CHAPTER: 13 - Two Variables, One Line

Exercise Set 13.1 — Standard form

Q1. Construct a two-variable linear equation with a = 3, b = 0, c = −1/5 in ax + by + c = 0.

Solution: Substitute the specified coefficients: 3x + 0y − 1/5 = 0. Therefore 3x − 1/5 = 0. The y-coefficient is zero, but it is still a linear equation in two variables because x and y are the permissible variables and at least one of a, b is nonzero.

Q2. Convert each equation into ax + by + c = 0 and identify a, b and c: (i) y − 15 = √2x; (ii) 3y − 2x = 0; (iii) 5x = 3y; (iv) x = 8; (v) 3y = 1.

Solution: (i) √2x − y + 15 = 0 gives (a,b,c)=(√2,−1,15).

(ii) −2x + 3y + 0 = 0 gives (−2,3,0).

(iii) 5x − 3y + 0 = 0 gives (5,−3,0).

(iv) x + 0y − 8 = 0 gives (1,0,−8).

(v) 0x + 3y − 1 = 0 gives (0,3,−1). Multiplying all three coefficients by the same nonzero constant gives an equivalent standard form.

Q3. (i) A notebook costs twice as much as a pen. With prices t and p respectively, decide whether t=2p or p=2t represents this statement. (ii) Two batsmen scored 176 runs altogether. Is x+y=176 an appropriate model of their individual scores?

Solution: (i) The notebook price is 2 times the pen price, so t=2p is correct. For example, p=₹10 implies t=₹20.

(ii) Yes. If one batsman scored x and the other y, the total is x+y=176. Since runs cannot be negative and are whole numbers, add x,y ≥ 0 with integer values.

Exercise Set 13.2 — Solutions and graphs

Q1. Test whether (4,3) satisfies the equation 5x − 6y = 2.

Solution: Replace x by 4 and y by 3. LHS = 5(4) − 6(3) = 20 − 18 = 2, equal to the RHS.

Hence (4,3) is a solution.

Q2. Give any two ordered-pair solutions of (i) 7x − 3y = 21 and (ii) 2x + 3y = 5.

Solution: (i) Taking x=0 gives −3y=21 and y=−7: (0,−7). Taking y=0 gives 7x=21 and x=3:

(3,0).

(ii) Taking x=1 gives 2+3y=5, so y=1: (1,1). Taking x=−2 gives −4+3y=5, so y=3: (−2,3).

Other pairs are possible.

Q3. Both equations 2mx+3y=7 and 4x+ny=−10 have (2,−1) as a solution. Determine m and n.

Solution: First equation: 2m(2)+3(−1)=7 ⇒ 4m−3=7 ⇒ m=10/4=5/2.

Second equation: 4(2)+n(−1)=−10 ⇒ 8−n=−10 ⇒ n=18. Thus m=5/2 and n=18.

Q4. For each equation, choose two solutions in different quadrants and name those quadrants: (i) 5x+3y=7; (ii) 5x−3y=7; (iii) −5x+3y=7; (iv) −5x−3y=7. Represent their lines on graph paper.

Solution: (i) (2,−1) is in IV and (−1,4) in II: 10−3=7 and −5+12=7.

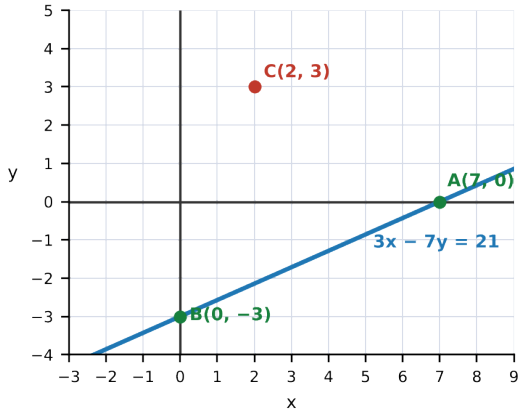
(ii) (2,1) is in I and (−1,−4) in III: 10−3=7 and −5+12=7.

