ESJMRT13PH056

Section-I(SC)

1. The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum errors in the measurement of force and length are respectively, 4% and 2%, then the maximum error in the measurement of pressure is

(A) 1 % (B) 2 % (C) 6 % (D) 8 %

1. (D)

1. $\therefore p = \frac{F}{A} = \frac{F}{l^2}$, so maximum error in pressure (p) is $\left(\frac{\Delta p}{p} \times 100\right)_{max} = \frac{\Delta F}{F} \times 100 + 2\frac{\Delta l}{l} \times 100$

$= 4\% + 2 \times 2\%$

$= 8\%$

2. Given that $\vec{P} + \vec{Q} + \vec{R} = \vec{0}$. Which of the following statement is true ?

(A) $|\vec{P}| + |\vec{Q}| = |\vec{R}|$ (B) $|\vec{P} + \vec{Q}| = |\vec{R}|$

(C) $|\vec{P}| - |\vec{Q}| = |\vec{R}|$ (D) $|\vec{P} - \vec{Q}| = |\vec{R}|$

2. (B)

2. $\vec{P} + \vec{Q} + \vec{R} = \vec{0} \Rightarrow \vec{P} + \vec{Q} = -\vec{R} \Rightarrow |\vec{P} + \vec{Q}| = |\vec{R}|$

3. A train moving with uniform speed passes a pole in 10 sec. and a bridge of length 1200 m in 130sec. Speed of the train is :

(A) 90 km/hr (B) 72 km/hr (C) 36 km/hr (D) 54 km/hr

3. (C)

3. Let the length of train = x

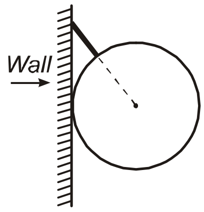
so x = vt $\Rightarrow$ x = 10v (Here v is speed of train) to cross a bridge train have to move (1200 + x)

so (1200 + x) = vt

(1200 + 10v) = v(130)

v = 10 m/s or 36 km/hr

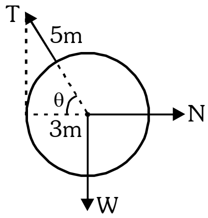
4. A uniform sphere of weight w and radius 3 m is being held by a string of length 2 m. attached to a frictionless wall as shown in the figure. The tension in the string will be:



(A) 5 w/4 (B) 15 w/4 (C) 15 w/16 (D) none of these

4. (A)

4. $T \sin \theta = W$



$T = \frac{5W}{4}$

5. A block takes time t to reach the bottom of an inclined plane of angle θ with the horizontal. If the plane is made rough, time taken now is 2t. The coefficient of friction of the rough surface is:

(A) $\frac{3}{4}\tan\theta$

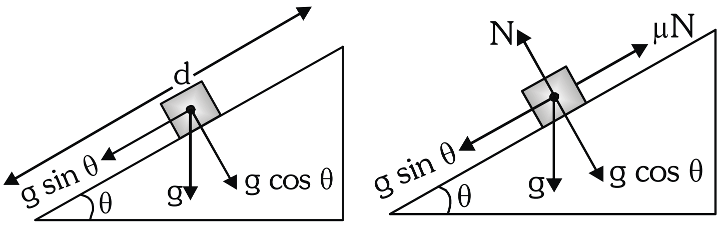
(B) $\frac{2}{3}\tan\theta$

(C) $\frac{1}{4}\tan\theta$

(D) $\frac{1}{2}\tan\theta$

5. (A)

5.



When the body moves on frictionless surface, then

$$d = \frac{1}{2} g \sin\theta\, t^2$$

When the body moves on rough inclined surface,

$$d = \frac{1}{2} g(\sin\theta - \mu\cos\theta)(2t)^2$$

$\therefore \ \frac{1}{2} g \sin\theta t^2 = \frac{1}{2} g(\sin\theta - \mu\cos\theta)(2t)^2$

or $\sin\theta = 4(\sin\theta - \mu \cos\theta)$

or $\frac{\sin\theta}{4} = \sin\theta - \mu\cos\theta$

or $\mu\cos\theta = \frac{3}{4}\sin\theta$

or $\mu = \frac{3}{4}\tan\theta$

6. The potential energy for a conservative system is given by : $U = ax^2 - bx$.Where a and b are positive constants. The law of the force governing the system is :

(A) F = constt. (B) $F = bx - 2a$

(C) $F = b - 2ax$ (D) $F = 2ax$

6. (C)

6. In a conservative field, $F = -\frac{dU}{dr}$

$\therefore \ F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$

7. A projectile is fired with velocity u making an angle θ with the horizontal. What is the angular momentum of the projectile at the highest point about the starting point (Given the mass of the projectile is m):-

(A) $\frac{m\cos\theta}{2g}$ (B) $\frac{mu^2\sin^2\theta\cos\theta}{2g}$

(C) $\frac{mu^3\cos^2\theta}{2g}$ (D) $\frac{mu^3\sin^2\theta\cos\theta}{2g}$

7. (D)

7. h = maximum height = $\frac{u^2\sin^2\theta}{2g}$

Linear momentum = mu cos θ at the highest point.

Angular momentum of the projectile at the highest point = linear momentum × perpendicular distance

$= mu\cos\theta \times h = mu\cos\theta \times \frac{u^2\sin^2\theta}{2g}$

$= \frac{mu^3\sin^2\theta\cos\theta}{2g}$

8. The two bodies of masses m_1 and m_2 $(m_1 > m_2)$ respectively are tied to the ends of a string which passes over a light frictionless pulley. The masses are initially at rest and released. The acceleration of the centre of mass is :

(A) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$ (B) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

(B) g (D) zero

8. (A)

8. The equation of motion of the centre of mass is,

$M\vec{a}_{CM} = \vec{F}_{ext.}$ And as there is no external force in horizontal direction, so the centre of mass of the system does not change along horizontal direction. For vertical motion of the centre of mass,

$$(a_{CM})_y = \frac{F_{ext.}}{M} = \frac{(m_1 + m_2)g - 2T}{(m_1 + m_2)} \quad \text{...(i)}$$

Further, $a_{CM} = \frac{m_a a_1 + m_2 a_2}{m_1 + m_2} = \frac{m_1 - m_2}{m_1 + m_2} a$ $[\because \vec{a}_1 = a \text{ and } \vec{a}_2 = -a]$

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g \qquad \left[\because \; a = \frac{m_1 - m_2}{m_1 + m_2} g\right]$$

However, the equations of motion of two blocks are

$m_2 g - T = m_2 a$

$T - m_1 g = m_1 a$

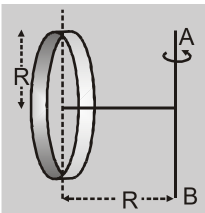
Eliminating a, we get;

$$T = \frac{2m_1 m_2}{m_1 + m_2} g \qquad \text{......(ii)}$$

Putting eqn. (ii) in eqn. (i), we get:

$$(a_{CM})_y = \frac{(m_1 + m_2)^2 - 4m_1 m_2}{(m_1 + m_2)^2} g = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

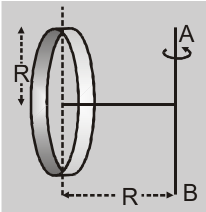
9. Find the moment of inertia of the ring about the axis AB .



