CHAPTER: 14 - Math of Space: Surface Area and Volume

Exercise Set 14.1 — Cuboids and Cubes

Q1. A cube occupies 64 cm³. Calculate the area of all six faces together.

Solution:

Let the edge of the cube be a cm. Volume = $a^3 = 64$ cm³.

Taking the cube root gives a = 4 cm.

Each face is a 4 cm by 4 cm square, so its area is $4^2 = 16$ cm².

A cube has six equal faces. Total surface area = 6 × 16 = 96 cm².

Answer: 96 cm².

Q2. Find how many cubes of edge 20 cm fit exactly inside a cubical box of edge 2 m.

Solution:

Convert the box edge into centimetres: 2 m = 200 cm.

Each small cube has edge 20 cm. Along one edge of the box, 200 ÷ 20 = 10 cubes fit.

There are 10 rows across the width and 10 layers vertically as well.

Total cubes = 10 × 10 × 10 = 1,000. Since 200 is exactly divisible by 20, no gaps remain.

Answer: 1,000 cubes.

Q3. A warehouse is 40 m long, 25 m wide and 10 m high. How many 2 m × 1.25 m × 1 m boxes can fill it?

Solution:

First compute the volume of the warehouse: 40 × 25 × 10 = 10,000 m³.

A single box has volume 2 × 1.25 × 1 = 2.5 m³.

The volume ratio suggests 10,000 ÷ 2.5 = 4,000 boxes.

Check that the boxes can actually be placed without wasted space: 40 ÷ 2 = 20, 25 ÷ 1.25 = 20 and 10 ÷ 1 = 10.

Thus 20 × 20 × 10 = 4,000 boxes fit exactly.

Answer: 4,000 boxes.

Q4. Two cubes, each having volume 125 cm³, are attached face-to-face. Determine the exposed surface area of the resulting cuboid.

Solution:

Write $a^3 = 125$ for the side a of each cube; thus a = 5 cm.

When two cubes join face-to-face, the new solid is a cuboid measuring 10 cm × 5 cm × 5 cm.

For a cuboid, total surface area = 2(lw + wh + hl).

Substituting: 2(10 × 5 + 5 × 5 + 10 × 5) = 2(50 + 25 + 50) = 250 cm².

An alternative check: two separate cubes have area 300 cm², but the two touching faces contribute 2 × 25 = 50 cm² that is no longer exposed.

Answer: 250 cm².

Q5. A 4 cm cube is cut into 1 cm cubes. Compare the outer surface area before cutting with the sum of surface areas after cutting. The total volume stays unchanged.

Solution:

Surface area of the original cube = 6 × 4^2 = 96 cm².

A side of length 4 cm gives 4 one-centimetre cubes along each edge, so $4^3 = 64$ small cubes are formed.

Each 1 cm cube has area 6 × 1^2 = 6 cm². Together their exposed surface area is 64 × 6 = 384 cm².

Required ratio (original : cut cubes) = 96 : 384 = 1 : 4.

Cutting exposes many new interior faces, although the total volume is unchanged.

Answer: 1 : 4.

Q6. Three mutually adjoining faces of a cuboid have areas 6 cm², 15 cm² and 10 cm². Obtain its volume.

Solution:

Let the three edge lengths be l, w and h.

The areas of the three adjoining faces are lw = 6, wh = 15 and hl = 10 (cm²).

Multiply the equations: (lw)(wh)(hl) = (lwh)² = 6 × 15 × 10 = 900.

Since volume is positive, lwh = √900 = 30 cm³.

For example, the corresponding edge lengths can be 2 cm, 3 cm and 5 cm.

Answer: 30 cm³.

Q7. ★ A cube of edge 5 cm is painted on every face and divided into unit cubes. Count the pieces having (i) exactly 3 painted faces, (ii) exactly 2, (iii) exactly 1, and (iv) none.

Solution:

There are 5 unit cubes along each edge, so total small cubes = 5^3 = 125.

(i) Three painted faces: only the 8 corner cubes touch three painted outer faces.

(ii) Two painted faces: each of the 12 edges has 5 − 2 = 3 cubes excluding its corners, giving 12 × 3 = 36.

(iii) One painted face: each of the 6 faces has a central (5 − 2) × (5 − 2) array, giving 6 × 3^2 = 54.

(iv) No painted face: the interior is a $(5 - 2)^3 = 3^3$ = 27 cube.

Check: 8 + 36 + 54 + 27 = 125, matching the total.

Answers: (i) 8; (ii) 36; (iii) 54; (iv) 27.

Q8. ★ Find every cuboid with positive integral edge lengths in centimetres whose total surface area is exactly 100 cm². Address (i) whether several exist, (ii) enumerate them, and (iii) justify completeness.

Solution:

Let integer edge lengths satisfy 1 ≤ a ≤ b ≤ c.

Total surface area 2(ab + bc + ca) = 100 gives ab + bc + ca = 50.

Because ab, bc and ca are each at least a^2, we have $3a^2 \leq 50$, so only a = 1, 2, 3 or 4 is possible.

