CHAPTER: 10 - How Quantities Combine: Understanding Data

Exercise Set 10.1 (Solutions)

Q1. Two sections wrote the same test. Section A has 30 students with an average of 72, and Section B has 25 students with an average of 76. Find the combined average of the two sections.

Solution:

Total marks of A = 72 × 30 = 2160, and total marks of B = 76 × 25 = 1900. Number of students = 30 + 25 = 55.

Combined average = (2160 + 1900)/55 = 4060/55 ≈ **73.82**. This lies between 72 and 76 and is closer to 72, because Section A has more students.

Q2. A farmer mixes equal quantities of three fertilisers. Their nitrogen fractions are 1/10, 9/50 and 3/60. What fraction of the mixture is nitrogen?

Solution:

The quantities are equal, so the fraction in the mixture is the simple average of the three fractions: 1/10 = 0.10, 9/50 = 0.18 and 3/60 = 0.05.

Average = (0.10 + 0.18 + 0.05)/3 = 0.33/3 = 0.11 = 11/100. So the mixture is **11% nitrogen**.

Q3. In ancient India, a gold purity of 16 varṇa meant pure gold, and k varṇa meant that k/16 of the alloy was gold. A goldsmith melts three pieces together: 9 units of 12 varṇa, 5 units of 10 varṇa and 17 units of 11 varṇa. Find the purity of the combined gold.

Solution:

The purity of the mixture is the weighted mean of the purities, weighted by the amount of each piece:

(9 × 12 + 5 × 10 + 17 × 11)/(9 + 5 + 17) = (108 + 50 + 187)/31 = 345/31 ≈ **11.13 varṇa**.

Q4. The average rainfall per day in May, June and July in a place is 3.5 mm, 10 mm and 8.7 mm. Write an expression for the combined average per day, and evaluate it.

Solution:

The months have 31, 30 and 31 days, so each average is weighted by the number of days:

Combined average = (3.5 × 31 + 10 × 30 + 8.7 × 31)/(31 + 30 + 31) = (108.5 + 300 + 269.7)/92 = 678.2/92 ≈ **7.37 mm per day**.

Q5. Find the concentration of spice mix when kadha of different strengths is combined.

(i) 100 mL at 5%, 200 mL at 10% and 300 mL at 15%.

(ii) 300 mL at 5%, 200 mL at 10% and 100 mL at 15%.

Solution:

(i) Spice = 5 + 20 + 45 = 70 mL in 600 mL, so the concentration is 70/600 ≈ **11.67%**.

(ii) Spice = 15 + 20 + 15 = 50 mL in 600 mL, so the concentration is 50/600 ≈ **8.33%**.

The same three concentrations give different results, because the amounts change: in (i) most of the mixture is the strong 15% kadha, and in (ii) most is the mild 5% kadha.

Exercise Set 10.2 (Solutions)

Q1. Savitri scored 35 out of 50 in internal tests, 44 out of 60 in the project and 80 out of 100 in the final exam. The annual percentage combines the three in the ratio 3 : 4 : 5. Which of these expressions gives her annual percentage?

- (i) (35 × 3 + 44 × 4 + 80 × 5)/(3 + 4 + 5)
- (ii) [(35/100) × 3 + (44/100) × 4 + (80/100) × 5]/(3 + 4 + 5)
- (iii) [(35/50) × 3 + (44/60) × 4 + (80/100) × 5]/(3 + 4 + 5) × 100
- (iv) [(35/50 × 100) × 3 + (44/60 × 100) × 4 + (80/100 × 100) × 5]/(3 + 4 + 5)

Solution:

The three marks are out of different totals (50, 60, 100), so each must first be turned into a fraction or percentage of its own total: 35/50 = 70%, 44/60 = 73.33% and 80/100 = 80%. Only then can the weights 3 : 4 : 5 be used.

- (i) **Wrong.** It uses raw marks that are out of different totals.

- (ii) **Wrong.** It divides every mark by 100, although the marks are out of 50 and 60, and it does not give a percentage.
- (iii) **Correct.** It weights the correct fractions and multiplies by 100 at the end.
- (iv) **Correct.** It converts to percentages first. This is the same as (iii), with the × 100 done earlier.

Annual percentage = (3 × 70 + 4 × 73.33 + 5 × 80)/12 = (210 + 293.33 + 400)/12 ≈ **75.28%**.

Exercise Set 10.3 (Solutions)

Q1. A stationery shop earned ₹8000 from selling books, of which 30% was profit, and ₹1000 from selling covers, of which 50% was profit. What is the percentage of profit on the total sales?

Solution:

Profit on books = 30% of 8000 = ₹2400. Profit on covers = 50% of 1000 = ₹500. Total profit = ₹2900 on total sales of ₹9000.

Profit percentage = 2900/9000 × 100 ≈ **32.22%**. This is a weighted mean of 30% and 50%, with weights 8000 and 1000, so it is close to 30%.

