^2-c}=2\sqrt{\frac{25}{4}+14}=\sqrt{25+56}=9$

$y-$ intercept $=2\sqrt{f^2-c}=2\sqrt{\frac{169}{4}+14}=\sqrt{169+56}=15$

Q9. $\int \frac{\operatorname{cosec}^2 x-2017}{\cos^{2017} x}dx$ is equal to

(A) $-\frac{\operatorname{cosec} x}{(\cos x)^{2016}}+C$

(B) $\frac{\cot x}{(\cos x)^{2017}}+C$

(C) $-\frac{\cot x}{(\cos x)^{2016}}$

(D) $\frac{\tan x}{(\cos x)^{2017}}+C$

Ans. (A)

Sol. $I=\int (\cos x)^{-2017}\cdot \operatorname{cosec}^2 x dx-2017\int (\cos x)^{-2017}dx$

$=(-\cot x)\cdot(\cos x)^{-2017}-\int(-2017)\cdot(\cos x)^{-2018}\cdot(-\sin x)(-\cot x)dx-2017\int(\cos x)^{-2017}dx$

$I=\frac{-\cot x}{(\cos x)^{2017}}+2017\int(\cos x)^{-2017}dx-\int(2017)(\cos x)^{-2017}dx$

Q10. Function $f(x)=e^{2x}-(a+1)e^x+2x$ is monotonically increasing for all $x\in R$, if a belongs to

(A) (3,4) (B) $(3,\infty)$ (C) $(-\infty,3]$ (D) $(2,\infty)$

Ans. (C)

Sol. $f(x)=e^{2x}-(a+1)e^x+2x$

$f'(x)=2e^{2x}-(a+1)e^x+2$

Now, $2e^{2x}-(a+1)e^x+2\geq 0$ for all $x\in R$

i.e. $2\left(e^x+\frac{1}{e^x}\right)-(a+1)\geq 0$ for all $x\in R$

i.e. $4-(a+1)\geq 0$ i.e. $a\leq 3$

Q11. If α and β are the roots of equation $x^2 - 3x + 1 = 0$ and $a_n = \alpha^n + \beta^n, n \in N$ then value of

$4\left(\frac{a_7 + a_5}{a_6}\right)$ is

(A) 8 (B) 3 (C) 12 (D) 4

Ans. (C)

Sol. $\left(\alpha^{n-1} + \beta^{n-1}\right)(\alpha + \beta)$

$= \alpha^n + \beta^n + \alpha\beta\left(\alpha^{n-2} + \beta^{n-2}\right)$

$a_{n-1}\left(-\frac{b}{a}\right) = a_n + \frac{c}{a}a_{n-2}$

$aa_n + ba_{n-1} + ca_{n-2} = 0$

$a_n - 3a_{n-1} + a_{n-2} = 0$

$\frac{a_n + a_{n-2}}{a_{n-1}} = 3$

put n = 7.

Q12. Direction cosines ℓ, m, n of a line which are connected by the relations $\ell + m + n = 0, 2mn + 2m\ell - n\ell = 0$ may be

(A) $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$

(B) $\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

(C) $\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$

(D) $\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

Ans. (A)

Sol. $\ell + m + n = 0, 2mn + 2m\ell - n\ell = 0$

$\Rightarrow 2m(-\ell - m) + 2m\ell - (-\ell - m)\ell = 0$ $\Rightarrow -2m^2 + \ell^2 + m\ell = 0$

$\Rightarrow \ell^2 + m\ell - 2m^2 = 0$ $\Rightarrow \ell^2 + 2m\ell - m\ell - 2m^2 = 0$

$\Rightarrow \ell(\ell + 2m) - m(\ell + 2m) = 0$ $\Rightarrow (\ell + 2m)(\ell - m) = 0$

$\ell = m, \ell = -2m$

when $\ell = m, m + m + n = 0 \Rightarrow n = -2m$ direction ratio

$m, m, -2m \quad \Rightarrow 1, 1, -2$ Direction cosine $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \Rightarrow -2m, m, m$

$-2, 1, 1 \quad \Rightarrow$ Direction cosines $\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

Q13. The value of λ for which the vector $\vec{r} = (\lambda^2 - 9)\hat{i} + 2\hat{j} - (\lambda^2 - 16)\hat{k}$ makes acute angle with the positive direction of coordinate axis.