(A) $\frac{1}{2} MR^2$ (B) $\frac{3}{2} MR^2$

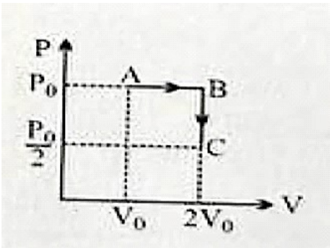
(C) $\frac{3}{4}MR^2$ (D) MR^2

9. (B)

9. 

$$I_{AB} = I_{dia} + MR^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

10. One mole of an ideal monoatomic gas is taken from point A to C along the path ABC. The initial temperature at A is T_0. For the process $A \rightarrow B \rightarrow C$ Change in internal energy of gas is



(A) 1 (B) 2 (C) 3 (D) 0

10. (D)

10. $\because T_A = T_C$

Hence $\Delta U = n\frac{f}{2}RdT = 0$

11. A Zener diode undergoes breakdown for an electric field of $10^6\ V\,m^{-1}$ at the depletion region of p-n junction. If the width of the depletion region is $2.5\ \mu m$, what should be the reverse-biased potential for the breakdown to occur ?

(A) 3.5 V (B) 2.5 V

(C) 1.5 V (D) 0.5 V

11. (B)

11. Reverse biased potential for the zener breakdown

$$V_r = Ed = 10^6 \times 2.5 \times 10^{-6} = 2.5 \text{ volt}$$

12. In a hypothetical hydrogen like atom, if transition of electron from n = 4 to n = 3 produces visible light, then the possible transition to obtain infrared radiation is

(A) n = 5 to n = 3 (B) n = 4 to n = 2

(C) n = 3 to n = 1 (D) n = 5 to n = 4

12. (D)

12. ΔE should be less than $E_4 - E_3$

$\Rightarrow n_1 = 5,\ \ n_2 = 4$

13. When Uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as:

$_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3x + Q$

where three particles named x are produced and energy Q is released. What is the name of the particle x?

(A) α-particle (B) electron

(C) neutron (D) neutrion

13. (C)

13. $_{92}U^{235} + _0n \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3\,_z^AX + Q$

$92 = 56 + 36 + Z$

$Z = 0$

$235 + 1 = 144 + 92 + 3A$

$A = 1$

$_z^AX = _o^1n$ = neutron

14. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecules at 367ºC ?

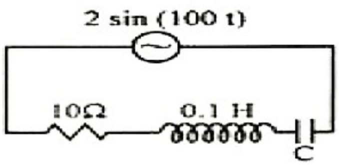
(A) 80 K (B) 40 K (C) 60 K (D) 100 K

14. (B)

14. $\frac{(V_{rms})_1}{(V_{rms})_2} = \sqrt{\frac{T_1M_2}{T_2M_1}} = 1 \quad \left[V_{rms} = \sqrt{\frac{3RT}{M_W}}\right]$

$T_1M_2 = T_2M_1 \Rightarrow T_2 = \frac{640 \times 2}{32} = 40K$

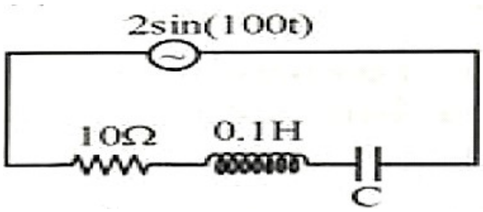
15. The power factor of the given circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(A) 400 μF (B) 300 μF

(C) 500 μ F (D) 200 μ F

15. (C)

15. 

$w = 100$

$L = 0.1\text{H}$

$X_L = wL = 100 \times 0.1$

$X_L = 10\Omega$

$R = 10\Omega$

$X_C = \frac{1}{wC} = \frac{1}{100C}$

Impedence of the circuit

$Z = \sqrt{R^2 (X_L - X_C)^2}$

$= \sqrt{10^2 + (10 - X_C)^2}$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$

$Z = \sqrt{2}R$

$Z^2 = 2R^2$

$10^2 + (10 - X_c)^2 = 2(10)^2$

$(10 - X_C)^2 = 100$

$10 - X_C = -10$

$X_c = 20$

$\frac{1}{100C} = 20 \Rightarrow C = \frac{1}{2000} F = 500\mu\text{F}$

16. Critical angle for light going from medium (1) to (2) is θ. The speed of light in medium (1) in

v, then speed in medium (2) is :-

(A) $v(1 - \cos\theta)$ (B) $v/\sin\theta$

(C) $v/\cos\theta$ (D) $v(1 - \sin\theta)$

16. (B)

16. $\boxed{\mu = \sin\theta}$

$$\therefore\ v_2 = \frac{v}{\sin\theta}$$

By snell's law

$$\mu_1 \sin\theta = \mu_2 \Rightarrow \sin\theta = \frac{\mu_2}{\mu_1} .....(i)$$

$$v \propto \frac{1}{\mu}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{v}{v_2} = \sin\theta \text{ [from (i)]}$$

$$\therefore\ v_2 = \frac{v}{\sin\theta}$$

17. Two plano-convex lenses of focal lengths 20 cm and 30 cm are placed together to form a double convex lens. The final focal length will be

(A) 12 cm (B) 60 cm

(C) 20 cm (D) 30 cm

17. (A)

17. Equivalent focal length

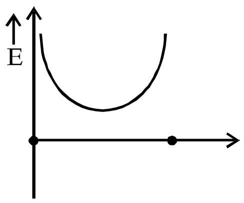
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{20} + \frac{1}{30}$$

$$\therefore F = \frac{20 \times 30}{20 + 30}$$

$$= \frac{600}{50} = 12\,\text{cm}$$

18. Two point charges a & b, whose magnitudes are same are positioned at a certain distance from each other with a at origin. Graph is drawn between electric field strength at points between a & b and distance x from a. E is taken positive if it is along the line joining from a to b. From the graph, it can be decided that :-



(A) a is positive, b is negative
(B) a and b both are positive
(C) a and b both are negative
(D) a is negative, b is positive

18. (A)

18. a & b can't be both +ve or both –ve otherwise field would have been zero at their mid point. b can't be positive even, otherwise the field would have been in –ve direction to the right of mid point.

19. A steel wire is rigidly fixed at both ends. Its cross sectional area is $10^{-6}\ \text{m}^2$. If the temperature of the wire is lowered by 20°C. Find the tension developed in the wire. [coefficient of linear expansion of steel $= 1.21 \times 10^{-5} /^{\circ}\text{C}$ and Young's modulus $= 2 \times 10^{11}\ \text{N/m}^2$].