Q2. A white stork averaged 44.5 km per day over 20 days, and flew 55 km on day 21. What is its average daily distance over the 21 days? Guess first.

Solution:

Guess: one day of 55 km is only a little above 44.5 km and is one day out of 21, so the average rises only a little, to about 45 km.

Distance in 20 days = 44.5 × 20 = 890 km. Distance in 21 days = 890 + 55 = 945 km. New average = 945/21 = **45 km per day**.

Q3. A 600 mL solution with 5% salt is mixed with a 300 mL solution with 8% sugar. What are the concentrations of salt and sugar in the mixture? Choose the correct option.

Options: (i) salt 5%, sugar 8%; (ii) salt 13%, sugar 3%; (iii) salt 6%, sugar 6%; (iv) salt 5.55%, sugar 8.88%; (v) salt 3.33%, sugar 2.67%; (vi) salt 4.1%, sugar 7.08%.

Solution:

Salt = 5% of 600 = 30 mL, and sugar = 8% of 300 = 24 mL. The total volume is 600 + 300 = 900 mL.

Salt concentration = 30/900 × 100 ≈ 3.33%. Sugar concentration = 24/900 × 100 ≈ 2.67%. Both go down, because each substance is now spread over the whole 900 mL. The correct option is **(v)**.

Q4. The pani at a panipuri stall has 8% spice. How much plain water must be added to 10 litres of the pani to bring the spice level down to three-fourths of the original concentration?

Solution:

Target concentration = (3/4) × 8% = 6%. Spice in 10 L of pani = 0.08 × 10 = 0.8 L, and this amount stays the same when water is added. Let w litres of water be added:

0.8/(10 + w) = 0.06, so 10 + w = 0.8/0.06 = 40/3, and w = 40/3 − 10 = 10/3.

Add **10/3 ≈ 3.33 litres** of water. Check: 0.8 ÷ (40/3) = 0.06 = 6%.

Q5. In a fitness test, the marks for strength, flexibility and agility are combined in the ratio 4 : 5 : 6. Rashi scored 60, 65 and 70. Keerthi scored 55, 65 and 75.

(i) Who has the higher total? Answer without calculating.

(ii) Find their final marks.

Solution:

(i) Both have the same flexibility score. Rashi is 5 marks ahead in strength (weight 4), and Keerthi is 5 marks ahead in agility (weight 6). The 5 marks that Keerthi gains carry more weight, so **Keerthi** has the higher total.

(ii) Rashi = (60 × 4 + 65 × 5 + 70 × 6)/15 = (240 + 325 + 420)/15 = 985/15 ≈ **65.67**.

Keerthi = (55 × 4 + 65 × 5 + 75 × 6)/15 = (220 + 325 + 450)/15 = 995/15 ≈ **66.33**.

Q6. Ten customers rated a restaurant from 1 to 5 stars. The number of customers giving each rating is in the table. Find the overall rating if food, ambience and service are combined with weights 6 : 5 : 4.

	5 stars	4 stars	3 stars	2 stars	1 star
Food	5	3	2	0	0
Ambience	0	4	5	1	0
Service	1	2	2	4	1

Solution:

Average rating of each aspect (weighted by the number of customers): Food = (5 × 5 + 4 × 3 + 3 × 2)/10 = 43/10 = 4.3. Ambience = (4 × 4 + 3 × 5 + 2 × 1)/10 = 33/10 = 3.3. Service = (5 × 1 + 4 × 2 + 3 × 2 + 2 × 4 + 1 × 1)/10 = 28/10 = 2.8.

Overall = (4.3 × 6 + 3.3 × 5 + 2.8 × 4)/15 = (25.8 + 16.5 + 11.2)/15 = 53.5/15 ≈ **3.57 stars**.

Q7. Langurs in an animal facility have an average weight of 16.5 kg for males, 13.8 kg for females and 14.925 kg for all 60 langurs.

(a) Without calculating, are there more male or more female langurs?

(b) Which equation describes the situation, with x males and y females? (A) (16.5x + 13.8y)/(x + y) = 14.925; (B) (16.5x + 13.8y)/60 = −14.925; (C) (16.5x + 13.8y)/2 = −14.925; (D) (16.5x + 13.8y)/(16.5 + 13.8) = −14.925.

(c) How many male langurs are there?

(d) A 15.2 kg female is admitted. What is the new average weight of the females?

(e) Two males of 16.9 kg and 16.1 kg are released. What is the new average weight of the males?

(f) One of the remaining males loses 1 kg. What is the new average weight of the males?

Solution:

(a) The overall average 14.925 is closer to the female average 13.8 (it is 1.125 away) than to the male average 16.5 (1.575 away). So the females pull the average more, and there are **more females**.

(b) Only **(A)** is correct: it is the total weight divided by the number of langurs. In (B), (C) and (D) the right side is negative, which is impossible for an average weight. In addition, (C) divides by 2 and (D) divides by 16.5 + 13.8, and neither is the number of langurs.