(iii) (1,4) in I, (−2,−1) in III: −5+12=7 and 10−3=7.

(iv) (1,−4) in IV, (−2,1) in II: −5+12=7 and 10−3=7. Join the points of each pair to plot its equation.

Q5. In the graph of 3x−7y=21, examine whether C(2,3) lies on the line, and whether an off-line point could satisfy the equation.

Solution: On substituting C: 3(2)−7(3)=6−21=−15, not 21. Thus C does not satisfy the equation and cannot lie on the graph. Every point on the line satisfies 3x−7y=21 by definition, and no point outside the line does.



Q6. Say True or False with a reason: (i) a two-variable linear equation has one solution; (ii) its graph must pass through the origin; (iii) it cannot have rational solutions; (iv) x=3 is a valid two-variable linear equation; (v) 2x+3y=7 has infinitely many solutions; (vi) (1,2) solves 2x+3y=7.

Solution: (i) False: one line contains infinitely many points, e.g. x+y=2.

(ii) False: x+y=2 does not contain (0,0).

(iii) False: 3x+2y=12 has (0,6) and (1,9/2).

(iv) True: x+0y−3=0.

(v) True: for any x choose y=(7−2x)/3.

(vi) False: 2(1)+3(2)=8, not 7.

Q7. (i) Compare the solution sets of 3x+4y=7 and 6x+8y=14. (ii) Prove that ax+by=c and kax+kby=kc have identical solution sets when k≠0.

Solution: (i) The second equation is obtained by multiplying the first by 2. Every solution of the first therefore satisfies the second, and dividing the second by 2 gives the first. They describe the same line.

(ii) Multiplying ax+by=c by nonzero k yields kax+kby=kc. Since k≠0 the operation can be reversed by dividing by k, so the sets of ordered-pair solutions coincide.

Exercise Set 13.3 — Slope

Q1. Three ski slopes have rise/run measurements A: 60/70, B: 60/110, C: 80/100. Put them in increasing order of steepness.

Solution: Steepness is absolute rise divided by run. A=60/70≈0.857; B=60/110≈0.545;

C=80/100=0.8. Hence the order from least to most steep is B, C, A.

Q2. A loading-ramp rises 2.5 m while moving 4 m horizontally. Calculate its slope.

Solution: Slope=rise/run=2.5/4=0.625=5/8. This positive ratio means a rise of 0.625 m per metre of horizontal movement.

Q3. Read slopes of four lines: (i) horizontal; (ii) line through (0,0) and (−2,2); (iii) line through (−2,−1) and (1,1); (iv) vertical.

Solution: (i) Horizontal: rise=0, slope 0.

(ii) (2−0)/(−2−0)=−1.

(iii) (1−(−1))/(1−(−2))=2/3.

(iv) Vertical: run=0, so division by zero is undefined. The slopes are 0, −1, 2/3, undefined.

Q4. A wheelchair ramp must rise only 1 cm for every 12 cm horizontally. What horizontal distance is needed for an 18 cm height?

Solution: Let horizontal length be L. The required slope is 1/12=18/L. Cross-multiplying gives L=18×12=216 cm, or 2.16 m.

Exercise Set 13.4 — Forming equation pairs

Q1. Movie tickets cost ₹x and snack boxes cost ₹y. Write equations for purchases of two tickets and three boxes costing ₹850, and four tickets and one box costing ₹1100.

Solution: The first bill is $2x+3y=850$. The second is $4x+y=1100$. If prices are also required, substituting $y=1100-4x$ gives $2x+3300-12x=850$, so $x=245$, $y=120$.

Q2. Taxi fare has fixed charge ₹x and charge ₹y/km. A 6-km ride is ₹122, while an 8-km ride is ₹160. Form the equations.

Solution: Each ride costs the fixed charge plus distance times the kilometre charge. Thus $x+6y=122$ and $x+8y=160$. Subtract to get $2y=38$, $y=19$, then $x=8$.