(A) 48.4 N
(B) 24.2 N
(C) 20 N
(D) 0

19. (A)

19. $Y = \dfrac{\text{stress}}{\text{strain}}$

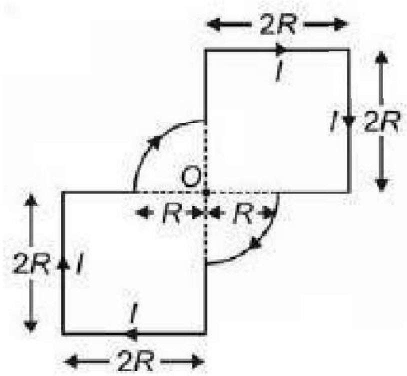
$$\frac{\Delta L}{L} = \frac{\text{stress}}{y} = \alpha \Delta T$$

$$T = \alpha A Y \Delta T$$

$$= \left(1.21 \times 10^{-5}\right)\left(10^{-6}\right)\left(2 \times 10^{11}\right)(20)$$

$= 48.4\,N = 48\,N$

20. A current-carrying loop is shown in the figure. The magnitude of the magnetic field produced at a point O is



(A) $\frac{\mu_0 I}{4R}\left(1+\frac{\sqrt{2}}{\pi}\right)$

(B) $\frac{\mu_0 I}{4R}\left(1-\frac{2}{\pi}\right)$

(C) $\frac{\mu_0 I}{2R}\left(1+\frac{\sqrt{2}}{\pi}\right)$

(D) $\frac{\mu_0 I}{4R}\left(1+\frac{2\sqrt{2}}{\pi}\right)$

20. (A)

20. $B_0 = \frac{\mu_0 I}{2R}\times\frac{1}{2} + 4\times\frac{\mu_0 I}{4\pi\times(2R)}\times\frac{1}{\sqrt{2}}$

$B_0 = \frac{\mu_0 I}{4R}\left(1+\frac{\sqrt{2}}{\pi}\right)$

Section-II(NV)

21. In a resonance tube experiment, to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in the pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound (in ms^{-1}) in the air at room temperature.

Ans. 336

Sol. $\frac{v}{4[L+0.6r]} = 480$

$\Rightarrow v = 480 \times 4 \times [L + 0.6r]$

$\Rightarrow v = 336\,m\,s^{-1}$

22. In a Wheatstone's network, $P = 2\Omega$, $Q = 2\Omega$, $R = 2\Omega$ and $S = 3\Omega$. Find the resistance $(\text{in}\Omega)$ with which S is to be shunted, in order that the bridge gets balanced.

Ans. 6

Sol. Let a resistance $r\,\Omega$ be shunted with resistance S, so that the bridge is balanced. Let S 'be the resultant resistance of S and r, then In balanced position

$$\frac{P}{Q} = \frac{R}{S'}$$

$$\frac{2}{2} = \frac{2}{S'}$$

$$\therefore S' = 2\Omega$$

Now,

$$\frac{1}{S'} = \frac{1}{S} + \frac{1}{r}$$

$$\frac{1}{r} = \frac{1}{S'} - \frac{1}{S} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6}$$

$$\frac{1}{r} = \frac{1}{6}$$

$$r = 6\Omega$$

Q23. A machine gun fires a bullet of mass 40 gm with a valocity $1200\,ms^{-1}$. The man holding it can exert a maximum force of 144 N on the gun. The maximum number of bullets that the man can fire per second is

Ans. 3

Sol. Suppose he can fire n bullets per second

$\therefore$ *Force* = Change in momentum per second

$$144 = n \times \left(\frac{40}{1000}\right) \times (1200)$$

$$n = \frac{144 \times 1000}{40 \times 1200}$$

$$n = 3$$

Q24. A galvanometer coil has a resistance 90Ω and fullscale deflection current 10 mA. A 910Ω resistance is connected in series with the galvanometer to make a voltmeter. If the least count of the voltmeter is 0.1 V, the number of divisions on its scale is

Ans, 100

Sol. $V_{max} = (I_{max})(R_g + R_s)$

$= \left(10 \times 10^{-3}\right)(1000) = 10$

$\text{L.C.} = 0.1\,\text{V}$

$N(\text{ L.C. }) = V_{max}$

$\Rightarrow N = \dfrac{10}{0.1} = 100$

Q25. An 8 kg metal block of dimensions $16\,\text{cm} \times 8\,\text{cm} \times 6\,\text{cm}$ is lying on a table with its face of largest area touching the table. If $g = 10\,\text{ms}^{-2}$, then the minimum amount of work done in making it stand with its length vertical is (in J) :

Ans. 4

Sol. When face of the largest area is touching the table, height of center of gravity above the table $h_1 = \dfrac{6}{2} = 3\,\text{cm}$ With its length vertical, height of Centre of gravity would become $h_2 = \dfrac{16}{2} = 8\,\text{cm}$

Minimum work required

$W = (\text{ P.E. })_2 - (\text{ P.E. })_1 = mg\left(h_2 - h_1\right)$

$W = (\text{ P.E. })_2 - (\text{ P.E. })_1 = mg\left(h_2 - h_1\right)$

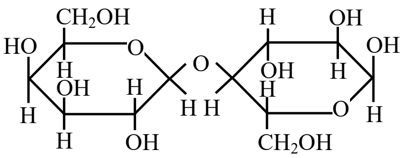
$W = \dfrac{8 \times 10(8-3)}{100} = 4\,\text{J}$

CHEMISTRY

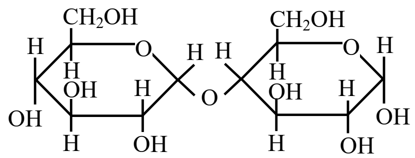
Section-I(SC)

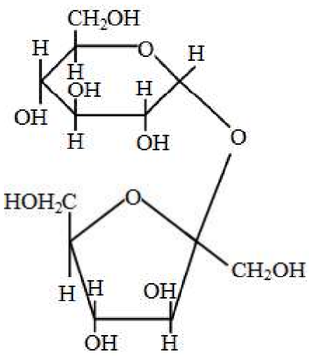
Q1. Which of the following is/are non reducing sugars -

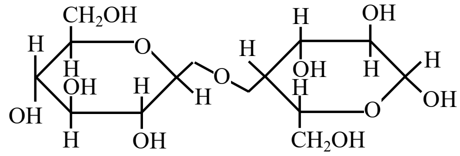
(i)



(ii)



(iii) 

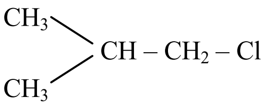
(iv) 

(A) (i) & (iv) (B) (i), (ii) and (iv) (C) (iii) (D) (iii) & (iv)

Ans. [C]