(c) Put y = 60 − x in (A): 16.5x + 13.8(60 − x) = 14.925 × 60 = 895.5, so 2.7x + 828 = 895.5, giving x = 25. There are **25 males and 35 females**.

(d) Total female weight = 35 × 13.8 = 483 kg. New average = (483 + 15.2)/36 = 498.2/36 ≈ **13.84 kg**.

(e) Total male weight = 25 × 16.5 = 412.5 kg. New average = (412.5 − 16.9 − 16.1)/23 = 379.5/23 = **16.5 kg**, unchanged, because the two released males also average (16.9 + 16.1)/2 = 16.5 kg.

(f) New average = (379.5 − 1)/23 = 378.5/23 ≈ **16.46 kg**.

Q8. Dorjee has 1 litre of Dead Sea water. Salinities: Dead Sea ≈ 34%, other seas and oceans ≈ 3.5%, groundwater ≈ 0.01%, purified drinking water ≈ 0.001%.

(i) Find the salinity if he mixes it with 2 litres of purified water.

(ii) Can he get the salinity of groundwater by adding purified water? If yes, how much is needed?

(iii) Can he get the salinity of purified water by adding groundwater?

Solution:

(i) Salinity = (1 × 34 + 2 × 0.001)/(1 + 2) = 34.002/3 ≈ **11.33%**.

(ii) **Yes.** A mixture's salinity always lies between the salinities of the liquids mixed, and 0.01% lies between 0.001% and 34%. Let w litres of purified water be added: (34 + 0.001w)/(1 + w) = 0.01. Then 34 + 0.001w = 0.01 + 0.01w, so 33.99 = 0.009w and w ≈ 3776.67. About **3777 litres** are needed.

(iii) **No.** Purified water (0.001%) is less salty than both the Dead Sea water (34%) and groundwater (0.01%). A mixture can never be less salty than the least salty liquid in it.

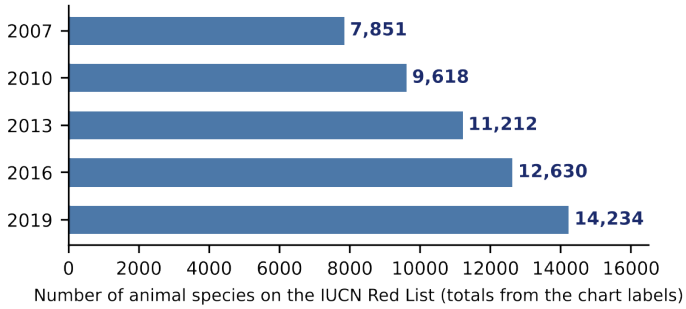
Exercise Set 10.4 (Solutions)

Q1. A stacked column chart shows the number of animal species on the IUCN Red List, by class, for 2007 to 2019. The totals above the columns are given below.

(i) What does 14,234 above the 2019 column represent?

(ii) About how many reptile species were listed in 2016?

(iii) Which classes grew relatively little between 2007 and 2019?



Solution:

(i) 14,234 is the **total number of animal species** of all classes on the Red List in 2019, that is, the height of the whole stacked column.

(ii) In the 2016 column, the reptile band is about 1,100 species thick, so there were **about 1,100 reptile species**. Readings from a chart are approximate, so an answer of about 1,000 to 1,300 is acceptable.

(iii) The bands of **mammals, birds and amphibians** have nearly the same thickness in 2007 and in 2019, so their counts grew relatively little.

Q2. The wickets taken by a bowler in international cricket till 2025 are in the table. Complete two stacked bar charts.

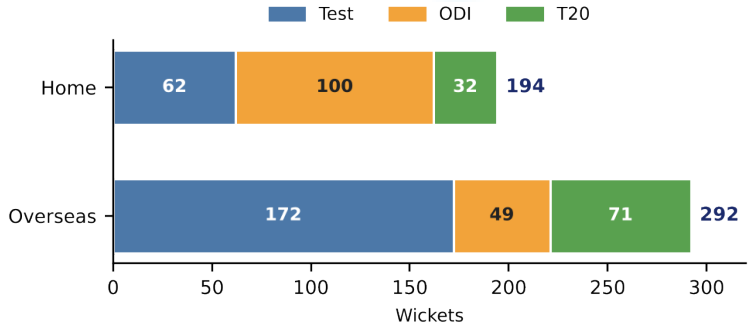
	Test	ODI	T20
Home	62	100	32
Overseas	172	49	71

(i) The total wickets at home and overseas.

(ii) The total wickets in each format.

Solution:

(i) Home total = 62 + 100 + 32 = **194**. Overseas total = 172 + 49 + 71 = **292**. Each bar is split into Test, ODI and T20.



(ii) Test = 62 + 172 = **234**. ODI = 100 + 49 = **149**. T20 = 32 + 71 = **103**. Each bar is split into home and overseas.