Q3. There were 200 attendees at a sports meet. Adult tickets cost ₹150, children's tickets ₹100, and total receipts were ₹25,000. Let the numbers of adult and child tickets be x and y; write a pair of equations.

Solution: Headcount: $x+y=200$. Revenue: $150x+100y=25000$. Divide the second by 50 to obtain $3x+2y=500$. Subtract $2(x+y)=400$ to get $x=100$, and $y=100$.

Exercise Set 13.5 — Solving pairs

Q1. Solve the ten situations: (i) sum 5, difference −21; (ii) difference 26 and one number triple the other; (iii) seven bats and six balls cost ₹8880, three bats and five balls ₹4000; (iv) taxi 10 km ₹155, 15 km ₹220, then price for 25 km; (v) fraction becomes 9/11 after adding 2 to top and bottom, and 5/6 after adding 3; (vi) fraction becomes 1 after numerator +1 and denominator −1, and 1/2 after denominator +1 alone; (vii) Nuri is three times Sonu's age five years ago, twice Sonu's age ten years later; (viii) digit sum 9 and nine times the number equals twice its reversal; (ix) ₹2000 using 25 notes of ₹50 and ₹100; (x) library fixed fee for three days plus per-extra-day fee, ₹27 for 7 days and ₹21 for 5 days.

Solution: (i) $x+y=5$, $x-y=-21 \Rightarrow 2x=-16 \Rightarrow x=-8$, $y=13$.

(ii) $x-y=26$, $x=3y \Rightarrow y=13$, $x=39$.

(iii) $7x+6y=8880$ and $3x+5y=4000$. Multiplication by 5 and 6 then subtraction gives $17x=20400 \Rightarrow$ bat ₹1200; ball ₹80.

(iv) $f+10r=155$, $f+15r=220 \Rightarrow r=13$ and $f=25$; 25 km costs ₹350.

(v) $11(x+2)=9(y+2)$, $6(x+3)=5(y+3) \Rightarrow 11x-9y=-4$ and $6x-5y=-3$. Elimination gives $x=7, y=9 \Rightarrow 7/9$.

(vi) $(x+1)/(y-1)=1$ and $x/(y+1)=1/2 \Rightarrow x-y=-2$, $2x-y=1 \Rightarrow x=3, y=5 \Rightarrow 3/5$.

(vii) $N-5=3(S-5)$, $N+10=2(S+10) \Rightarrow N=50$, $S=20$.

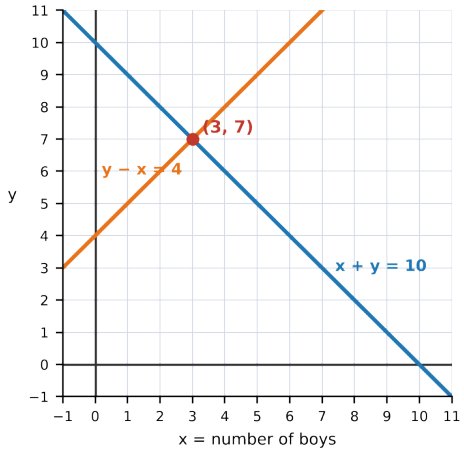
(viii) tens digit x, units digit y: $x+y=9$, $9(10x+y)=2(10y+x) \Rightarrow y=8x \Rightarrow x=1, y=8$; number 18.

(ix) $x+y=25$; $50x+100y=2000 \Rightarrow x=10$ notes of ₹50, $y=15$ notes of ₹100.

(x) $f+4r=27$, $f+2r=21 \Rightarrow r=3$, $f=15$.

Q2. Ten students entered a maths quiz. Girls outnumber boys by 4. Form two equations and find the counts graphically.

Solution: Let boys be x and girls y. $x+y=10$ and $y-x=4$. Their graphs intersect where adding gives $2y=14 \Rightarrow y=7$ and $x=3$. The two lines meet at (3,7): 3 boys and 7 girls.