Sucrose with anomeric–OH groups in glycosidic linkage & thus non-Reducing

Q2. The IUPAC name of

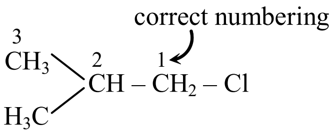


is -

(A) 2-Chloro-2-methylpropane (B) 2-Chloro-1-methylpropane

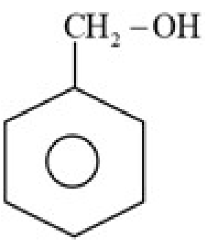
(C) 1-Chloro-2-methylpropane (D) 2-Chloro-2-methyl butane

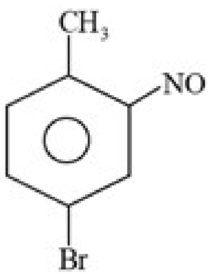
Ans.[C]

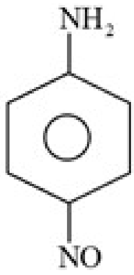


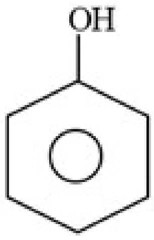
Q3. $PhCH_2COOH \xrightarrow[ii)KOBr]{i)NH_3/\Delta} X \xrightarrow{dil.HNO_2} Y$

Product (Y) is:

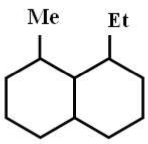
(A) 

(B) 

(C) 

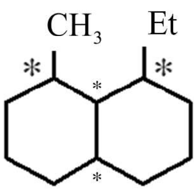
(D) 

Ans. **(A)**

Q4. Number of chiral centres in  is/are –

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **[D]**

Sol. 

Q5. The compound

$$CH_3 - CH_2 - CH = C\begin{matrix} CH_3 \\ CH_3 \end{matrix} \xrightarrow[\text{ii)}H_2O]{\text{i)}O_3} \text{product, here product is}$$

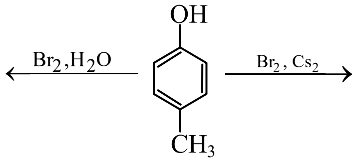
(A) CH_3COOH & $CH_3 - \overset{\overset{\displaystyle O}{||}}{C} - CH_3$

(B) $CH_3 - CH_2 - COOH$ & $CH_3 - \overset{\overset{\displaystyle O}{||}}{C} - CH_3$

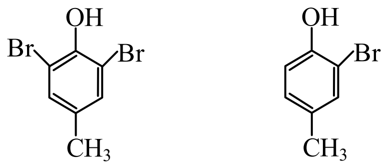
(C) $CH_3 - CH_2 - COOH$ only

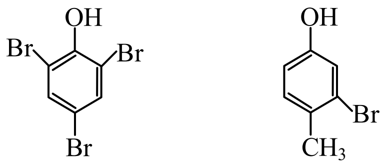
(D) $HCOOH$ & $CH_3 - - CH_3$

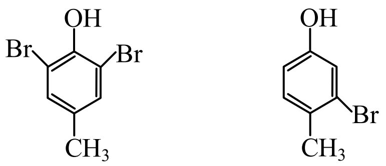
Ans. **[B]**

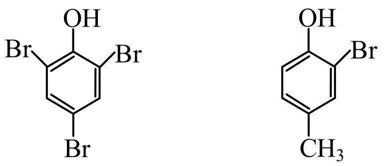
Q6. A  B Predominant

'A' and 'B' are respectively :

(A) 

(B) 

(C) 

(D) 

Ans. [A]

Q7. The major product in the given reaction.

$$CH_3CH_2CHClCH_3 \xrightarrow[\Delta]{alc.KOH}$$

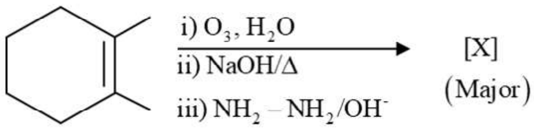
(A) $CH_3-CH=CH-CH_3$

(B) $CH_3-CH_2-CH=CH_2$

(C) $CH_3-\overset{\overset{\large CH_3}{|}}{C}=CH_2$

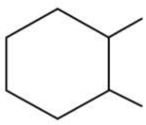
(D) $CH_3-C\equiv C-CH_3$

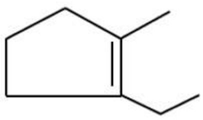
Ans. [A]

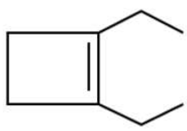
Q8. 

In given reaction product (x) is :

(A) 

(B) 

(C) 

(D) 

Ans. (C)

Q9. The equivalent mass of H_3PO_3 is?

(A) $\frac{M}{3}$ (B) $\frac{M}{2}$ (C) $\frac{M}{4}$ (D) $\frac{M}{1}$

Ans. (B)

Sol: H_3PO_3 is dibasic acid

Q10. Which of the following does not exist at normal condition.

(A) $HClO_4$ (B) $HBrO_4$ (C) HIO_4 (D) HFO_4

Ans. **[D]**

Sol. Fluorine cannot produce HFO_4 at normal condition.

Q11. Which of the set of species have same hybridisation state but different shapes:

(A) NO_2^+, NO_2, NO_2^- (B) ClO_4^-, SF_4, XeF_4

(C) NH_4^+, H_3O^+, OF_2 (D) SO_4^{-2}, PO_4^{-3}, ClO_4^-

Ans. **[C]**

Sol. No. of lone pairs are different and same hybridization sp^3.

Q12. Graphite is good conductor of current but diamond is non-conductor because :

(A) Diamond is hard and graphite is soft

(B) graphite and diamond have different atomic configuration

(C) Graphite is composed of positively charged carbon ions

(D) Graphite has hexagonal layer structure with mobile -electrons while diamond has continuous tetrahedral covalent structure with no free electrons

Ans. **[D]**

Q13. Which of the following bonds/forces is weakest?

(A) Covalent bond (B) Ionic bond

(C) Hydrogen bond (D) London force

Ans. ()

Q14. The oxidation state of Fe in its complex species $Na_2[Fe(CN)_5(NO^+)]$ is

(A) + 2 (B) + 3 (C) + 4 (D) + 5

Ans. **[B]**

Q15. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas is evolved which turns $K_2Cr_2O_7$ solution (acidified with dilute H_2SO_4):

(A) Black (B) Red (C) Green (D) Blue

Ans. (C)

Sol. $Cr_2O_7^{2-} + SO_3^{2-} \xrightarrow{H^+} \underset{\text{Green}}{Cr^{3+}} + SO_4^{2-}$

Q16. Which of the following hydrides of nitrogen family is the most basic ?