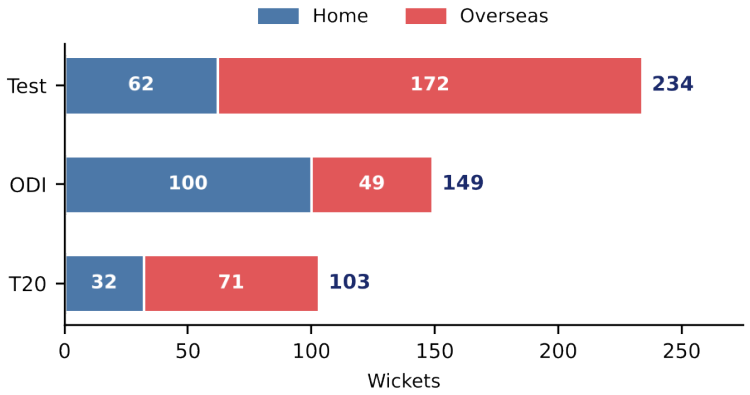


Chart (i) shows that most wickets came overseas, mainly in Tests. Chart (ii) shows that Tests give the most wickets, while in ODIs the larger part came at home.

Exercise Set 10.5 (Solutions)

Q1. A smart watch tracked the average hours per day (Monday to Friday) that five people spent lying down, sitting, and standing or moving.

Person	Lying down (h)	Sitting (h)	Standing or moving (h)
Pavani (patient)	20	3	1
Raghu (nurse)	7	4	13
Zakir (teacher)	8	6	10
Sahana (student)	?	?	?
Julie	8	10	6

(i) Guess Sahana's hours and fill her row.

(ii) Guess what work Julie could be doing.

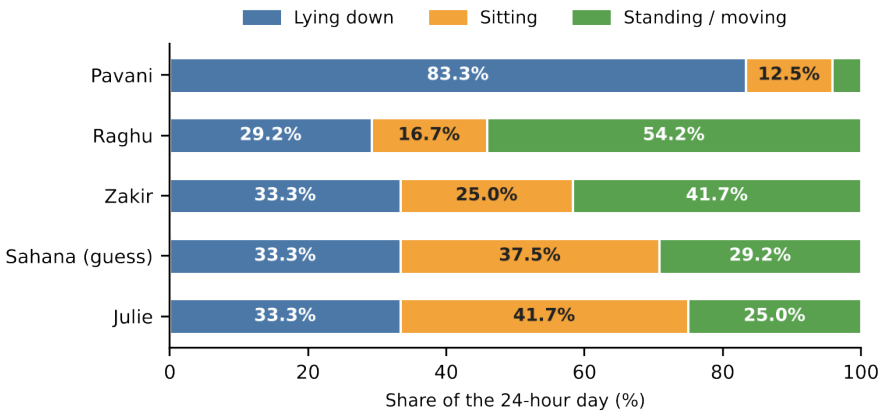
(iii) Complete the 100% stacked bar chart.

Solution:

(i) A student sleeps about 8 hours and sits for most of the school day, homework and meals. A reasonable guess is **8 h lying down, 9 h sitting and 7 h standing or moving** (the total must be 24 h). Other sensible guesses are also acceptable.

(ii) Julie sits for 10 hours, more than anyone else, and moves for only 6. She could work in a **desk job** such as an office worker, accountant, bank cashier or receptionist. Answers may vary.

(iii) Every total is 24 h, so each share = hours/24 × 100. The bars below use Sahana's guessed values.



End-of-Chapter Exercises (Solutions)

Q1. The run rate in cricket is the average number of runs per over. A T20 team scored 6 runs in the first over, so the run rate is 6.

(i) In the second over they scored 12 runs. What is the run rate now?

(ii) After 19 overs the run rate was 6. What is the run rate after 20 overs, if 12 runs were scored in the last over?

Solution:

(i) Runs = 6 + 12 = 18 in 2 overs, so the run rate is 18/2 = **9**.

(ii) Runs after 19 overs = 6 × 19 = 114. After the last over, runs = 114 + 12 = 126, so the run rate is 126/20 = **6.3**.

One over changes the run rate a lot after 1 over (6 to 9), but only a little after 19 overs (6 to 6.3): a new value has less effect on an average when there are many values already.

Q2. Five friends noted the prices of nylon (N) and feather (F) shuttlecocks (see the table). Yusuf collected 6 prices, Srikanth 4, Kashvi 5, Prasanna 4 and Gracy 4.

Friend	Prices in ₹ (N = nylon, F = feather)
Yusuf	40N, 105N, 383F, 108N, 165F, 116F
Srikanth	194N, 85N, 93N, 121N
Kashvi	49N, 297F, 105N, 275F, 40N
Prasanna	124N, 333F, 182N, 258F
Gracy	220F, 458F, 129F, 183N

(i) Each friend finds the average of his or her own prices: a_y, a_s, a_k, a_p, a_g. Write the combined average.