(A) $(-\infty, -3) \cup (3, \infty)$ (B) $(4, 4)$

(C) $(-4, -3) \cup (3, 4)$ (D) None of these

Ans. (C)

Sol. $\vec{r} \cdot \hat{i} > 0 \Rightarrow \lambda^2 - 9 > 0 \Rightarrow \lambda \in (-\infty, -3) \cup (3, \infty)$

and $\vec{r} \cdot \hat{k} > 0 \quad \Rightarrow \quad -(\lambda^2 - 16) > 0$

$\Rightarrow \quad \lambda \in (-4, 4) \Rightarrow \lambda \in (-4, -3) \cup (3, 4)$

Q14. [x] denotes the greatest integer less than or equal to x. If $f(x) = [x][\sin \pi x]$ in $(-1, 1)$, then $f(x)$ is:

(A) continuous at $x = 0$ (B) continuous in $(-1, 0)$

(C) differentiable in $(-1, 1)$ (D) none

Ans. (B)

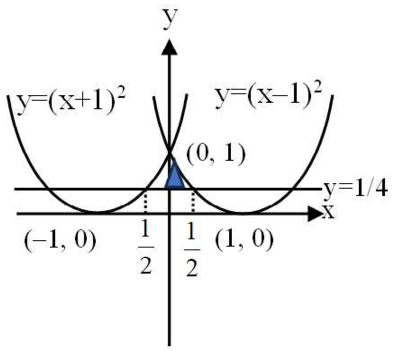
Sol. $f(x) = [x][\sin \pi x], x \in (-1, 1) = \begin{cases} 1; & x \in (-1, 0) \\ 0; & x \in [0, 1) \end{cases}$

$f(x)$ is continuous in $(-1, 0)$

Q15. The area bounded by the curves $y = (x-1)^2, y = (x+1)^2$ and $y = \frac{1}{4}$ is

(A) $\frac{1}{3}$ sq unit   (B) $\frac{2}{3}$ sq unit   (C) $\frac{1}{4}$ sq unit   (D) $\frac{1}{5}$ sq unit

Ans. (A)

Sol. 

Required area = $2\int_0^{1/2}(x-1)^2 dx - \frac{1}{4}$

$= \frac{2}{3}\left(\left(x-1^3\right)\right)_0^{1/2} - \frac{1}{4}$

$= \frac{2}{3}\times\frac{7}{8} - \frac{1}{4} = \frac{1}{3}$

Q16. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1,2,3,4,5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Ans. (A)

Sol. Required Probability

= 1 - Probability of D_4 showing a number different from D_1, D_2 and D_3

$= 1 - \frac{{}^6C_1 \cdot 5^3}{6^4} = \frac{91}{216}$

Q17. The outcome of each of 30 items was observed, 10 items gave an outcome $\frac{1}{2}-d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}+d$ each. If the variance of this outcome data is 4/3, then |d| equals

(A) $\frac{2}{3}$ (B) $\frac{\sqrt{5}}{2}$ (C) $\sqrt{2}$ (D) 2

Ans. (C)

Sol. Variance = $\frac{\sum x_i^2}{30} - \left(\frac{\sum x_i}{30}\right)^2$

$$\Rightarrow \frac{4}{3}=\frac{10\left(\frac{1}{2}-d\right)^2+10x\left(\frac{1}{2}\right)^2+10x\left(\frac{1}{2}+d\right)^2}{30}-\left(\frac{1}{2}\right)^2$$

$$\Rightarrow \frac{4}{3}+\frac{1}{4}=\frac{3\times\frac{1}{4}+2\cdot d^2}{3}$$

$$\Rightarrow 2d^2=4$$

$$\Rightarrow |d|=\sqrt{2}$$

Q18. The population p(t) at a time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt}=0.5p(t)-450$. If $p(0)=850$, then the time at which the population becomes zero is

(A) $\frac{1}{2}\ln 18$ (B) $\ln 18$ (C) $2\ln 18$ (D) ln

Ans. (C)

Sol. Let, $p=p(t)$

$$\frac{dp}{dt}=0.5p-450=\frac{p-900}{2}$$

$$\Rightarrow \int_{850}^{p}\frac{2}{p-900}dp=\int_{0}^{t}dt$$

$$\Rightarrow 2\left|\log|p-900|\right|_{850}^{p}=t$$

$$\Rightarrow 2\,|\log|p-900|-\log|-50||=t$$

$$\Rightarrow 2\log\left|\frac{p-900}{-50}\right|=t\ldots(1)$$

$$\Rightarrow \left|\frac{p-900}{-50}\right|=e^{t/2}$$

$$\Rightarrow p=900-50e^{t/2}$$

Let, when $t=T, p=0$ (using (1))

$$\Rightarrow T=2\ln 18$$

Q19. If $y=2+\sqrt{\sin x+2+\sqrt{\sin x+2+\sqrt{\sin x+\ldots\infty}}}$,then the value of $\frac{dy}{dx}$ at $x=0$ is

(A) 0 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Ans. (D)