(A) NH_3 (B) PH_3 (C) AsH_3 (D) SbH_3

Ans. **[A]**

Sol. Because of the –I effect of F, NF_3 is least basic

Q17. pk_b for NH_4OH at certain temperature is 4.74. The pH of basic buffer containing equimolar concentration of NH_4OH and NH_4Cl will be:-

(A) 7.74 (B) 4.74 (C) 2.37 (D) 9.26

Ans. (D)

Sol. $pOH = pk_b \log \frac{[NH_4Cl]}{[NH_4OH]}$

$= 4.74 + \log 1 \; (\because [NH_4Cl] = [NH_4OH])$

$= 4.74$

pH = 14 - pOH = 14-4.74 = 9.26

Q18. The reduction potential of hydrogen electode $\left(P_{H_2} = 1\text{bar}, \left[H^+\right] = 0.1\text{M}\right)$ at 25° will be-

(A) 0 V (B) -0.059 V

(C) 0.118 V (D) 0.059 V

Ans. (B)

Sol. $2H^+ + 2e^- \rightarrow H_2(g)$

$$E_{H^+/H_2} = E^0_{H^+/H_2} - \frac{0.059}{2} \log \left\{ \frac{P_{H_2}}{\left[H^+\right]^2} \right\}$$

$$= 0 - \frac{0.059}{2} \log \frac{1}{(0.1)^2}$$

$$= -\frac{0.059}{2} \times 2 = -0.059\,V$$

Q19. The temperature of an ideal gas increases in an:

(A) adiabatic compression (B) adiabatic expansion

(C) isothermal expansion (D) isothermal compression

Ans. (A)

Sol. For adiabatic Process :- q = 0

$\Delta U = w$

For compression :- w > 0

$\Delta U > 0$

$\Rightarrow \Delta T > 0$ $\quad (\because \Delta U = nC_{v,m}\Delta T)$

$\Rightarrow T_f > T_i$

Q20. Bond dissociation energy of Cl_2 is $240\,kJ/mol$. The longest wavelength of photon that can break this bond would be $\left[N_A = 6\times10^{23}, h = 6.6\times10^{-34}\,J/s\right]$

(A) $4.95\times10^{-7}\,m$ (B) $9.9\times10^{-7}\,m$ (C) $4.95\times10^{-6}\,m$ (D) $9.9\times10^{-6}\,m$

Ans. (A)

Sol. $E = N_A \times \frac{hC}{\lambda}$

$$240\times10^3 = \frac{6\times10^{23}\times6\cdot6\times10^{-34}\times3\times10^8}{\lambda}$$

$\lambda = 4.95\times10^{-7}\,m$

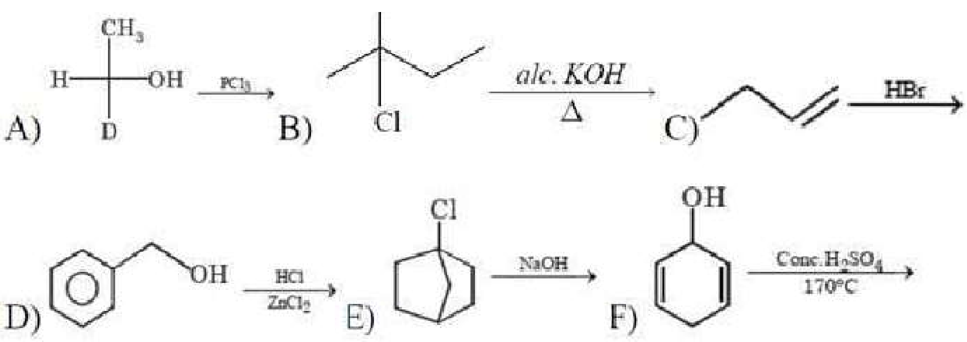
Section-II(NV)

Q21. The oxidation state exhibited by the transition metal which is present in Ziegler-Natta catalyst is

Ans. (4)

Sol. Ziegler-Natta catalyst is $TiCl_4 + Al(C_2H_5)_3$

Q22. Identify number of nucleophilic substitution reactions in the given reactions?



Ans. (2)

Sol. A, D

Use the concept explained below on each of the reaction to identify the nucleophilic substitution reaction:

In a nucleophilic substitution reaction, the leaving group (nucleophile) is replaced by an electron rich compound (nucleophile). The whole molecular entity of which the electrophile and the leaving group are part is Usually called the substrate. The nucleophile essentially attempts to replace the leaving group as the primary substituent in the reaction itself, as a part of another molecule.

Nuc: $+R-LG \rightarrow R-Nuc+LG$

The electron pair (:) from the nucleophile (Nuc) attacks the substrate (R-LG) forming a new bond, while the leaving group (LG) departs with an electron pair. The principal product in this case is R-Nucleophile. The nucleophile may be electrically neutral or negatively charged, where as the Substrate is typically neutral or positively charged. An example of nucleophilic substitution is the hydrolysis of an alkyl bromide, R-Br, under basic conditions, where the attacking nucleophile is theOH– and the leaving group is $Br-$

$R-Br+OH^{-} \rightarrow R-OH+Br^{-}$

Q23. Electrons jump from 3^{rd} excited state to ground state in H -atom sample. Find maximum number of lines in Lyman series

Ans. 3

Sol: No. of lines = 4 - 1 = 3

Q24. The following redox reaction occurs in basic medium :

$NO_3^- + Zn(s) \rightarrow Zn^{2+} + NH_4^+$

When the above reaction is balanced such that the stoichiometric coefficients are in smallest whole number ratio, then the difference of stoichiometric coefficient of Zn(s) and OH^- ion will be ?

Ans. (6)

Sol. Balanced reaction is :-

$4Zn + NO_3^- + 7H_2O \rightarrow 4Zn^{2+} + NH_4^+ + 10OH^-$

Q25. Equilibrium constant K_p for the reaction $CaCO_{3(s)} \rightleftharpoons CaO_{(s)} + CO_{2(g)}$ is 0.82 atm at $727^{\circ}C$, If 1 mole of $CaCO_3$ is placed in a closed container of 20 L and heated to this temperature, what amount of (in gm) $CaCO_3$ would dissociate at equilibrium? $R = 0.082 \frac{atm \times \ell}{mol \times k}$.