(ii) If a_n and a_f are the average prices of nylon and feather shuttlecocks, write the combined average. Is it equal to the answer in (i)?

Solution:

(i) The friends collected 6, 4, 5, 4 and 4 prices (23 in all), so each average is weighted by the number of prices: combined average = $(6a_y + 4a_s + 5a_k + 4a_p + 4a_g)/23$.

(ii) There are 13 nylon and 10 feather prices (13 + 10 = 23), so combined average = $(13a_n + 10a_f)/23$.

Yes, they are equal. Both expressions equal (sum of all 23 prices)/23; they only group the same prices in two ways. Check: the nylon prices add up to ₹1429 and the feather prices to ₹2634, so the average is (1429 + 2634)/23 = 4063/23 ≈ **₹176.65** by either method.

Q3. Shreyas has 25 shares and Vaishnavi has 5 shares of a company, each at an average price of ₹150.

(i) Shreyas buys 10 more shares at ₹30 each. What is his new average price per share?

(ii) Vaishnavi buys some shares at ₹30 each, and her new average price per share is ₹70. How many shares did she buy?

Solution:

(i) New average = (25 × 150 + 10 × 30)/(25 + 10) = (3750 + 300)/35 = 4050/35 ≈ **₹115.71** per share.

(ii) Let her buy k shares: (5 × 150 + 30k)/(5 + k) = 70. So 750 + 30k = 350 + 70k, that is, 400 = 40k and k = **10**. Check: (750 + 300)/15 = 70.

Q4. A pool 30 hastas long is dug in five sections of lengths 4, 5, 6, 7 and 8 hastas, to depths of 9, 7, 7, 3 and 2 hastas. Find the mean depth of the pool.

Solution:

Each depth is weighted by the length of its section: mean depth = (4 × 9 + 5 × 7 + 6 × 7 + 7 × 3 + 8 × 2)/(4 + 5 + 6 + 7 + 8) = (36 + 35 + 42 + 21 + 16)/30 = 150/30 = **5 hastas**.

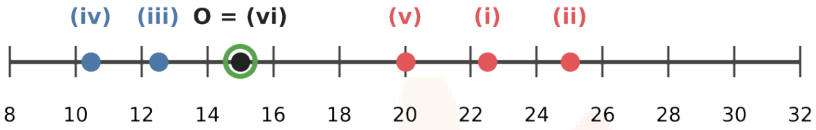
The simple average of the depths, (9 + 7 + 7 + 3 + 2)/5 = 5.6, would be too high, because the deepest parts are the shortest sections.

Q5. Suvarna has 1 g of gold bought at ₹15,000 per gram (₹15k). She makes one more purchase in each of six situations: (i) 1 g at ₹30k; (ii) 2 g at ₹30k; (iii) 1 g at ₹10k; (iv) 10 g at ₹10k; (v) 0.5 g at ₹30k; (vi) 0.5 g at ₹15k. Find her new average price in each case.

Case	Calculation (₹ thousand per gram)	New average
(i)	(15 + 30)/2	22.5
(ii)	(15 + 2 × 30)/3	25
(iii)	(15 + 10)/2	12.5
(iv)	(15 + 10 × 10)/11 = 115/11	≈ 10.45
(v)	(15 + 0.5 × 30)/1.5	20
(vi)	(15 + 0.5 × 15)/1.5	15 (no change)

Solution:

If she buys w grams at price p (in ₹ thousand per gram), the new average is (1 × 15 + w × p)/(1 + w).



Average price of gold (₹ thousand per gram); O is the starting average, 15

Each new average lies between the old average (15) and the new price, and it is closer to whichever price has more grams.

Q6. What happens to a weighted average if all the weights are doubled? Experiment, and justify with algebra.

Solution:

Experiment: values 60, 64, 73 with weights 3, 2, 5 give (180 + 128 + 365)/10 = 67.3. With doubled weights 6, 4, 10: (360 + 256 + 730)/20 = 1346/20 = 67.3. No change.

Algebra: with weights $2w_1$, $2w_2$, ..., $2w_n$, the average is $(2w_1x_1 + \ldots + 2w_nx_n)/(2w_1 + \ldots + 2w_n) = 2(w_1x_1 + \ldots + w_nx_n)/[2(w_1 + \ldots + w_n)]$. The factor 2 cancels, so **the weighted average does not change**. The same is true if all weights are multiplied by any positive number, because only the ratio of the weights matters.

Q7. A chart shows the cumulative number of objects orbiting Earth by launch year (as of 24 April 2026), split into payload objects and other objects.

(i) About how many objects were in orbit in 2003, and in which year did this number double?