Add a^2 and factor: $(a + b)(a + c) = 50 + a^2$.

Case a = 1: 51 = 3 × 17 gives b = 2 and c = 16.

Case a = 2: 54 = 6 × 9 gives b = 4 and c = 7.

Case a = 3: 59 is prime, so no admissible pair with b ≥ 3.

Case a = 4: 66 has no admissible factor pair with a + b ≥ 8 and a + b ≤ a + c.

Both proposed cuboids satisfy ab + bc + ca = 50, and all possible smallest edges have been checked.

Answers: (i) Yes; (ii) 1 × 2 × 16 and 2 × 4 × 7 cm; (iii) the four-case factor test proves completeness.

Exercise Set 14.2 — Cylinders

Q1. Cylinder B has twice the base radius but only half the height of cylinder A. Compare (a) their curved surface areas and (b) their volumes.

Solution:

Let cylinder A have radius r and height h.

Cylinder B then has radius 2r and height h/2.

Curved surface area of A = $2\pi rh$. For B it is $2\pi(2r)(h/2) = 2\pi rh$.

Therefore CSA(A) : CSA(B) = 1 : 1.

Volume(A) = $\pi r^2 h$, while Volume(B) = $\pi(2r)^2(h/2) = 2\pi r^2 h$.

Therefore Volume(A) : Volume(B) = 1 : 2.

Answers: curved area 1 : 1; volume 1 : 2.

Q2. Two cylinders have radii in ratio 2 : 3 and heights in ratio 3 : 2. Calculate the ratios of (a) their volumes and (b) their curved areas.

Solution:

Represent the two radii by 2k and 3k and the respective heights by 3m and 2m.

(a) Cylinder volume is $\pi r^2 h$, so the volume ratio is $(2^2 \times 3) : (3^2 \times 2) = 12 : 18 = 2 : 3$.

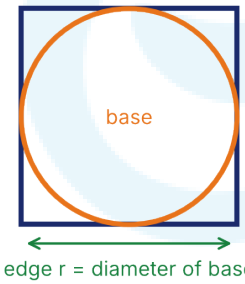
(b) Curved surface area is $2\pi rh$, so the area ratio is $(2 \times 3) : (3 \times 2) = 6 : 6 = 1 : 1$.

The constants π and 2 cancel in the ratios.

Answers: (a) 2 : 3; (b) 1 : 1.

Q3. A cube has edge r cm. Find the volume of the largest right circular cylinder that can be cut from it.

Solution:



A cylinder placed inside a cube must fit its circular base inside a square face.

For the largest possible circular base, the circle is inscribed in the square, so its diameter is r cm and its radius is r/2 cm.

The largest available cylinder height equals the cube edge r cm.

Volume = $\pi \times (r/2)^2 \times r = \pi r^3/4$ cm³.

This occupies about 78.5% of the cube volume r^3.

Answer: $\pi r^3/4$ cm³.

Q4. ★ A cylinder's radius increases by 10%, while its height decreases by x%. Determine x if its volume is unchanged.

Solution:

Let the original radius and height be r and h. Original volume V = $\pi r^2 h$.

After a 10% rise in radius, the new radius is 1.1r. After an x% fall in height, the new height is $h(1 - x/100)$.

Unchanged volume means $\pi(1.1r)^2h(1 - x/100) = \pi r^2h$.

Cancel πr^2h to get $1.21(1 - x/100) = 1$.

Thus $1 - x/100 = 1/1.21 = 100/121$, so $x/100 = 21/121$.

Therefore $x = 2100/121 \approx 17.36\%$.

Answer: the height must decrease by approximately 17.36%.

Q5. A metallic cube of edge 12 cm is recast as cylindrical rods of radius 2 cm and height 12 cm. Find (i) cube volume, (ii) one rod volume, (iii) the number of complete rods. Use $\pi \approx 22/7$.

Solution:

(i) Volume of the metal cube = $12^3 = 1,728$ cm³.

(ii) One cylindrical rod has radius 2 cm and height 12 cm.

Volume of one rod = $\pi r^2h = (22/7) \times 2^2 \times 12 = 1056/7 \approx 150.86$ cm³.

(iii) Number of such rods by volume = $1,728 \div (1056/7) \approx 11.45$.

Only complete rods count, so round down rather than to the nearest whole number.

Answers: (i) 1,728 cm³; (ii) about 150.86 cm³; (iii) 11 complete rods.

Exercise Set 14.3 — Cones

Q1. Find the total area of a cone having slant height 21 m and base diameter 24 m.

Solution:

Diameter of base = 24 m, hence radius r = 12 m. The slant height l = 21 m.

Total surface area of a cone = curved area + circular base = $\pi rl + \pi r^2 = \pi r(l + r)$.

Substitute: TSA = $\pi \times 12 \times (21 + 12) = 396\pi$ m².

Using $\pi = 22/7$ gives $396 \times 22/7 \approx 1,244.57$ m². Using 3.14 would give 1,243.44 m².