Ans. 20

Sol. $CaCO_3 \rightleftharpoons CaO + CO_2$

$K_p = p_{CO_2} = 0.82\,atm$

$0.82\,atm \times 20\,L = n_{CO_2} \times 0.082 \times 1000$

$\frac{200}{1000} = \frac{1}{5} = n_{CO_2}$

No. of moles CO_2 = no. of moles of $CaCO_3$ decomposed $= \frac{1}{5}$ mole

Amount of $CaCO_3$ decomposed

$= \frac{1}{5} \times 100 = 20\,g$

MATH

Section-I(SC)

Q1. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(A) 2550 (B) 1050 (C) 3050 (D) none of these

1. (C)

1. Sum of the integer divided by 2 = 2 + 4 +...........+ 98 + 100 $= \frac{50}{2}$[2.2 + (50 – 1)2] = 50[51] = 2550

Sum of the integer divided by 5 = 5 + 10 + ... 95 + 100 = $\frac{20}{2}$ [5 + 100] = 1050

Sum of the integer divided by 10 $\Rightarrow \frac{10}{2}[10 + 100] = 550$

Sum of the integers divided by 5 or $10 = 2550 + 1050 - 550 = 3050$

Q2. The solution of the differential equation $y\frac{dy}{dx} = x - 1$ satisfying $y(1) = 1$ is-

(A) $y^2 = x^2 - 2x + 2$ (B) $y^2 = 2x^2 - x - 1$

(C) $y = x^2 - 2x + 2$ (D) none of these

2. (A)

2. $y\frac{dy}{dx} = x - 1$

$\Rightarrow ydy=(x-1)dx \Rightarrow \frac{y^2}{2}=\frac{x^2}{2}-x+c$

for x = 1, y = 1

$\Rightarrow \frac{1}{2}=\frac{1}{2}-1+C \quad \Rightarrow \quad C=1$

Thus $\frac{y^2}{2}=\frac{x^2}{2}-x+1 \Rightarrow y^2=x^2-2x+2$

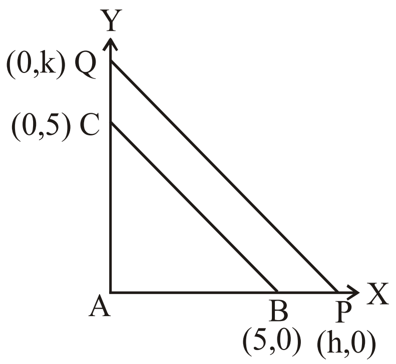
Hence (c) is the correct answer.

Q3. If the equal sides AB and AC (each equal to 5 units) of a right-angled isosceles triangle ABC are produced to P and Q such that $BP.CQ=AB^2$, then the line PQ always passes through the fixed point (where A is the origin and A B, A C lie along the positive x and positive y -axis respectively)

(A) (7,6) (B) (6,5) (C) (5,5) (D) (6,6)

3. (C)

3. Let, $AP=h, AQ=k$



Equation of the line PQ is $\frac{x}{h}+\frac{y}{k}=1 \ldots (1)$

Given, $BP.CQ=AB^2$

$\Rightarrow (h-5)(k-5)=25$

or, $5h+5k=hk$

or, $\frac{5}{h}+\frac{5}{k}=1$

From (2), it follows that line (1), i.e., PQ passes through the fixed point (5,5)

Q4. Bag I contains 4 blue and 3 white balls and Bag II contains 2 blue and 6 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be a white ball. Then, the probability, that the transferred ball is blue, is

(A) $\frac{7}{15}$ (B) $\frac{8}{15}$ (C) $\frac{1}{6}$ (D) $\frac{3}{10}$

4. (B)

4. $P\left(\frac{B_1}{w_2}\right)=\frac{P(B_1\cap w_2)}{P(w_2)}$

$$=\frac{P(B_1\cap w_2)}{P(B,\cap w_2)+P(w_1\cap w_2)}$$

$$=\frac{\frac{4}{7}\times\frac{6}{9}}{\frac{4}{7}\times\frac{6}{9}+\frac{3}{7}\times\frac{7}{9}}$$

$$=\frac{8}{15}$$

Q5. $\ell \underset{x\to 2}{\text{im}}\frac{(x^3+8)\ell\text{n}(x-1)}{(x^3-8)}$ is equal to

(A) 16 (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{-4}{3}$

5. (C)

5. Required $=\lim\limits_{x\to 2}\left((x^3+8)\frac{\ell n(1+x-2)}{(x-2)}\times\frac{1}{x^2+2x+4}\right)$

$$=(8+8)\times 1\times\frac{1}{4+4+4}=\frac{4}{3}$$

Q6. Let f(x) = x^5+x^3+2x+4 and g(x) is inverse function of f(x), then g'(0) =

(A) 0.2 (B) 0.1 (C) 0.3 (D) 0.4

6. (B)

6. (6) we know that, $g(f(x))=x$

$$\Rightarrow g'(f(x))=f'(x)=1$$

$$\Rightarrow g'(f(x))=\frac{1}{f'(x)}$$

$$\Rightarrow g''(f(x))=-\frac{1}{(f'(x))^3}f''(x)$$

put x = –1

$$g'(0)=\frac{1}{f'(-1)}$$

$\Rightarrow g'(0) = \frac{1}{10}$

Q7. The intercepts made on the x, y and z axes, by the plane which bisects the line joining the points (1,2,3) and (–3,4,5) at right angles, are a, b and c respectively, then the ordered triplet (a, b, c) is

(A) $\left(\frac{-9}{2}, 9, 9\right)$ (B) $\left(\frac{9}{2}, 9, 9\right)$ (C) $\left(9, \frac{-9}{2}, 9\right)$ (D) $\left(9, \frac{9}{2}, 9\right)$

7. (A)

7. Let, equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Mid-point M of line joining $P(1,2,3)$ and $Q(-3,4,5)$ is (–1,3,4).

It lies on the plane i.e. $\frac{-1}{a} + \frac{3}{b} + \frac{4}{c} = 1$

Also, PQ is parallel to the normal of the plane, so,

$$\frac{\frac{1}{a}}{-4} = \frac{\frac{1}{b}}{2} = \frac{\frac{1}{c}}{2} = \lambda$$

$$\Rightarrow \frac{1}{a} = -4\lambda, \frac{1}{b} = 2\lambda, \frac{1}{c} = 2\lambda$$

$$\Rightarrow 4\lambda + 6\lambda + 8\lambda = 1$$

$$\Rightarrow \lambda = \frac{1}{18}$$

i.e. $a = -\frac{9}{2}, b = 9, c = 9$

Hence, ordered pair is $\left(\frac{-9}{2}, 9, 9\right)$

Q8. $\int_1^{2024} \frac{2^x}{2^x + 2^{2025-x}} dx$ is equal to:

(A) 2025 (B) 1012 (C) $1012\frac{1}{2}$ (D) $1011\frac{1}{2}$

8. (D)

8. $I = \int_1^{2024} \frac{2^x}{2^x + 2^{2025-x}} dx$

using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_1^{2024} \frac{2^{2025-x}}{2^{2025-x} + 2^x} dx$$

$$\therefore 2I = \int_{1}^{2024} 1dx$$

$$\Rightarrow I = \frac{2023}{2}$$

Q9. The chords of contact of the pair of tangents drawn from the point A(1, 2) to the circle $x^2 + y^2 = 1$ passes through the point -

(A) (2, 0) (B) (3, 0) (C) (4, 0) (D) (1, 0)

9. (D)

9. The COC is x + 2y = 1

Q10. If $(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, then $\left(1+\frac{a_1}{a_0}\right)\left(1+\frac{a_2}{a_1}\right)\left(1+\frac{a_3}{a_2}\right)\ldots\left(1+\frac{a_n}{a_{n-1}}\right)$ is equal to

(A) $\frac{n^n}{n!}$ (B) $\frac{(n+1)^n}{n!}$ (C) $\frac{n^{n+1}}{(n+1)!}$ (D) None of these.