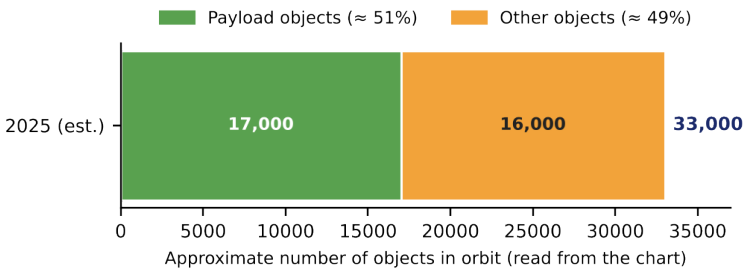
(ii) Estimate the number and share of payload objects and other objects in 2025.

(iii) Describe how the numbers and the shares changed over time.

Solution:

(i) The 2003 column reaches about **10,000** objects. Double this, about 20,000, is reached around **2021**.

(ii) In 2025 the total is about 33,000. Payload objects are about 17,000 (≈ 51%) and other objects about 16,000 (≈ 49%).



Approximate number of objects in orbit (read from the chart)

(iii) **Numbers:** both groups keep growing; payload objects rise sharply after about 2019. **Shares:** for most of the period the payload objects are a smaller share, and after about 2019 their share rises quickly to about half by 2025, while the share of other objects falls, even though the number of other objects keeps growing.

The values are read from the chart, so answers close to these are acceptable.

Q8. An infographic gives the percentage of schools with a playground in each state and union territory (see the table for the values listed).

State or union territory	Schools with a playground (%)
Haryana	90
Uttarakhand	78
Arunachal Pradesh	68
Punjab	98
Maharashtra	93
Telangana	74
Dadra & Nagar Haveli and Daman & Diu	99.8

(i) Which inferences are correct? (a) More schools have a playground in Haryana than in Uttarakhand. (b) About 2 out of every 3 schools in Arunachal Pradesh have a playground. (c) Punjab has the highest number of schools with a playground. (d) Maharashtra has more schools than Telangana, so more of its schools have a playground.

(ii) Can we find the national percentage from this information? If not, what else is needed?

Solution:

(i)

- (a) **Cannot be inferred.** Haryana's percentage (90%) is higher, but the statement is about the *number* of schools, and the number of schools in each state is not given.
- (b) **Correct.** 68% is close to 2/3 ≈ 67%.
- (c) **Incorrect.** The map gives percentages, not numbers. Even by percentage, Punjab (98%) is not the highest: Dadra & Nagar Haveli and Daman & Diu shows 99.8%.
- (d) **Correct.** Maharashtra has a higher percentage (93% against 74%) and, as given, more schools, so it has more schools with a playground.

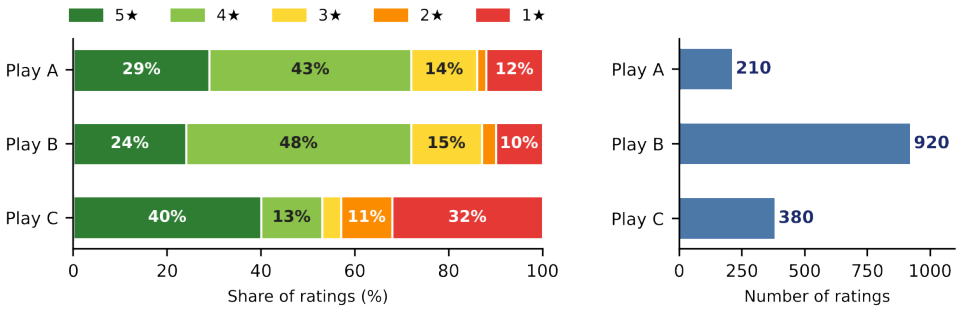
(ii) **No.** The national percentage is a weighted average of the state percentages, weighted by the number of schools in each state or union territory. A simple average would treat a small union territory like a large state. We also need the **number of schools in each state and union territory**. Then national percentage = Σ (schools in the state × state percentage)/Σ (schools in the state).

Q9. Decision dilemma. A 100% stacked chart shows the share of ratings of three plays (A, B and C), and a stacked chart shows the number of ratings. The values below are read from the two charts and are approximate.

Play	5★	4★	3★	2★	1★	Number of ratings
A	29%	43%	14%	2%	12%	≈ 210
B	24%	48%	15%	3%	10%	≈ 920
C	40%	13%	4%	11%	32%	≈ 380

(i) Which play would you choose from the 100% stacked chart?

(ii) Would you change your decision after seeing the stacked chart of the number of ratings?



Solution:

(i) From the 100% chart alone, **Play A** looks best: 72% of its ratings are 4★ or 5★ (the same as Play B), and it has a slightly larger share of 5★. Play C has the most 5★ ratings but also about one-third 1★ ratings, so opinions about it are split.

(ii) The second chart shows that Play B has about 920 ratings, more than four times the 210 of Play A. Their shares are almost the same, but B's shares come from many more people, so they are more reliable. A few more ratings could change A's shares noticeably. So many people would now choose **Play B**. Percentages alone hide the number of ratings.