Sol. Given equation can be rewritten as

$$y = 2 + \sqrt{\sin x + y}$$

$$\Rightarrow (y-2)^2 = \sin x + y$$

$$\Rightarrow y^2 - 4y + 4 = \sin x + y \ldots (i)$$

Putting x = 0 in the equation (i), we get,

$$y^2 - 4y + 4 = 0 + y \Rightarrow y^2 - 5y + 4 = 0$$

$$\Rightarrow (y-1)(y-4) = 0$$

$$y = 1 \text{ or } y = 4$$

$$\because y > 2 \Rightarrow y = 4$$

Now differentiating equation (i) with respect to x, we get,

$$\frac{dy}{dx} = \frac{\cos x}{2y - 5}$$

Putting $x = 0, y = 4$

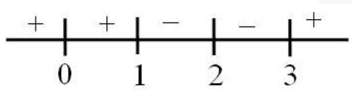
$$\frac{dy}{dx} = \frac{\cos(0)}{2(4) - 5} = \frac{1}{3}$$

Q20. The function $f(x) = \int_{-1}^{x} t\left(e^t - 1\right)(t-1)(t-2)^4(t-3)^5\,dt$ has a local minimum at x equals

(A) 0 (B) 1 (C) 2 (D) 3

Ans. (D)

Sol. $f^1(x) = x(e^x - 1)(x-1)(x-2)^4(x-3)^5$



$\therefore f(x)$ has local minima at x = 3

SECTION-II(NT)

Q21. Let $f(x) = x^{105} + x^{53} + x^{27} + x^{13} + x^3 + 3x + 1$. If g(x) is inverse of function f(x), then the value of $3g'(1)$ is

Ans. 1

Sol. Since $g(x)$ is inverse of $f(x)$

$g(f(x)) = x$

differentiating w.r.t.

$g'(f(x)) \cdot f'(x) = 1$

$x = 0, f(0) = 1$

$\therefore g'(f(0)) \cdot f'(0) = 1$

$g'(1) = \frac{1}{f'(0)} (\because f(0) = 3) \quad \therefore g'(1) = \frac{1}{3}$

Q22. If $2^{2020} + 2021$ is divided by 9, then the remainder obtained is

Ans. 3

Sol. $2^{2020} = 2.2^{2019} = 2.\left(2^3\right)^{673} = 2(9-1)^{673}$

$= 2\left(9^{673} - {}^{673}C_1 \cdot 9^{672} + {}^{673}C_2 \cdot 9^{671} + {}^{673}C_{672}\, 9 - 1\right)$

= 2 (a multiple of 9 – 1)

$2^{2020} + 2021 = 9K + 2019$

9 K + 2019 when divided by 9 gives remainder 3

Q23. If f is a twice derivable function such that $f'(2010) = f(2010) = \frac{1}{2010}$ and $\int_0^{2010} f(x)dx = \frac{3}{2}$,

then $\int_0^{2010} x^2 f''(x)dx =$

Ans. (2011)

Sol. $\int_0^a x^2 f''(x)dx = a^2 f'(a) - \int_0^a 2xf'(x)dx$

$= a^2 f'(a) - 2\left[af(a) - \int_0^a f(x)dx\right] = a^2 f'(a) - 2af(a) + 2\int_0^a f(x)dx$

put a = 2010

$\int_0^{2010} x^2 f''(x)dx = 2010 - 2 + 3 = 2011$

Q24. If $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$, then the value of ab is equal to

Ans. 3.75

Sol. Using the expansion, we get,

$$\lim_{x\to0}\frac{x+ax\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-......\right)-b\left(x-\frac{x^3}{3!}+......\right)}{x^3}=1$$

$$\lim_{x\to0}\frac{(1+a-b)x+\left(\frac{-a}{2}+\frac{b}{6}\right)x^3+......}{x^3}=1$$

For limit to exist numerator & denominator must be of the same degree

$\therefore 1+a-b=0\ldots(1)$

Also, $\frac{-a}{2}+\frac{b}{6}=1\Rightarrow 3a-b+6=0\ldots$

By equations (1) & (2), we get

$a=-\frac{5}{2}$ and $b=-\frac{3}{2}$

$\Rightarrow ab=\frac{15}{4}=3.75$

Q25. If $|Z-2|=2|Z-1|$, then the value of $\frac{\text{Re}(Z)}{|Z|^2}$ is (where Z is a complex number and $\text{Re}(Z)$ represents the real part of Z)

Ans. 0.75

Sol. Let, $Z=x+iy$

$\Rightarrow |(x-2)+iy|=2|(x-1)+iy|$

$\Rightarrow (x-2)^2+y^2=4\left((x-1)^2+y^2\right)$

$\Rightarrow 3\left(x^2+y^2\right)-4x=0$

$\Rightarrow \frac{x}{x^2+y^2}=\frac{3}{4}$

$\Rightarrow \frac{\text{Re}(Z)}{|Z|^2}=0.75$