Answer: approximately 1,244.57 m² (when $\pi = 22/7$).

Q2. A right cone has base radius 7 cm and slant height 10 cm. Determine its curved surface area.

Solution:

For a right circular cone, curved surface area = $\pi \times$ base radius $\times$ slant height.

Here r = 7 cm and l = 10 cm.

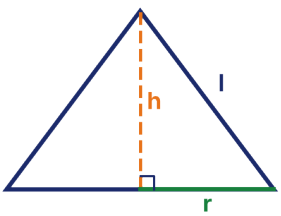
CSA = $(22/7) \times 7 \times 10 = 220$ cm².

Only the curved surface is required; the circular base is not added.

Answer: 220 cm².

Q3. The height and base radius of a cone are 16 cm and 12 cm. Find both its curved area and total area.

Solution:



$l^2 = h^2 + r^2$, CSA = πrl

Given vertical height h = 16 cm and radius r = 12 cm.

The slant height l forms a right triangle with h and r, so $l^2 = h^2 + r^2$.

Thus $l = \sqrt{(16^2 + 12^2)} = \sqrt{400} = 20$ cm.

Curved surface area = $\pi rl = \pi \times 12 \times 20 = 240\pi \approx 753.6$ cm².

Total surface area = $\pi rl + \pi r^2 = \pi r(l + r) = \pi \times 12 \times 32 = 384\pi \approx 1{,}205.76$ cm².

Answers ($\pi = 3.14$): CSA = 753.6 cm²; TSA = 1,205.76 cm².

Q4. A cone is 15 cm high and holds 1570 cm³. Calculate its base radius, using π = 3.14.

Solution:

Volume of a cone = $(1/3)\pi r^2 h$.

The given height is 15 cm and volume is 1,570 cm³.

Substituting $\pi = 3.14$: $1{,}570 = (1/3) \times 3.14 \times r^2 \times 15 = 15.7r^2$.

Therefore $r^2 = 1{,}570/15.7 = 100$.

Radius is positive, so $r = \sqrt{100} = 10$ cm.

Answer: 10 cm.

Q5. A cone has curved area 308 cm² and slant height 14 cm. Find (i) base radius and (ii) total area.

Solution:

Given curved surface area = 308 cm² and slant height l = 14 cm.

(i) Use CSA = πrl: $308 = (22/7) \times r \times 14 = 44r$.

Therefore $r = 308/44 = 7$ cm.

(ii) Circular base area = $\pi r^2 = (22/7) \times 7^2 = 154$ cm².

Total surface area = curved area + base area = 308 + 154 = 462 cm².

Answers: (i) 7 cm; (ii) 462 cm².

Q6. Ten cone-shaped joker caps are to be made from paper. Each has radius 7 cm and vertical height 24 cm. What sheet area is required?

Solution:

A cap is an open cone, so the sheet covers its curved surface but not a circular base.

Radius r = 7 cm; vertical height h = 24 cm.

Slant height $l = \sqrt{(r^2 + h^2)} = \sqrt{(49 + 576)} = 25$ cm.

Paper for one cap = $\pi rl = (22/7) \times 7 \times 25 = 550$ cm².

For ten caps: $10 \times 550 = 5{,}500$ cm². This excludes extra manufacturing waste.

Answer: 5,500 cm².

Q7. ★ How long a 3 m wide sheet of tarpaulin is required for a conical tent of radius 6 m and height 8 m? Allow another 20 cm of length for stitching and waste.

Solution:

The cone has vertical height 8 m and base radius 6 m.

Slant height $l = \sqrt{(8^2 + 6^2)} = \sqrt{100} = 10$ m.

Only the curved surface is covered, so required tarpaulin area = $\pi rl = 3.14 \times 6 \times 10 = 188.4$ m².

A 3 m wide sheet must therefore be $188.4/3 = 62.8$ m long.

Allow 20 cm = 0.2 m extra for seams and wastage: 62.8 + 0.2 = 63 m.

Answer: 63 m of 3 m wide tarpaulin.

Q8. A right triangle has side lengths 6, 8 and 10 cm. It is rotated completely around its 8 cm side. Compute the resulting solid's volume and curved area.

Solution:

The right triangle has perpendicular sides 6 cm and 8 cm, and hypotenuse 10 cm.

Rotating around its 8 cm side forms a cone whose height is 8 cm, radius 6 cm and slant height 10 cm.

Cone volume = $(1/3)\pi r^2 h = (1/3)\pi \times 36 \times 8 = 96\pi \approx 301.44$ cm³.

Cone curved surface area = $\pi rl = \pi \times 6 \times 10 = 60\pi \approx 188.4$ cm².

Answers ($\pi = 3.14$): volume ≈ 301.44 cm³ and CSA ≈ 188.4 cm².

Q9. ★ A conical cup is filled with water to half its total depth. What portion of the cup volume is occupied?

Solution:

Let the complete cup have height H and opening radius R, with its pointed end at the bottom.

At half the depth, the water forms a smaller similar cone whose height is H/2.