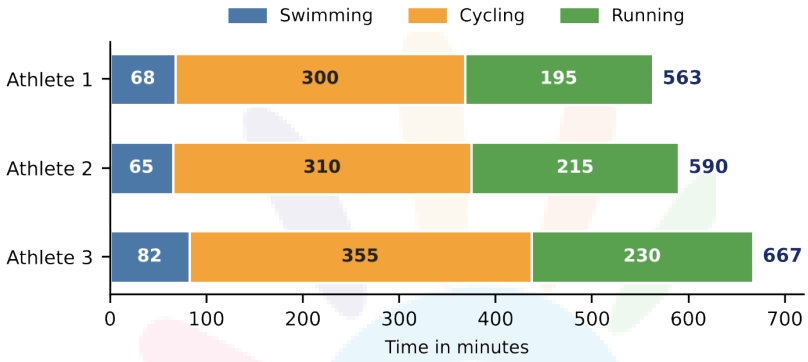
Q10. Three athletes finished a triathlon (swimming, cycling, running) in these times (hh:mm): Athlete 1: 01:08, 05:00, 03:15; Athlete 2: 01:05, 05:10, 03:35; Athlete 3: 01:22, 05:55, 03:50.

(i) Is a stacked or a 100% stacked bar chart more suitable? Why?

(ii) Which of these can be answered from a 100% stacked chart alone? (a) Who finished first? (b) What fraction of Athlete 1's time was spent cycling? (c) Who took the longest to run?

Solution:

In minutes: Athlete 1 = 68 + 300 + 195 = 563 (9 h 23 min). Athlete 2 = 65 + 310 + 215 = 590 (9 h 50 min). Athlete 3 = 82 + 355 + 230 = 667 (11 h 7 min).



(i) A **stacked bar chart**, because the total time decides the race. It shows the totals and how each total is made up. A 100% chart would make all bars equal in length and hide the finishing order. Here Athlete 1 finished first.

(ii) (a) **No**: all bars are equally long. (b) **Yes**: the share is 300/563 ≈ 53%, a little more than half. (c) **No**: it needs actual times. In fact a 100% chart could mislead: Athlete 2 has the largest running share (215/590 ≈ 36.4%), but Athlete 3 ran the longest (230 minutes). Only (b) can be answered.

Q11. A 100% stacked bar chart shows the distribution of disabled persons in India by type of disability for four age groups (Census 2011). What do you notice, and what do you wonder? Write your inferences.

Age group	Seeing	Hearing	Speech	Movement
0–19	18%	20%	9%	13%
20–39	15%	18%	9%	22%
40–59	19%	18%	8%	23%
60 and above	25%	19%	4%	25%

The approximate shares read from the chart for four of the types are given below (nearest 1%; approximate).

Solution:

- Seeing and movement disabilities take up a larger share in the older groups, together about half of all disabilities at 60 and above. This fits with eyesight and mobility problems becoming more common with age.
- The share of speech disability is highest among the young and falls to about 4% at 60 and above.
- Hearing disability has a nearly steady share (about 18% to 20%) in every age group.

What the chart cannot show: it gives shares *within* each age group, not numbers of people. Without the total number of disabled persons in each age group, we cannot say which group has the most affected people, or whether seeing disability is more common in number among the elderly. We may also wonder how the pattern differs between women and men, or between rural and urban areas.

Q12. Individual project (do at least one).

Person	Lying down (h)	Sitting (h)	Standing or moving (h)
Me (weekday)	8	10	6
Me (Saturday)	9	8	7
Me (Sunday)	10	7	7

(i) Record the hours you spent lying down, sitting and standing or moving on one day, ask two family members to do the same (or compare your weekday, Saturday and Sunday), and make a stacked bar chart.

(ii) Collect your household's spending for at least 3 months, by category, and make a 100% stacked bar chart with your observations.

Solution:

Your answer depends on your own data. Here is how to set it up, with a sample.

(i) Body-state record (each row must add up to 24 h):

Draw one bar for each row, split into the three body states. Because every total is 24 h, the same chart is also a 100% stacked bar. In your discussion, explain how you estimated the hours, for example by listing your activities hour by hour.

(ii) Household spending: (a) choose 5 to 7 categories, such as rent, food, education, transport, health, utilities and recreation; (b) note every bill or receipt under its category; (c) total each category for each month; (d) find share = category total ÷ month total × 100 and draw one 100% bar per month; (e) write your observations. For example, a category's share may rise in the month that school fees are paid. A 100% chart does not show whether the total spending went up or down, so use your monthly totals for that.

Q13. Small-group project. Choose one situation: (a) shopping at a cloth store, (b) a bus journey, (c) visiting a tourist spot, (d) a visit to a clinic or hospital.

(i) Choose 4 to 6 aspects to rate and justify them.

(ii) Decide the weights and justify them.

(iii) Collect or imagine ratings from at least 10 people on a scale of 1 to 5.