Q3. Classify each pair of lines as intersecting, coincident or parallel: (i) 5x−4y+8=0 and 7x+6y−9=0; (ii) 9x+3y+12=0 and 18x+6y+24=0; (iii) 6x−3y+10=0 and 2x−y+9=0.

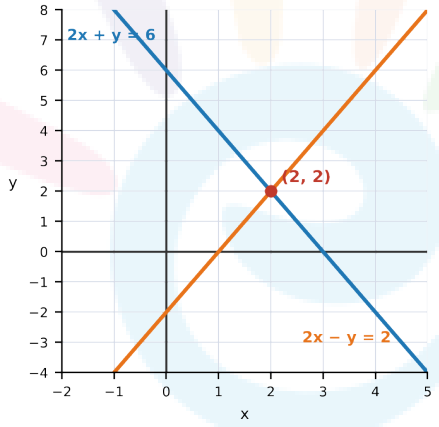
Solution: (i) 5/7 differs from −4/6; the slopes are different, so the lines intersect once.

(ii) 9/18=3/6=12/24=1/2; the equations are proportional, hence coincident.

(iii) 6/2=(−3)/(−1)=3 but 10/9≠3; same slope, different intercept: parallel.

Q4. Use the graphs to determine solutions for (i) x+y=5 and 2x+2y=10; (ii) x−y=8 and 3x−3y=16; (iii) 2x+y=6 and 4x−2y=4; (iv) 2x−2y=2 and 4x−4y=5.

Solution: (i) Identical equations, so infinitely many pairs, e.g. (0,5).



(ii) Rewriting the second as x−y=16/3 differs from x−y=8: parallel, no solution.

(iii) Simplify to 2x+y=6 and 2x−y=2. Add: 4x=8 ⇒ x=2, y=2: one intersection at (2,2).

(iv) x−y=1 and x−y=5/4 are parallel: no solution.

Q5. A rectangular garden has length 4 m more than width. Half its perimeter is 36 m. Find its length and width.

Solution: Let width=w and length=l. Given l+w=36 and l=w+4, we get 2w+4=36 ⇒ w=16.

Therefore l=20. Check: l+w=20+16=36 m.

Q6. Pair the equation 2x+3y−8=0 with another equation to illustrate (i) intersecting, (ii) parallel, (iii) coincident lines.

Solution: (i) x−y−1=0: its coefficients are not proportional, so the lines intersect.

(ii) 4x+6y−7=0: x and y coefficients are twice as large but the constant is not twice −8; the lines are distinct and parallel.

(iii) 4x+6y−16=0: exactly twice the original, so coincident.

Q7. A classical number puzzle gives 9 citrons and 7 wood-apples for 107, but 7 citrons and 9 wood-apples for 101. Determine each price without standard elimination, noting the swapped coefficients.

Solution: Write $9x+7y=107$ and $7x+9y=101$. Add: $16(x+y)=208 \Rightarrow x+y=13$. Subtract: $2(x-y)=6 \Rightarrow x-y=3$. Thus $x=8$ and $y=5$. Checking gives $72+35=107$ and $56+45=101$.

Q8. Five pencils and seven pens cost ₹50, while seven pencils and five pens cost ₹46. Find each unit price.

Solution: With pencil x and pen y: $5x+7y=50$ and $7x+5y=46$. Adding: $x+y=8$. Subtracting: $y-x=2$. Therefore pencil $x=₹3$ and pen $y=₹5$.

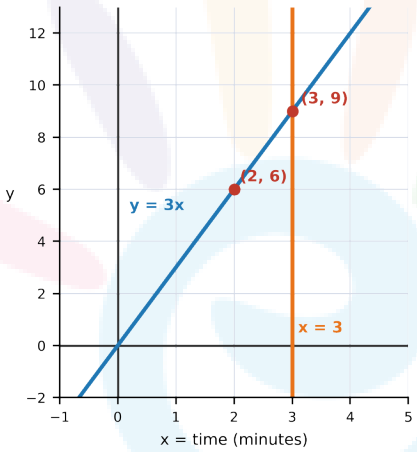
Q9. A cat on a stool makes the combined height 85 cm. With the cat beside the stool, the stool top stands 25 cm above the cat. Find the stool height.