By similarity of the triangular cross-sections (or the midpoint theorem), its radius is R/2.

Full cup capacity = $(1/3)\pi R^2 H$.

Water volume = $(1/3)\pi(R/2)^2(H/2) = (1/8) \times (1/3)\pi R^2 H$.

Therefore the water occupies one-eighth of the total volume, even though it reaches half the height.

Answer: 1/8 of the cup.

Exercise Set 14.4 — Spheres and Hemispheres

Q1. Find the area of the spherical surface of a ball bearing with radius 0.7 cm.

Solution:

Surface area of a sphere is $4\pi r^2$.

Given r = 0.7 cm, substitute $\pi = 22/7$.

Area = $4 \times (22/7) \times (0.7)^2 = 4 \times (22/7) \times 0.49 = 6.16$ cm².

Answer: 6.16 cm².

Q2. ★ Two homogeneous metal balls weigh 5920 g and 740 g. The smaller has diameter 5 cm. Determine the radius of the heavier ball.

Solution:

Both spheres are made from the same metal, so their density is the same.

Mass is therefore proportional to volume; for a sphere $V = (4/3)\pi r^3$, so mass is proportional to r^3.

The masses are 5920 g and 740 g, giving ratio 5920 : 740 = 8 : 1.

Hence $(R/r)^3 = 8$, which gives $R/r = 2$.

The smaller sphere has diameter 5 cm, so r = 2.5 cm.

Thus the larger sphere radius R = 2 × 2.5 = 5 cm.

Answer: 5 cm.

Q3. The Moon's diameter is approximately one quarter of Earth's. Assuming both are spheres, compare their surface areas (Moon : Earth).

Solution:

Let the diameter of the Moon be d and that of Earth be 4d.

Their radii are also in the same ratio, 1 : 4.

Sphere surface area = $4\pi r^2$, so the area ratio is the square of the radius ratio.

Moon : Earth surface areas = $1^2 : 4^2 = 1 : 16$.

Answer: 1 : 16.

Q4. For a hemisphere of radius 21 cm, find (i) curved surface area and (ii) total surface area including its circular base.

Solution:

A hemisphere has curved area $2\pi r^2$ and a circular base of area πr^2.

Given radius r = 21 cm.

(i) Curved area = $2 \times (22/7) \times 21^2 = 2 \times (22/7) \times 441 = 2{,}772$ cm².

(ii) Total area includes its base: $3\pi r^2 = 3 \times (22/7) \times 441 = 4{,}158$ cm².

Answers: (i) 2,772 cm²; (ii) 4,158 cm².

Q5. A 16 cm × 8 cm × 8 cm box contains sixteen metal balls of radius 2 cm, and remaining space is filled with liquid. Find liquid volume to nearest whole cm³.

Solution:

First calculate the capacity of the rectangular box: 16 × 8 × 8 = 1,024 cm³.

Each sphere has radius 2 cm, so its volume is $(4/3)\pi(2^3) = 32\pi/3$ cm³.

For sixteen spheres, total volume = $16 \times 32\pi/3 = 512\pi/3 \approx 536.17$ cm³.

The liquid fills the remaining space. Liquid volume = 1,024 − 536.17 ≈ 487.83 cm³.

Round to the nearest integer: 488 cm³.

Answer: approximately 488 cm³.

Q6. Explain quantitatively why a 10% increase in sphere radius increases its volume by 33.1%.

Solution:

Let the original sphere volume be $V = (4/3)\pi r^3$.

Increasing the radius by 10% makes it 1.1r.

New volume $V_{new} = (4/3)\pi(1.1r)^3 = (1.1)^3 V = 1.331V$.

Increase = $V_{new} - V = 0.331V$.

Percentage increase = $0.331 \times 100 = 33.1\%$.

Answer: the volume rises by 33.1%.

Q7. ★ A sphere's radius is raised by x%, increasing its volume by 72.8%. Determine x.

Solution:

Let the original radius be r. After an x% increase the radius is $r(1 + x/100)$.

Sphere volume changes in proportion to the cube of the radius.

A 72.8% volume increase means $V_{new}/V_{old} = 1 + 0.728 = 1.728$.

Therefore $(1 + x/100)^3 = 1.728 = (1.2)^3$.

Taking cube roots: $1 + x/100 = 1.2$, so x = 20.

Answer: the radius increases by 20%.

Q8. A hemispherical dome has base circumference 35.2 m. Painting costs ₹10 per 100 cm². Work out the price to paint only the curved outside of the dome.

Solution:

The dome is a hemisphere, so paint covers only its curved outer surface, not the circular base.

The circumference of its base is 35.2 m = $2\pi r$.

Using $\pi = 22/7$: $r = 35.2 \div (44/7) = 5.6$ m.

Curved area = $2\pi r^2 = 2 \times (22/7) \times 5.6^2 = 197.12$ m².

Convert to square centimetres: $197.12 \times 10{,}000 = 1{,}971{,}200$ cm².

At ₹10 per 100 cm², cost = (1,971,200/100) × 10 = ₹197,120.