(iv) Find each person's overall rating and the overall average.

(v) Make a 100% stacked bar chart.

(vi) Write what the scheme shows well and what it misses.

Solution:

The choices of your group will differ. Here is a sample for **(b) a bus journey**.

(i) and (ii) Aspects and weights: punctuality 4 (being on time matters most), safety of driving 4, cleanliness 2, seat comfort 2, behaviour of staff 1, ticket price 2. The total weight is 15.

(iii) and (iv) One passenger gives: punctuality 3, safety 5, cleanliness 2, comfort 3, staff 4, price 4. This passenger's weighted rating is $(3 \times 4 + 5 \times 4 + 2 \times 2 + 3 \times 2 + 4 \times 1 + 4 \times 2)/15 = (12 + 20 + 4 + 6 + 4 + 8)/15 = 54/15 = 3.6$. Find this for all 10 passengers, then take the simple average of the 10 ratings, since every passenger counts equally.

(v) For each aspect, count how many of the 10 passengers gave 5, 4, 3, 2 and 1 stars, and draw one 100% bar per aspect.

(vi) Sample note: the scheme gives the most weight to what passengers care about most (punctuality and safety), and the 100% bars show quickly which aspect needs improvement. It misses crowding at peak hours and the experience of elderly or disabled passengers, and a sample of 10 may not represent all passengers.

Q14. Whole-class project. Every student gives the average age of their family and the number of members. Find a way to get the average age of all the families in the class together.

Solution:

We cannot just average the family averages, because the families have different sizes (just like the seniors and juniors in the badminton example). Each family's average must be weighted by its number of members. If family i has n_i members with average age a_i:

Average age of all members = $(n_1a_1 + n_2a_2 + \ldots + n_ka_k)/(n_1 + n_2 + \ldots + n_k)$.

Steps: (1) each student finds $n_i \times a_i$, the total age of the family; (2) the class adds all these totals; (3) the class adds all the family sizes; (4) divide the total age by the total number of members.

Example: three families have 5 members with average 28, 4 members with average 35 and 6 members with average 30. Average = (140 + 140 + 180)/15 = 460/15 ≈ 30.67 years, and not (28 + 35 + 30)/3 = 31. If two classmates are from the same family, count that family only once.

Q15. (Challenge) If all the weights of a weighted average are increased by the same constant, say 1, what happens to the average? Experiment, and justify with algebra.

Solution:

Experiment: values 10 and 20 with weights 1 and 3 give (10 + 60)/4 = 17.5. With weights 2 and 4: (20 + 80)/6 ≈ 16.67. With weights 3 and 5: (30 + 100)/8 = 16.25. The average **changes**, moving towards the simple average, 15.

Algebra: let $W = w_1 + \ldots + w_n$, let $\bar{x}_w$ be the weighted average and $\bar{x} = (x_1 + \ldots + x_n)/n$ the simple average. With weights $w_i + 1$ the new average is $[(w_1x_1 + \ldots + w_nx_n) + (x_1 + \ldots + x_n)]/(W + n) = (W\,\bar{x}_w + n\,\bar{x})/(W + n)$.

This is itself a weighted average of the old weighted average $\bar{x}_w$ (with weight W) and the simple average $\bar{x}$ (with weight n), so it lies **between** them. Adding a constant to the weights pulls the average towards the simple average. It stays the same only when $\bar{x}_w = \bar{x}$, for example when all weights were already equal. This is different from Q6: multiplying the weights keeps their ratio, but adding a constant changes it (1 : 3 becomes 2 : 4 = 1 : 2).

Q16. (Challenge) On a farm, cows were 60%, sheep 25% and chickens 15% of the animals last year. This year the number of every kind of animal decreased. Which changes in their shares are possible?

(i) All three shares decreased.

(ii) All three shares increased.

(iii) All three shares stayed the same.

(iv) The share of cows decreased, of sheep increased, and of chickens stayed the same.

(v) The share of cows increased, of sheep increased, and of chickens decreased.

Solution:

The three shares always add up to 100%, so they cannot all fall and they cannot all rise. Take 600 cows, 250 sheep and 150 chickens (1000 animals) last year.

- (i) **Not possible.** If all three shares fell, they would add up to less than 100%.
- (ii) **Not possible.** If all three shares rose, they would add up to more than 100%.
- (iii) **Possible.** All numbers fall by the same fraction, for example 480, 200 and 120 (800 animals): the shares stay 60%, 25% and 15%.
- (iv) **Possible.** For example 460 cows, 220 sheep and 120 chickens (800 animals): the shares are 57.5%, 27.5% and 15%.
- (v) **Possible.** For example 490 cows, 210 sheep and 100 chickens (800 animals): the shares are 61.25%, 26.25% and 12.5%.

A share can rise even when the number falls, if the other groups fall faster. This is the difference between a stacked bar chart (numbers) and a 100% stacked bar chart (shares). Possible: (iii), (iv) and (v).