Solution: If stool=s and cat=c, then $s+c=85$ and $s-c=25$. Add: $2s=110 \Rightarrow s=55$ cm. Hence $c=30$ cm. The stool is 55 cm high.

End-of-Chapter Exercises — All 16

Q1. For the line $y=3x$ through (0,0) and (2,6): (i) find slope; (ii) find the intercept; (iii) water height after five minutes and time for height 21 cm; (iv) test (4,10); (v) locate the intersection with $x=3$.

Solution: (i) Slope$=(6-0)/(2-0)=3$.



(ii) y-intercept=0: the graph begins at the origin.

(iii) At $x=5$, $y=3\times5=15$ cm; $y=21$ implies $x=7$ minutes.

(iv) At $x=4$ the line gives $y=12$, not 10; the point is outside the line.

(v) $x=3$ gives $y=9$, so the intersection is (3,9).

Q2. For $F=(9/5)C+32$, (i) graph the conversion; (ii) convert 30°C; (iii) convert 95°F; (iv) find Fahrenheit at 0°C and Celsius at 0°F; (v) find the temperature at which numerical values agree.

Solution: (i) Plot (C,F) as (0,32), (100,212), (−40,−40) and draw a straight line.

(ii) $F=(9/5)\times30+32=86$°F.

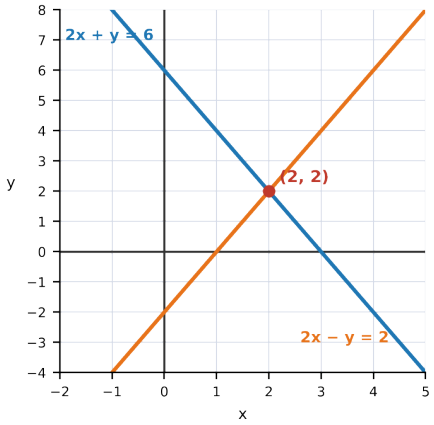
(iii) $95=(9/5)C+32 \Rightarrow 63=(9/5)C \Rightarrow C=35$°C.

(iv) 0°C corresponds to 32°F. For F=0, $C=-160/9\approx-17.8$°C.

(v) Set F=C: $C=(9/5)C+32 \Rightarrow -4C/5=32 \Rightarrow C=-40$; the common value is −40 degrees.

Q3. Find the common point of $2x+y=6$ and $2x-y-2=0$ using a graph.

Solution: Rewrite as $y=6-2x$ and $y=2x-2$. Draw the first using (0,6),(3,0), and the second using (0,−2),(1,0). They meet at (2,2). Check: $2(2)+2=6$ and $2(2)-2-2=0$.



Q4. Two lines on the chapter cover have equations 15x−10y−115=0 and 15x+100y=335. Find their intersection.

Solution: Dividing by 5: 3x−2y=23 and 3x+20y=67. Subtracting gives 22y=44 ⇒ y=2. Then 3x−4=23 ⇒ x=9. Thus the intersection is (9,2).

Q5. Give the x-intercept of y=mx+c and discuss the case m=0.

Solution: The x-axis has y=0. Hence 0=mx+c ⇒ x=−c/m when m≠0, giving (−c/m,0). If m=0 and c≠0, y=c is horizontal and never crosses the x-axis. If m=c=0, it coincides with the x-axis.

Q6. Compare a mobile plan charging ₹50 plus ₹0.20 per calling minute with one charging ₹30 plus ₹0.30. Which is cheaper, and at what call duration do they cost the same?

Solution: For t minutes A=50+0.20t and B=30+0.30t. Setting A=B gives 20=0.10t ⇒ t=200 minutes; each is ₹90 then. For t<200, B is cheaper; for t>200, A is cheaper.

Q7. How many lines (i) have a specified slope and (ii) have that slope and go through a specified point?