10. (B)

10. Clearly $a_r = {}^nC_r$

$$\Rightarrow \frac{a_r}{a_{r-1}} = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{(n-r+1)}{r}$$

$$\Rightarrow 1 + \frac{a_r}{a_{r-1}} = \frac{n+1}{r}$$

$$\Rightarrow \prod_{r=1}^{n}\left(1+\frac{a_r}{a_{r-1}}\right) = \prod_{r=1}^{n}\frac{(n+1)}{r} = \frac{(n+1)^n}{n!}$$

Q11. The locus of a point z represented by the equation $|z-1| = |z-i|$ on the argand plane is (where, $z \in C, i = \sqrt{-1}$)

(A) a circle of radius 1

(B) an ellipse with foci at 1 and $-i$

(C) a line passing through the origin

(D) a circle on the line joining 1 and -i as diameter

11. (C)

11. Given, $|z-1| = |z-i|$

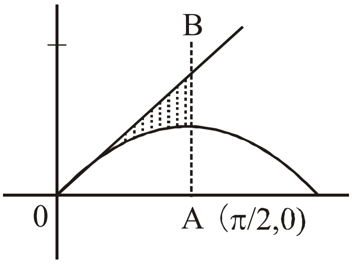
$\Rightarrow z$ lies on the perpendicular bisector of the line joining (1,0) and (0,1) and it is a straight line passing through the origin.

Q12. If the area bounded by $y = x, y = \sin x$ and $x = \frac{\pi}{2}$ is $\left(\frac{\pi^2}{k} - 1\right)$ sq. units, then the value of k is equal to

(A) 2 (B) 4 (C) 6 (D) 8

12. (D)

12. Required area $= ar(\Delta AOB) - \int_0^{\frac{\pi}{2}} \sin x dx$



$= \underbrace{\frac{1}{2}\cdot\frac{\pi}{2}\cdot\frac{\pi}{2}}_{\frac{1}{2}\cdot \text{base}-\text{height}} - 1$

$= \frac{\pi^2}{8} - 1$ sq. units

Q13. If $f(x) = |\log|x||$, then f(x) is-

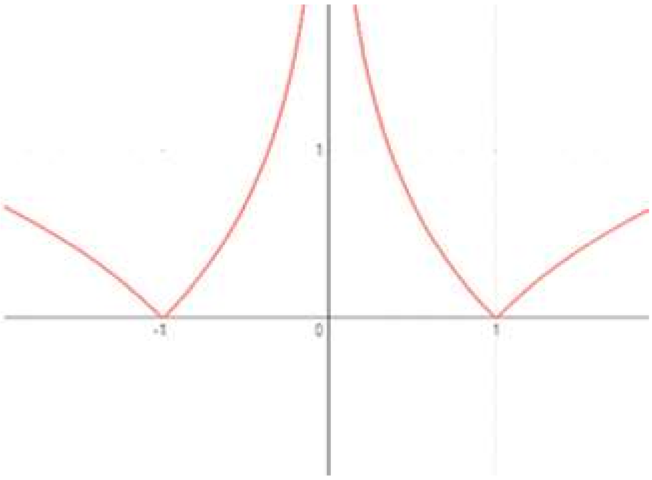
(A) Continuous & differentiable for all x in its domain

(B) discountinous at 2 points in its domain

(C) Continuous but non-differentiable at 2 points in its domain

(D) non-differentiable at 3 points in its domain

13. (C)

13. 

clearly, f(x) is non-differentiable at $x = \pm 1$.

Q14. A plane passes through the point (-2,-2,2) and contains the line joining the points (1,-1,2) and (1,1,1). Then the image of (-7,2,3) in the plane is

(A) (-8,5,9) (B) (-5,-4,-2) (C) (-6,-1,-3) (D) $\left(\frac{13}{23}, \frac{7}{23}, \frac{6}{23}\right)$

14. (C)

14. The plane passes through the points (-2,-2,2),(1,-1,2),(1,1,1)

Its equation is $\begin{vmatrix} x+2 & y+2 & z-2 \\ 3 & 1 & 0 \\ 3 & 3 & -1 \end{vmatrix} = 0$

$\Rightarrow x-3y-6z+8=0$

Image of (-7,2,3) in the plane is

$\frac{x+7}{1}=\frac{y-2}{-3}=\frac{z-3}{-6}=-2\left(\frac{-7-6-18+8}{1+9+36}\right)$

$\frac{x+7}{1}=\frac{y-2}{-3}=\frac{z-3}{-6}=\left(\frac{23}{23}\right)=1$

$x=-6, y=-1, z=-3$

Q15. The difference between the maximum and minimum values of the function $f(x)=\sin^3 x-3\sin x, \forall x \in \left[0,\frac{\pi}{6}\right]$ is

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{11}{8}$ (D) $\frac{7}{6}$

15. (C)

15. Let, $\sin x = t$

$\therefore$ If $x \in \left[0,\frac{\pi}{6}\right] \Rightarrow t \in \left[0,\frac{1}{2}\right]$

Now, $f(t)=t^3-3t$

$\Rightarrow f'(t)=3t^2-3=3(t-1)(t+1)$

Thus, f(t) is decreasing $\forall t \in \left[0,\frac{1}{2}\right]$

$\therefore (f(t))_{max}=f(0)=0$

$(f(t))_{min}=f\left(\frac{1}{2}\right)=\frac{1}{8}-\frac{3}{2}=-\frac{11}{8}$

$\therefore f_{max}-f_{min}=\frac{11}{8}$

Q16. Let $A_n=\int \tan^n x dx, \forall n \in N$ .If $A_{10}+A_{12}=\frac{\tan^m x}{m}+\lambda$ (where λ is an arbitrary constant), then the value of m is equal to

(A) 10 (B) 11 (C) 12 (D) 13

16. (B)

16. $A_{10}+A_{12}=\int \tan^{10} x dx+\int \tan^{12} x dx$

$= \int \left(\tan^{10} x + \tan^{12} x\right) dx$

$= \int \tan^{10} x \left(1 + \tan^2 x\right) dx$

$= \int \tan^{10} x \cdot \sec^2 x dx$

Let, $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore A_{10} + A_{12} = \int t^{10} dt = \frac{t^{11}}{11} + \lambda$

$= \frac{\tan^{11} x}{11} + \lambda$

Q17. The eccentricity of the hyperbola $x^2 - y^2 = 4$, is

(A) 2 (B) 3 (C) 4 (D) $\sqrt{2}$

17. (D)