Answer: ₹1,97,120.

End-of-Chapter Exercises

Q1. Estimate the number of ice-cream scoops you can obtain from a 10 cm × 15 cm × 20 cm cuboidal tub. Make and state a reasonable assumption about one scoop.

Solution:

The container has volume = 10 × 15 × 20 = 3,000 cm³.

A guesstimate needs an explicit scoop size. Suppose one scoop is roughly a sphere of radius 2.5 cm.

Its volume is (4/3)π(2.5)³ ≈ (4/3) × 3.14 × 15.625 ≈ 65.4 cm³.

Theoretical number = 3,000/65.4 ≈ 45.9 scoops.

Scooping leaves some ice cream behind, so 40–45 complete scoops is a practical estimate.

Answer: approximately 45 scoops; other justified scoop sizes are acceptable.

Q2. A cube of edge a is built from unit cubes. How many additional unit cubes are required to extend each edge to a + 1? Express your answer algebraically.

Solution:

The original cube contains a³ unit cubes because it has side length a.

After increasing the side length by one unit, the enlarged cube contains (a + 1)³ unit cubes.

Additional unit cubes = (a + 1)³ − a³.

Expand: (a + 1)³ = a³ + 3a² + 3a + 1.

Subtracting a³ leaves 3a² + 3a + 1.

This can also be seen as three face slabs (3a² cubes), three edge rods (3a cubes) and one corner cube.

Answer: 3a² + 3a + 1 unit cubes.

Q3. ★ Estimate (i) whether one person could drink the volume of a classroom in a lifetime and (ii) how many bricks make up classroom walls. State assumptions for both.

Solution:

(i) Make a reasonable classroom estimate: length 9 m, width 7 m and height 3.5 m.

Room capacity ≈ 9 × 7 × 3.5 = 220.5 m³ = 220,500 litres.

Suppose a person drinks 2 litres daily for 75 years: 2 × 365 × 75 = 54,750 litres.

Since 54,750 is far smaller than 220,500, the person would not fill such a classroom.

(ii) Using the same classroom, wall perimeter = 2(9 + 7) = 32 m.

For 3.5 m high walls of thickness 0.23 m, wall volume = 32 × 3.5 × 0.23 = 25.76 m³.

Subtract about 15% for doors and windows: 25.76 × 0.85 ≈ 21.90 m³.

If a brick with mortar takes 0.002 m³, count ≈ 21.90/0.002 = 10,950, or roughly 11,000 bricks.

Answers: (i) No, on these assumptions; (ii) around 11,000 bricks. Both are estimates.

Q4. Choose the option offering more chocolate: one 50 mm cube or one hundred separate 10 mm cubes. Compare the volumes.

Solution:

Option A is one cube with side 50 mm.

Volume A = 50³ = 125,000 mm³.

Option B has 100 cubes of side 10 mm.

Volume of one small cube = 10³ = 1,000 mm³; all 100 together = 100 × 1,000 = 100,000 mm³.

Difference = 125,000 − 100,000 = 25,000 mm³.

Answer: Option A provides more chocolate.

Q5. Give an order-of-magnitude estimate for the ground area occupied if all humans gathered in one densely packed crowd.

Solution:

An area estimate needs assumptions about the population and how tightly people stand.

Use a world population of approximately 8.2 billion people for this example.

Suppose each person requires 0.5 m × 0.5 m = 0.25 m^2 in a tightly packed crowd.

Total area ≈ $8.2 \times 10^9 \times 0.25 = 2.05 \times 10^9$ m^2.

Because 1 $km^2 = 10^6$ m^2, that is about 2,050 km^2.

A square of this area would have side $\sqrt{2050} \approx 45.3$ km.

Answer: roughly 2,000 km^2, depending on crowd density and population assumptions.

Q6. For 1600 students, milk is poured into cylindrical glasses of diameter 7 cm to depth 12 cm. Calculate the total volume required in litres (1000 cm^3 = 1 L).

Solution:

One cylindrical serving has diameter 7 cm, so radius r = 7/2 = 3.5 cm.

The milk height is h = 12 cm.

Volume per glass = $\pi r^2 h = (22/7) \times 3.5^2 \times 12 = 462$ cm^3.

For 1,600 students, total = 462 × 1,600 = 739,200 cm^3.

Convert using 1,000 cm^3 = 1 litre: 739,200/1,000 = 739.2 litres.

Answer: 739.2 litres of milk.

Q7. The surface of a sphere with radius 5 cm has five times the curved area of a cone whose radius is 4 cm. Find the cone height and volume.

Solution:

Surface area of the sphere of radius 5 cm = $4\pi(5^2) = 100\pi$ cm^2.

The sphere area is five times the cone curved area, hence $100\pi = 5 \times \pi \times 4 \times l$.

Therefore $100\pi = 20\pi l$ and slant height l = 5 cm.

For a right cone, $l^2 = h^2 + r^2$. With radius r = 4 cm, $h = \sqrt{(5^2 - 4^2)} = \sqrt{9} = 3$ cm.