Solution: (i) Infinitely many, since y=mx+d has the same slope m for any intercept d.

(ii) Exactly one. If (x_0,y_0) lies on y=mx+d, then $d=y_0-mx_0$ is uniquely determined.

Q8. Find all values of p for which 4x+py+8=0 and 2x+2y+2=0 intersect at exactly one point.

Solution: A unique solution requires $a_1b_2-a_2b_1\neq0$. Here 4×2−2p≠0 ⇒ p≠4. If p=4, the coefficients of x and y are proportional but the constants are not, so the lines are parallel.

Q9. For what a,b will (a+b)x−2by=5a+2b+1 and 3x−y=14 describe the same line?

Solution: To be proportional, a+b=3k, −2b=−k and 5a+2b+1=14k. From k=2b and a+b=6b we get a=5b. The last equality gives 25b+2b+1=28b, so b=1 and a=5.

Q10. Find k if x+2y=3 and (k−1)x+(k+1)y=k+3 represent coincident lines.

Solution: Proportional coefficients require (k−1)/1=(k+1)/2=(k+3)/3. From 2(k−1)=k+1 obtain k=3. Then the second equation becomes 2x+4y=6, exactly twice the first.

Q11. Two robots follow repeated horizontal-and-vertical moves. (i) Robot 1 starts (0,0) and moves right 3/up 4; Robot 2 starts (10,0) and moves right 1/up 2. (ii) Robot 1 starts (3,0) and moves right 5/up 5; Robot 2 starts (7,0) and moves right 5/down 3. Determine whether their actual paths meet.

Solution: (i) The straight lines through the end-of-cycle positions are y=4x/3 and y=2(x−10).

Their intersection is (30,40), a genuine shared end-of-cycle point (10 moves of Robot 1, 20 moves of Robot 2). The stepwise zigzag paths may also touch at other points, so do not mistake the line intersection for the only path intersection.

(ii) The straight lines meet at (4.5,1.5), but Robot 2 only moves right from x=7 and below the x-axis. The actual paths overlap along the starting x-axis from (7,0) to (8,0); they have no later intersection.

Q12. A watch showed a minutes after 2 o'clock, and 15 minutes later b minutes after 3 o'clock; a=6b. Determine the second reading.

Solution: The interval is (60−a)+b=15, so a−b=45. Since a=6b, 5b=45 and b=9, a=54. The first time is 2:54 and the second is 3:09.

Q13. A two-digit integer has digit sum 15. Reversing the digits increases its value by 9. Find it.

Solution: Let tens digit x, units digit y. Then x+y=15 and (10y+x)−(10x+y)=9 ⇒ y−x=1.

Solving gives x=7,y=8; the number is 78. The reversal 87 exceeds it by 9.

Q14. A cyclic quadrilateral has angles (x+7)°, (y+8)°, (3y+23)°, and (4x+12)° in order. Determine the four angles.

Solution: Opposite angles in a cyclic quadrilateral are supplementary. Thus (x+7)+(3y+23)=180 ⇒ x+3y=150, and (y+8)+(4x+12)=180 ⇒ 4x+y=160. Solving gives x=30, y=40, so the angles are 37°, 48°, 143°, 132°.

Q15. A train takes 6 fewer hours when its speed rises by 6 km/h, and 6 extra hours when its speed falls by 4 km/h. Find its original speed and travel distance.

Solution: Let original speed v and time t. Equal distances imply (v+6)(t−6)=vt ⇒ t−v=6, and (v−4)(t+6)=vt ⇒ 3v−2t=12. Substituting t=v+6 gives v=24 km/h, t=30 h. Therefore distance=24×30=720 km.

Q16. A father's age equals the combined ages of four children. After 20 years the children's combined age will be twice the father's. Find his age.

Solution: Write father age F and sum of children's ages S. Initially S=F. After 20 years, the four children add 80 years in total, while the father adds 20 years. Thus S+80=2(F+20). With S=F, F+80=2F+40, giving F=40 years.