17. a = 2, b = 2

Q18. If $\tan 25^\circ = x$, then $\frac{\tan 155^\circ - \tan 115^\circ}{1 + \tan 155^\circ \tan 115^\circ}$ is equal to

(A) $\frac{1-x^2}{2x}$ (B) $\frac{1+x^2}{2x}$ (C) $\frac{1+x^2}{1-x^2}$ (D) $\frac{1-x^2}{1+x^2}$

18. (A)

18. $\frac{\tan\left(180^\circ - 25^\circ\right) - \tan\left(90^\circ + 25^\circ\right)}{1 + \left(\tan\left(180^\circ - 25^\circ\right)\tan\left(90^\circ + 25^\circ\right)\right.} = \frac{-\tan 25^\circ + \frac{1}{\tan 25^\circ}}{2} = \frac{1-x^2}{2x}$

Q19. Function $f(x) = \frac{2x}{1+x^2} : R \to [-1,1]$ is

(A) Bijective (B) Injective but not surjective

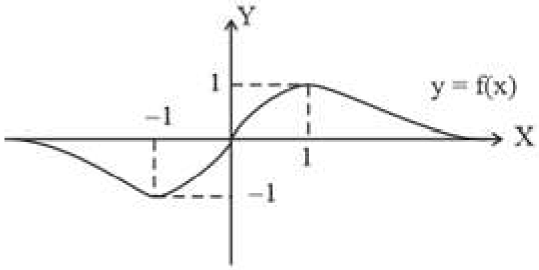
(C) Surjective but not injective (D) Neiter injective nor surjective

19. (C)

19. $f(x) = \frac{2x}{1+x^2}$

$\Rightarrow f'(x) = \frac{\left(1+x^2\right) \cdot 2 - 2x2x}{\left(1+x^2\right)^2}$

$\Rightarrow f'(x) = \dfrac{2(1+x)(1-x)}{\left(1+x^2\right)^2}$



Method-2

$f(x) = \dfrac{2x}{1+x^2} = \dfrac{2}{x + \dfrac{1}{x}}$

(i) $\because f(x) = f\left(\dfrac{1}{x}\right) \Rightarrow f(x)$ is many-one function

(ii) $x + \dfrac{1}{x} \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow f(x) \in [-1, 1]$

$\therefore f(x)$ is onto function.

Q20. The equation $(\cos P - 1)x^2 + (\cos P)x + \sin P = 0$ has real roots. Then P can take any value in the interval

(A) $(0, 2\pi)$ (B) $(0, \pi)$ (C) $(-\pi, 0)$ (D) $(-\pi/2, \pi/2)$

20. (B)

20. $D \geq 0$ $\cos^2 P - 4(\cos P - 1)\sin P \geq 0$
$\cos^2 P + 4(1 - \cos P)\sin P \geq 0$ for real roots $\sin P > 0 \; \therefore P \in (0, \pi)$

Section-II(NV)

Q21. $\int_0^\infty \left[\dfrac{2}{e^x}\right] dx$ is equal to ($[x] =$ greatest integer ℓnk, find k.

21. 2

21. We have, if $e^x > 2, \dfrac{2}{e^x} < 1$. Also $\dfrac{2}{e^x} > 0$

$\Rightarrow 0 < \dfrac{2}{e^x} < 1$

$\therefore$ If $x > \log_e 2, \left[\frac{2}{e^x}\right] = 0$

Again if $0 < x < \log_e 2$ then $1 < e^x < 2$

$\Rightarrow 1 > \frac{1}{e^x} > \frac{1}{2} \Rightarrow 2 > \frac{2}{e^x} > 1 or 1 < \frac{2}{e^x} < 2$

$\therefore \left[\frac{2}{e^x}\right] = 1$

$\therefore I = \int_0^\infty \left[\frac{2}{e^x}\right] dx = \int_0^\infty \left[2e^{-x}\right] dx$

$= \int_0^{\log 2} \left[2e^{-x}\right] dx + \int_{\log 2}^\infty \left[2e^{-x}\right] dx$

$= \int_0^{\log 2} (1) dx + \int_{\log 2}^\infty (0) dx = \log_e 2$

Q22. Let f(x) is a differentiable function on $x \in R$, such that $f(x + y) = f(x) f(y)$ for all $x, y \in R$ where $f(0) \neq 0$. If $f(5) = 10$, $f'(0) = 6$, then the value of $f'(5)$ is equal to

22. 60

22. $f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$

$= \lim_{h \to 0} \frac{f(5+h) - f(5+0)}{h}$

$= \lim_{h \to 0} \frac{f(5) \cdot f(h) - f(5) \cdot f(0)}{h}$

$[\because f(x+y) = f(x) \cdot f(y) \text{ for all } x, y]$

$= \left(\lim_{h \to 0} \frac{f(h) - f(0)}{h}\right) \cdot f(5)$

$= f'(0).f(5) = 6 \times 10$

$= 60$

Alternate Solution

$f(x+y) = f(x).f(y)$

diffrentiate both sides w.r.t. y

$f'(x+y) = f(x).f'(y)$

put $y = 0$ & $x = 5$

$f'(5) = f(5).f'(0)$

$f'(5) = 60$

Q23. If $A \neq B, AB = BA$ and $A^2 = B^2$, then the value of the determinant of matrix A + B is (where A and B are square matrices of order 3 × 3)

23. 0

23. $A^2 - B^2 = (A - B)(A + B) = 0$ (given $AB = BA$)

Since, $A \neq B \Rightarrow A - B$ is not a null matrix

Hence, A + B is a null matrix or $\det(A - B) = \det(A + B) = 0$

In both cases det (A + B) = 0

Q24. If $\int_0^{\pi/4} \log(1 + \tan x)\,dx = \frac{\pi}{a}\log b$, then the value of (a + b) is

24. (10)

24. $I = \int_0^{\pi/4} \log(1 + \tan x)\,dx$

$= \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$

$= \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$

$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right) dx$

$= \int_0^{\pi/4} \log 2\,dx - \int_0^{\pi/4} \log(1 + \tan x)\,dx$

. $\Rightarrow I = \frac{\pi}{4}\log 2 - I$

$\Rightarrow I = \frac{\pi}{8}\log 2$

Q25. The mean and variance of 7 observations are 5 and 10 respectively. If 5 observations are 1,3,5,7,9, then square of difference of remaining two observations is:

25. 60

25. mean $= \bar{x} = \frac{1+3+5+7+9+a+b}{7} = 5$

$\Rightarrow a + b = 10$ ...(1)

Variance $= \sigma^2 = \frac{\sum x_i^2}{7} - (\bar{x})^2 = 10$

$\Rightarrow \frac{1+9+25+49+81+a^2+b^2}{7} = 35$

$\Rightarrow a^2 + b^2 = 80$

$\Rightarrow (a+b)^2 - 2ab = 80$

$\Rightarrow ab = 10$ ...(2)

from (1) & (2)

$(a-b)^2 = 80 - 20$