Cone volume = $(1/3)\pi r^2 h = (1/3)\pi \times 16 \times 3 = 16\pi$ cm^3.

Using π = 22/7, volume ≈ 50.29 cm^3.

Answers: height 3 cm; volume $16\pi \approx 50.29$ cm^3.

Q8. Treat Earth, Jupiter and the Sun as spheres of radii 6370 km, 69,900 km and 695,700 km. Estimate Jupiter-to-Earth and Sun-to-Earth volume ratios.

Solution:

A sphere has volume $(4/3)\pi r^3$, so the factor $(4/3)\pi$ cancels when two volumes are compared.

(i) Jupiter-to-Earth ratio = $(69,900/6,370)^3$.

The radius ratio is about 10.97, so the volume ratio is approximately $10.97^3 \approx 1,321 : 1$.

(ii) Sun-to-Earth ratio = $(695,700/6,370)^3$.

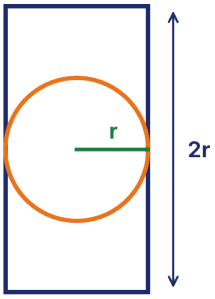
The radius ratio is around 109.2, so the volume ratio is about $1.30 \times 10^6 : 1$.

These are estimates because the heavenly bodies are not perfect spheres and input radii are approximate.

Answers: Jupiter : Earth ≈ 1,321 : 1; Sun : Earth ≈ 1,300,000 : 1.

Q9. Prove a sphere occupies two-thirds the volume of the smallest enclosing circular cylinder.

Solution:



Consider a sphere of radius r.

The smallest cylinder around it has the same radius r and height equal to the sphere diameter 2r.

Cylinder volume = $\pi r^2 \times 2r = 2\pi r^3$.

Sphere volume = $(4/3)\pi r^3$.

Divide: sphere volume/cylinder volume = $[(4/3)\pi r^3]/[2\pi r^3] = 4/6 = 2/3$.

Thus the sphere occupies two-thirds of the enclosing cylinder volume, as required.

Q10. ★ A second circular string is 1 m longer than one touching the equator of a sphere, yet stays at constant clearance everywhere. Find its clearance, and compare Earth, Moon, Jupiter and a volleyball.

Solution:

Let the original spherical body have radius R and the uniform gap under the longer string be h.

The tight string has length $2\pi R$. The raised string has length $2\pi(R + h)$.

It is exactly 1 m longer: $2\pi(R + h) - 2\pi R = 1$.

Subtract to get $2\pi h = 1$, hence $h = 1/(2\pi)$ m ≈ 0.159 m ≈ 15.9 cm.

R cancels completely, so the same clearance is obtained for Earth, Moon, Jupiter and a volleyball.

The result seems surprising because these bodies have very different circumferences; a 1 m change in circumference always changes radius by $1/(2\pi)$ m.

Answer: about 16 cm for each of the four objects.

Q11. ★ A square-based pyramid sits inside a cylinder of radius r and height h. Its square base is inscribed in the circular base and its apex touches the top plane. Compare pyramid volume to cylinder volume.

Solution:

The cylinder has base radius r, height h and volume $\pi r^2 h$.

The square base of the pyramid is inscribed in the circular cylinder base, so the square diagonal equals the circle diameter 2r.

Square area = $\text{diagonal}^2/2 = (2r)^2/2 = 2r^2$.

The pyramid height is h because its apex reaches the cylinder top.

Pyramid volume = $(1/3) \times 2r^2 \times h = (2/3)r^2h$.

Ratio (pyramid : cylinder) = $[(2/3)r^2h]/[\pi r^2 h] = 2/(3\pi)$.

Answer: $2 : 3\pi$ (approximately $0.212 : 1$).

Q12. Determine the volume change when (i) a cuboid length l increases by 1 with w,h fixed; (ii) cylinder radius r increases by 1 with h fixed; (iii) ★ sphere radius r decreases by 1. Include the given multiple-choice values for (i) and (ii): (i) 1, lwh+1, wh, lw, lh; (ii) 1, $\pi r^2 h$, πr^2, $2\pi rh+2\pi h$, $2\pi rh+\pi h$.

Solution:

(i) Original cuboid volume = lwh. New length is l + 1, so new volume = (l + 1)wh.

Increase = (l + 1)wh − lwh = wh cubic units: option (c).

(ii) Original cylinder volume = $\pi r^2 h$; new radius is r + 1, giving $\pi(r + 1)^2 h$.

Increase = $\pi h[(r + 1)^2 - r^2] = \pi h(2r + 1) = 2\pi rh + \pi h$: option (e).

(iii) For r ≥ 1, original sphere volume = $(4/3)\pi r^3$ and new volume = $(4/3)\pi(r - 1)^3$.

The decrease is $(4/3)\pi[r^3 - (r - 1)^3] = (4/3)\pi(3r^2 - 3r + 1)$.

Answers: (i) wh; (ii) $2\pi rh + \pi h$; (iii) decrease of $(4/3)\pi(3r^2 - 3r + 1)$ cubic units.

Q13. ★ Express the total surface area S of a cube in terms of its given volume V.

Solution:

Let the cube side be a. Its volume is $V = a^3$.

Taking cube roots yields a = V^(1/3).

Total surface area $S = 6a^2$.

Substitute the expression for a: $S = 6[V^{(1/3)}]^2 = 6V^{(2/3)}$.

Answer: S = 6V^(2/3).

Q14. ★ Express a cube's volume V using its given total surface area S.

Solution:

Let the cube side be a. Total surface area $S = 6a^2$.

Rearrange: $a^2 = S/6$. Since side length is positive, $a = \sqrt{(S/6)}$.

Volume $V = a^3 = [\sqrt{(S/6)}]^3$.

This can also be written as V = (S/6)^(3/2).

Answer: V = (S/6)^(3/2).

Q15. A cylindrical glass of radius 4 cm contains water to 16 cm, but a crow needs 20 cm. How many spherical marbles of radius 1 cm must be submerged to raise the water level?

Solution:

The glass radius is 4 cm. The water has to rise from 16 cm to 20 cm, a height increase of 4 cm.

The extra volume required below that level is a cylinder of height 4 cm: $\Delta V = \pi \times 4^2 \times 4 = 64\pi$ cm³.

Each submerged marble is a sphere of radius 1 cm and displaces $(4/3)\pi(1^3) = 4\pi/3$ cm³.

Number of marbles = $64\pi \div (4\pi/3) = 64 \times 3/4 = 48$.

This assumes all marbles can be placed inside the glass and are completely submerged.

Answer: 48 marbles.

Q16. Estimate how much ink a fresh ballpoint-pen refill contains. Choose plausible measurements and state them.

Solution:

The ink in a ballpoint pen lies in a narrow tube, which can be approximated by a cylinder.

Measure the inner radius and length of the ink column for an actual pen.

For an example, suppose the inner diameter is 2 mm = 0.2 cm, so radius is 0.1 cm, and the ink length is 10 cm.

Volume $= \pi r^2 h \approx 3.14 \times 0.1^2 \times 10 = 0.314$ cm³.

Since 1 cm^3 = 1 mL, this is roughly 0.31 mL.

Answer: about 0.3 mL for the assumed measurements; actual refill sizes vary.

Q17. Estimate (i) the number of chapatis obtainable from dough shaped as a ball of radius 6 cm; (ii) ★ the volume of edible flesh in a coconut or muskmelon after measuring its cross-section.

Solution:

(i) Treat the dough ball as a sphere of radius 6 cm.

Dough volume = $(4/3)\pi(6^3) = 288\pi \approx 904$ cm^3.

Assume a typical chapati is a thin cylinder of radius 8 cm and thickness 0.2 cm.

Volume per chapati $\approx \pi \times 8^2 \times 0.2 = 12.8\pi \approx 40.2$ cm^3.

Estimated number = 904/40.2 ≈ 22 chapatis.

(ii) For a coconut, model edible flesh as a spherical shell. Suppose its outer radius is 5.5 cm and cavity radius is 4.5 cm.

Flesh volume $\approx (4/3)\pi(5.5^3 - 4.5^3) = (4/3)\pi \times 75.25 \approx 315$ cm^3.

For a muskmelon, subtract the estimated seed-cavity volume and any rind from the whole sphere volume.

Answers are measurement-dependent: about 22 chapatis and about 315 cm^3 of coconut flesh for these assumptions.

Q18. Sheela asks (i) if sheets of the Grade 9 Part II book could cover a classroom floor when spread out and (ii) the greatest possible number of books fitting a 15 ft × 20 ft × 30 ft storeroom. Estimate using measured book dimensions.

Solution:

Assume the book measures about 16 cm × 23.5 cm, contains around 80 physical sheets, and is 1 cm thick.

(i) Area of one sheet ≈ 16 × 23.5 = 376 cm^2.

Area of all 80 sheets laid out flat = 80 × 376 = 30,080 $cm^2 \approx 3.01$ m^2.

A sample classroom floor of 60 m^2 is far larger, so the sheets would not cover it.

(ii) Storeroom volume = 15 × 20 × 30 = 9,000 ft^3.

Using approximately 28,317 cm^3 per ft^3, volume ≈ 254.85 million cm^3.

Volume of one book ≈ 16 × 23.5 × 1 = 376 cm^3.

The volume-based upper bound ≈ 254,850,000/376 ≈ 678,000 books; actual packing is lower due to gaps and shelf arrangement.

Answers: (i) No; (ii) around 6.8 lakh books as an ideal upper bound.

Q19. ★ Suppose everyone on Earth entered one large cube. Estimate the side length needed, stating how much space each person occupies.

Solution:

This is an estimate, so specify the volume allowed per person.

Use an example population of about 8.2 billion and an allocated standing volume of 0.5 m^3 per person.

Combined volume $\approx 8.2 \times 10^9 \times 0.5 = 4.1 \times 10^9$ m^3.

For a cube of side a, $a^3 = 4.1 \times 10^9$ m^3.

Taking cube roots: a ≈ 1,600 m = 1.6 km.

If people are packed more tightly, the required side is smaller. State the assumed packing method.

Answer: about 1.6 km per side on the stated assumption.

Q20. ★ Imagine Earth's 1.38 billion km³ of water is a uniform spherical shell over an Earth of radius about 6371 km. (i) Derive the shell-thickness equation. (ii) Evaluate it numerically.

Solution:

Let Earth radius R = 6,371 km and total water volume V = 1.38×10^9 km³.

(i) If a uniform layer has thickness t, its outer radius is R + t.

Water occupies the difference between two spherical volumes: $V = (4/3)\pi[(R + t)^3 - R^3]$.

Rearrange: $(R + t)^3 = R^3 + 3V/(4\pi)$.

Therefore $t = \sqrt[3]{[R^3 + 3V/(4\pi)]} - R$.

(ii) Because t is much smaller than R, shell volume is also approximately surface area × thickness: $V \approx 4\pi R^2 t$.

So $t \approx V/(4\pi R^2) \approx (1.38 \times 10^9)/(4\pi \times 6{,}371^2) \approx 2.7$ km.

The exact cube-root formula agrees closely.

Answer: thickness about 2.7 km.

Q21. ★ Find two cuboids so that the second has half the volume but twice the outer surface area of the first. Give actual dimensions.

Solution:

We need two cuboids: the second must have half the first volume and twice the first total surface area.

Start with cuboid A of dimensions 2 cm × 3 cm × 4 cm.

Volume(A) = 2 × 3 × 4 = 24 cm³.

Area(A) = 2(2×3 + 3×4 + 2×4) = 2(6 + 12 + 8) = 52 cm².

Choose cuboid B of dimensions 4 cm × 12 cm × 0.25 cm.

Volume(B) = 4 × 12 × 0.25 = 12 cm³ = 24/2.

Area(B) = 2(4×12 + 12×0.25 + 4×0.25) = 2(48 + 3 + 1) = 104 cm² = 2 × 52.

This example meets both requirements. Flattening can increase exposed area while decreasing volume.

Answer: 2 × 3 × 4 cm and 4 × 12 × 0.25 cm.

Q22. Project: Estimate the volume of your own house using measurements or sensible approximations; show the method clearly.

Solution:

This is a project and there is no unique numerical answer; record your actual measurements.

Step 1: Divide the house into simple shapes such as rectangular rooms. Treat each room approximately as a cuboid.

Step 2: Measure length, breadth and ceiling height for every room. For a sloping roof, use a triangular prism or an appropriate simple shape.

Step 3: Calculate each room volume as length × breadth × height, then add the volumes without double-counting overlapping spaces.

Worked example: Two bedrooms each 4 × 3.5 m, a hall 5 × 4 m, a kitchen 3 × 2.5 m and a bathroom 2 × 1.5 m.

Total floor area = 2(14) + 20 + 7.5 + 3 = 58.5 m².

With common ceiling height 3 m, total enclosed interior volume ≈ 58.5 × 3 = 175.5 m³.

Step 4: State whether walls, balconies, staircases and the roof are included and explain all approximations.

Illustrative answer: about 175.5 m³; calculate your own home from its dimensions.

Think and Reflect — Detailed Notes

Why stacked layers give the cuboid formula

Solution:

Think of many identical thin sheets of length l, breadth w and thickness t.

Every sheet has volume lwt. If n sheets make up the solid, total height h = nt.

The full volume is n × lwt = lw(nt) = lwh. Thus it equals base area multiplied by height.

For a cube of side a, volume = a^3 and total surface area = $6a^2$.

Choosing a school swimming pool

Solution:

Option A: 8 × 50 × 20 = 8,000 ft^3. At 28.3 L/ft^3 it holds about 226,400 litres.

Option B: 6 × 100 × 15 = 9,000 ft^3, approximately 254,700 litres.

Option B is shallower (which may improve safety) and longer for practice, though it uses 28,300 more litres of water.

Compare campus space, water cost, tiles required, intended users and adult supervision before choosing.

Relating sphere volume to surface area

Solution:

Imagine the sphere partitioned into very many tiny surface patches.

Connect each patch to the centre. The resulting thin pyramids approximately have height r.

Add their volumes: (1/3) × r × (total patch area) = (r/3) × $4\pi r^2$.

Therefore sphere volume = $(4/3)\pi r^3$.

Why offset layers hold more spherical balls

Solution:

A square grid of balls wastes space because each ball has gaps beside and above it.

When the next layer rests in hollows of the preceding layer, the vertical distance between rows can be reduced.

For 6 cm diameter balls in a roughly 900 × 900 × 450 cm space, a simple rectangular count is 150 × 150 × 75 = 1,687,500.

Densely arranged offset layers can yield roughly 2.3–2.4 million by idealised packing estimates; walls and edges reduce the practical result.

The estimates depend on the arrangement and boundary losses.